Problem 1

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(a) The Kaon is the lowest mass particle, which contains an s-quark. Since only the weak interactions don't conserve flavor, the Kaon can only decay via weak interactions.

(b) $P|K^0\rangle = -|K^0\rangle$, $P|\bar{K}^0\rangle = -|\bar{K}^0\rangle$

$C|K^0\rangle = |\bar{K}^0\rangle$, $C|\bar{K}^0\rangle = |K^0\rangle$

$CP|K^0\rangle = -|\bar{K}^0\rangle$, $CP|\bar{K}^0\rangle = -|K^0\rangle$

(c) $|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$ with $CP|K_1\rangle = |K_1\rangle$ and

$|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$ $CP|K_2\rangle = -|K_2\rangle$.

(d) The 2-pion states are parity-even, while the 3-pion states are parity-odd (in the ground state).

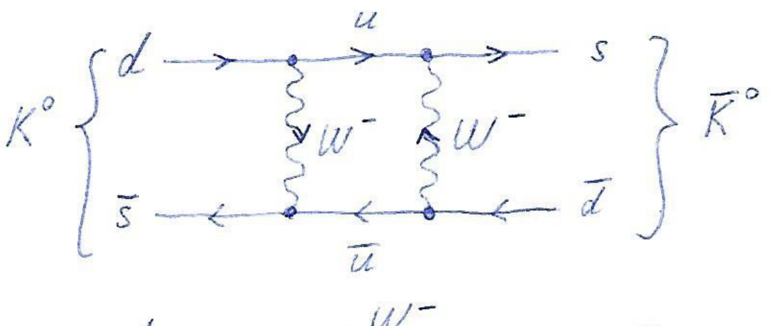
$\Rightarrow CP|\pi\pi\rangle = |\pi\pi\rangle$, $CP|\pi\pi\pi\rangle = -|\pi\pi\pi\rangle$

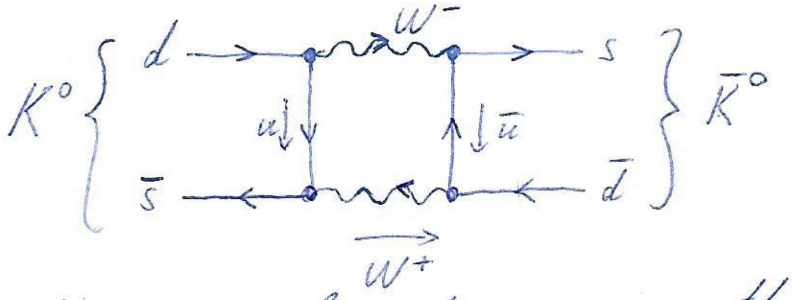
$\hookrightarrow K_1 \to 2\pi$, $K_2 \to 3\pi$

So if we observe $K^0 \to 2\pi$, then actually only the component $|K_1\rangle$ of $|K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_1\rangle + |K_2\rangle\right)$ contributes to the decay.

(e) Now the component $|K_2\rangle$ contributes.

(f)





$\hookrightarrow$ There are also diagrams with c, t instead of u, which are more suppressed due to smaller CKM-factors.

(g) • The diagonal elements of $H_{weak} = \begin{pmatrix} m-\frac{i}{2}\Gamma & -p^2 \\ -q^2 & m-\frac{i}{2}\Gamma \end{pmatrix}$ describe the quantum mechanical (independent) time-evolution of $|K^0\rangle$ and $|\bar{K}^0\rangle$, with the imaginary piece $\sim i\Gamma$ due to decay.

• The off-diagonal elements $\langle K^0|H_{weak}|\bar{K}^0\rangle = -p^2$, $\langle \bar{K}^0|H_{weak}|K^0\rangle = -q^2$ are generated by the above loop diagrams
$\hookrightarrow$ p and q encode the (non-symmetric) dependence on the CKM-elements.

• The eigenvalues of H_{weak} are $\lambda_{\mp} = m - \frac{i}{2}\Gamma \mp pq$, with the corresponding eigenvectors $v_{\mp} = \begin{pmatrix} p \\ \pm q \end{pmatrix}$, so we see that $|K_S^0\rangle$ and $|K_L^0\rangle$ are the eigenstates of H_{weak}.

• We can read off the masses and life-times from $\lambda_{\mp}$:

K_S^0: $m_S = m - \mathrm{Re}[pq]$, $\tau_S = \left(\Gamma + 2\,\mathrm{Im}[pq]\right)^{-1}$

K_L^0: $m_L = m + \mathrm{Re}[pq]$, $\tau_L = \left(\Gamma - 2\,\mathrm{Im}[pq]\right)^{-1}$

$\hookrightarrow$ In deriving $\lambda_{\mp}$, we chose the signs in $m - \frac{i}{2}\Gamma - \lambda = \pm pq$, such that the imaginary part of $\sqrt{p^2q^2}$ is positive, so we know that $\tau_S < \tau_L$. Experiment shows that $m_S < m_L$, so $\mathrm{Re}[pq] > 0$.

Problem 2

(a) The CKM matrix appears in the W-boson interaction terms, after going from the "gauge basis" to the "mass-basis". The CKM matrix reads

$$V_{CKM} = (L^u)^\dagger L^d \quad , \qquad (1)$$

with L^u, L^d the transformation matrices of u_L & d_L, respectively. Since $M^{(u)}$ is already diagonal, the CKM-matrix is given by

$$V_{CKM} = L^d \quad . \qquad (2)$$

To find it, we need to actually diagonalize $M^{(d)}$,

$$M^{(d)} = V_{CKM}\, M^{(d)}_{\text{diagonal}}\, V_{CKM}^\dagger \quad . \qquad (3)$$

V_{CKM} comprises as columns the normalized eigenvectors of $M^{(d)}$.

A straightforward exercise reveals that

$$V_{CKM} = \begin{pmatrix} 0 & \frac{a}{c} & -\frac{b}{c} \\ 0 & b/c & a/c \\ 1 & 0 & 0 \end{pmatrix}, \qquad (4)$$

with $c = \sqrt{a^2+b^2}$.

Comparing (4) to the rotation matrix

$$\begin{pmatrix} 0 & \cos\theta_{mix} & -\sin\theta_{mix} \\ 0 & \sin\theta_{mix} & \cos\theta_{mix} \\ 1 & 0 & 0 \end{pmatrix}, \qquad (5)$$

we immediately conclude that

$$\theta_{mix} = \arctan(b/a) \,. \qquad (6)$$

(b) Since the matrix is real, there is no CP violation.

Problem 3

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(a) Once again we have a diagonal up-quark mass matrix, meaning that V_{CKM} is such that

$$M^{(d)} = V_{CKM} M^{(d)}_{\text{diagonal}} V^{\dagger}_{CKM} \, . \quad (1)$$

We find

$$V_{CKM} = \frac{1}{\sqrt{a^2+4b^2}} \begin{pmatrix} 2b & a \\ -a & 2b \end{pmatrix} , \quad (2)$$

meaning that

$$\theta_{mix} = \arctan\left(\frac{a}{2b}\right) \, , \quad (3)$$

(b) Since

$$m_s = \lambda_1 \simeq 2mb \, , \quad m_d = \lambda_2 \simeq -\frac{a^2 m}{2b} \, , \quad (4)$$

we find

$$\theta_{mix} \simeq \arctan\sqrt{\frac{m_d}{m_s}} \simeq 12{,}5^\circ \, . \quad (5)$$