LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

Fakultät für Physik im SoSe 2022
TMP - TA3: Condensed Matter Many-Body-Physics and Field Theory I
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https://www2.physik.uni-muenchen.de/lehre/vorlesungen/sose_22/TMP-TA3/index.html

Sheet 12:

Hand-out: Friday, July 15, 2022[1]

Problem 1 Feynman diagrams 1

In this problem we use the linked-cluster theorem to expand the ground state energy E_0 of an interacting electron gas to first order:

$$E_0 = iV\Sigma\{\text{linked-cluster diagrams in momentum space}\} \tag{1}$$

(2.a) Consider an interaction $V(q)$ independent of spin, and derive all first-order Feynman diagrams contributing to the linked-cluster expression for the ground state energy.

(2.b) Calculate these so-called Hartree-Fock diagrams, which consist of direct and exchange terms.

(2.c) In real-space, the first-order perturbative result for E_0 can be written

$$E_0 = \frac{1}{2}\sum_{\sigma,\sigma'}\int d^3x d^3y\; V(\boldsymbol{x}-\boldsymbol{y})\; C_{\sigma\sigma'}(\boldsymbol{x}-\boldsymbol{y}). \tag{2}$$

Write out the corresponding real-space Feynman diagrams for E_0; next derive and calculate Feynman diagrams for the real-space correlations $C_{\sigma\sigma'}(\boldsymbol{x}-\boldsymbol{y})$.

Problem 2 Feynman diagrams: large-N limit

In this problem we learn a powerful approximation technique to drop certain types of diagrams. The basic idea is to consider a system with $N = 2S+1$ spin components and consider the limit $N \to \infty$ where certain classes diagrams vanish. Neglecting the same diagrams even for small values of N (down to $N = 2$) usually yields a systematic simplification of a given theory.

(2.a) Consider interacting fermions with $N = 2S+1$ spin-degeneracy and interaction strength $V(\boldsymbol{q}) = \frac{1}{N}U(\boldsymbol{q})$. Draw the Feynman diagrams expansion for the ground state energy and identify leading and sub-leading terms in the $1/N$ expansion.

(2.b) Discuss which classes of diagrams in the linked-cluster expansion of the ground state energy vanish.

(2.c) $N\chi^{(0)}(q) = \langle\delta\rho(q)\delta\rho(-q)\rangle_0$ is the susceptibility of the non-interacting Fermi gas. Draw the corresponding diagrams for the polarization bubble $\chi(q)$, up to order $1/N$ and where $q = (\boldsymbol{q}, \nu)$.

(2.d) Derive the self-energy of the system in the large-N limit and extract an effective interaction between the fermions in the large-N limit.

[1] If you would like to present your solution(s), feel free to send them to Felix Palm until Fri, July 22.

Problem 3 Electrons in a disordered potential

In this problem we consider free electrons in a disorder potential:

$$\hat{\mathcal{H}} = \sum_{\boldsymbol{k}} \varepsilon_{\boldsymbol{k}} \hat{\psi}^\dagger_{\boldsymbol{k}} \hat{\psi}_{\boldsymbol{k}} + \hat{V}_{\rm dis}, \qquad \hat{V}_{\rm dis} = \int d^3x \; U(\boldsymbol{x}) \; \hat{\psi}^\dagger(\boldsymbol{x}) \hat{\psi}(\boldsymbol{x}). \tag{3}$$

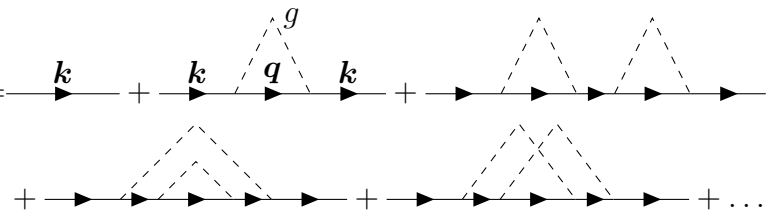
We will perform disorder averages and assume white-noise correlations of the disorder:

$$\overline{\delta U(\boldsymbol{x}) \delta U(\boldsymbol{x}')} = g \; \delta(\boldsymbol{x} - \boldsymbol{x}'), \quad \text{where} \quad \delta U(\boldsymbol{x}) = U(\boldsymbol{x}) - \overline{U(\boldsymbol{x})} = U(\boldsymbol{x}) - \Delta. \tag{4}$$

(3.a) Express the disorder potential $\hat{V}_{\rm dis}$ in Fourier space $\hat{\psi}^{(\dagger)}_{\boldsymbol{k}}$, and write down a diagrammatic expression for the Green's function. Explain, why the Green's function depends explicitly on two momenta.

(3.b) Translate the diagrammatic expression for the Green's function to an algebraic one and perform the disorder average under the assumption $\Delta = 0$. Comment on the momentum dependence of the disorder averaged Green's function.

We interpret the second order contribution in terms of an effective interaction of the electrons and rewrite the algebraic expression of the disorder averaged Green's function as

$\mathcal{G}(\boldsymbol{k}; i\omega_n) = \overline{G(\boldsymbol{k}, \boldsymbol{k}'; i\omega_n)} =$ 

(5)

(3.c) Reorganize this series similar to the reordering for the self energy of an interacting electronic system. Use this reordering to find a compact expression of the disorder averaged Green's function involving the "self energy" $\Sigma(\boldsymbol{k}, i\omega_n)$.

A typical approach would now include a self-consistent calculation of the self energy to ultimately calculate quantities like the spectral function. For the sake of time, we won't do this here.