Neutrino Physics Course

Lecture XXIV

22/7/2022

LMU

Summer 2022

e – μ oscillations?

oscillation ⟺

$$\Delta m^2 \ll \sigma_m^2$$

$$\sigma_m^2 \simeq E\,\sigma_E \simeq E\,\Gamma$$

in order to have e – μ

⇒ large E

- $W_R \to N\ell$ $\quad \ell = e, \mu$

$$N = N_1, N_2$$

$$\begin{pmatrix} N_1 \\ \ell_1(\theta) \end{pmatrix}_R ; \begin{pmatrix} N_2 \\ \ell_2(\theta) \end{pmatrix}_R \longleftrightarrow W_R$$

N_1, N_2 = mass eigenstates

$$\ell_1(\theta) = \cos\theta \, e + \sin\theta \, \mu$$
$$\ell_2(\theta) = -\sin\theta \, e + \cos\theta \, \mu$$

$$\Delta m_N^2 > \sigma_{m^2} = E\Gamma \simeq M_R \Gamma_R$$

$$\Gamma_R \simeq \alpha M_R$$

(a) $$\boxed{\Delta m_N^2 > \alpha M_R^2}$$

$m_N \propto v_R$

$M_{W_R} \equiv M_R \propto v_R$

(b) $$\ell_{coh}(N) \equiv \frac{E^2}{\Delta m_N^2} \frac{1}{\Gamma}$$

$$\simeq \frac{M_R^2 \alpha^{-1}}{\Delta m_N^2 M_R} \simeq 100 \frac{M_R}{\Delta m_N^2}$$

$$\Delta m_N^2 \simeq M_R^2 \Rightarrow$$

$$l_{coh}(N) \simeq 10^2/M_R \lesssim \underbrace{10^{-2}\,10^{-14}\,cm}$$

$$M_R \gtrsim 10^4\,GeV$$

$$\Rightarrow \boxed{N_{1,2} \neq \text{coherent}}$$

- Q. Can $l_1(\theta) \longrightarrow l_2(\theta)$?

(a) $\Delta m_{e\mu}^2 \simeq m_\mu^2 \ll \sigma_m^2$

$$\boxed{m_\mu^2 \ll \alpha\, M_R^2}$$

$\cancel{\checkmark}$

$$l_{coh} \simeq \frac{E^2}{\Delta m^2_{e\mu}} \frac{1}{\Gamma_R} \sim \frac{M_R\, \alpha^{-1}}{m_\mu^2}$$

$$\gtrsim \underline{\underline{100\ m}} \implies \underline{\text{oscillations}}$$

$$\frac{M_R\, \alpha^{-1}}{m_\mu^2} \gtrsim 10^4\, \text{cm}$$

$$M_R \gtrsim 10^2\, 10^{-2}\, \text{GeV}^{-2}\, \text{cm}$$

$$\Uparrow$$

$$(m_\mu \simeq 10^{-1}\, \text{GeV})$$

$$\text{GeV cm} \simeq 10^{14}$$

$$\Downarrow$$

$$\boxed{M_R \gtrsim 10^{14}\, \text{GeV}}$$

• $\tau_\mu \simeq 10^{-6}\,\mathrm{sec} \simeq 10^{4}\,\mathrm{cm} \simeq 100\,\mathrm{m}$

$$\tau_\mu(E) \simeq \frac{E}{m_\mu}\,\tau_\mu(\mathrm{rest})$$

$$E \simeq M_R \simeq 10^{14}\,\mathrm{GeV}$$

$$\Rightarrow \quad E/m_\mu \simeq 10^{15}$$

$$\boxed{\tau_\mu(E = M_R) \simeq 10^{19}\,\mathrm{cm}}$$

Untangling seesaw

$$\underbrace{M_\nu = - M_D^T \frac{1}{M_N} M_D}_{\text{not complete in LRSM}}$$

$$\to \quad M_N = Y_{\Delta_R}^* v_R \qquad M_{W_R} = g v_R$$

$$M_D = Y_D v_{SM} \qquad M_{W_L} = \frac{g}{2} v_{SM}$$

$$\to \quad v_L \simeq \frac{v_{SM}^2}{v_R}$$

induced

$$\mathcal{L}_Y(\Delta) = l_L^T C Y_{\Delta_L} \Delta_L l_L + l_R^T C Y_{\Delta R} \Delta_R l_R + h.c.$$

$$\langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix} \Rightarrow$$

$$\boxed{\delta M_\nu = Y_{\Delta_L} v_L}$$

LR

- P: $\Delta_L \longleftrightarrow \Delta_R$
  $\Phi \longleftrightarrow \Phi^+$ $\Leftarrow$ $\boxed{f_L \leftrightarrow f_R}$

$$\Downarrow$$

$$Y_{\Delta_L} = Y_{\Delta_R}$$

$$Y_D = Y_D^+$$

- C: $f_L \longrightarrow C \bar{f}_R^T = i \gamma_2 f_R^*$

$$\Downarrow$$

$$\left.\boxed{\begin{aligned} Y_{\Delta_L} &= Y_{\Delta_R}^{*} \\ Y_D &= Y_D^{T} \end{aligned}}\right\}$$ verify!

$\Downarrow$ neutrino mass

$$\boxed{M_\nu = Y_{\Delta_L} v_L - M_D^{T} \frac{1}{M_N} M_D}$$

complet neutrino mass matrix

$\Downarrow$ $LR = C$

$$M_\nu = Y^*_{\Delta_R} v_L - M_D \frac{1}{M_N} M_D$$

$$M_N = M^*_{\nu_R} = Y^*_{\Delta_R} v_R$$

$$\Downarrow \quad LR = C$$

$$\boxed{\begin{aligned} M_\nu &= \varepsilon M_N - M_D \frac{1}{M_N} M_D \\ \varepsilon &\equiv \frac{v_L}{v_R} \end{aligned}}$$

Nemevšek, Tello, G. S.

'2012

$$\Downarrow$$

$$\frac{1}{M_N} M_\nu = \varepsilon - \left(\frac{1}{M_N} m_D\right)^2$$

$$\left(\frac{1}{M_N} m_D\right)^2 = \varepsilon - \frac{1}{M_N} M_\nu$$

$$\Downarrow$$

$$\boxed{m_D = M_N \underset{(I)}{\sqrt{\varepsilon - \frac{1}{M_N} \underset{(I)}{M_\nu}}}}$$

$$\Rightarrow \quad \Theta = \frac{1}{M_N} m_D = \sqrt{\varepsilon - \frac{1}{M_N} M_\nu}$$

LHC

oscill.

$$\boxed{\begin{aligned} \underline{M}_N &= V_R\, m_N\, V_R^T \\ \underline{M}_\nu &= V_L^*\, m_\nu\, V_L^+ \end{aligned}}$$

Message (LRSM)

$S\underline{M}$ + ? (minimal)

(a) $\underbrace{SM + \nu_R}_{\text{semantics}}$ (Type I seesaw)

Why?

$$\underline{M}_\nu = - M_D{}^T \frac{1}{\underline{M}_N} \underline{M}_D$$

$$\Downarrow$$

$$\underline{M}_D = i\sqrt{M_N}\, O \sqrt{M_\nu}$$

$$O O^T = 1, \quad O \in C$$

(b) $\underbrace{SM + \Delta_L \equiv \Delta}$ $\quad \Delta \to U \Delta U^+$

$$\Downarrow$$

predictive

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_\Delta + \mathcal{L}_{\Delta, SM}$$

$$\mathcal{L}_\Delta = \mathrm{Tr}\,(D_\mu \Delta)^\dagger (D^\mu \Delta) - V(\Delta)$$

$$\to \mathcal{L}_{\Delta, SM} = \underbrace{l_L^T C\, y_\Delta\, \Delta\, l_L} + h.c. \qquad (1)$$

$$Y(\Delta) = -2\,Y(l_L) = +2$$

$$Y(\phi) = +1$$

$$\cancel{+\ \Phi^T \Delta\, \Phi}$$

$$+\ \mu\ \phi^T i\sigma_2 \Delta^* \phi \ + h.c.$$

$$Y: \quad 1 \qquad -2 \quad 1 \quad = 0$$

$$\Uparrow$$

$$\mathrm{Tr}\ \Delta_L^\dagger\, \Phi\, \langle \Delta_R \rangle\, \Phi^\dagger$$

$$V(\Delta) = + m_\Delta^2 \, Tr \, \Delta^+ \Delta + - - - -$$

$$+ \mu \, \phi^T i \sigma_2 \Delta^* \phi + h.c.$$

$$\Delta = \begin{pmatrix} \delta^+ & \boxed{\delta^{++}} \\ \delta^0 & -\delta^+ \end{pmatrix}$$

$$\Downarrow \qquad \phi = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$V(\delta^0) = + m_\Delta^2 \delta_0 \delta_0^* +$$

$$+ \mu v^2 \delta_0^* + h.c.$$

$$\frac{\partial V}{\partial \delta_0^*} = \mu v^2 + m_\Delta^2 \delta_0 + \cdots = 0$$

$$\Downarrow$$

$$\boxed{\langle \delta_0 \rangle = \mu \frac{v^2}{m_\Delta^2}}$$

$$m_\Delta \to \infty \Rightarrow \langle \delta_0 \rangle \to 0$$

from (1) $\Rightarrow$ $\boxed{M_\nu = Y_\Delta \langle \delta^0 \rangle}$

Type II seesaw

Magg, Wetterich

Lazarides, Shafi,

Mohapatra, G.S.

$\Downarrow$

$$\delta^{++} (e_i)^T C (Y_\Delta)_{ij} e_j =$$

$$= \delta^{++} e_i^T C \frac{(M_\nu)_{ij}}{\langle \Delta \rangle} e_j$$

$$\Downarrow$$

$$\boxed{\begin{array}{c} \delta^{--} \longrightarrow e_i e_j \\ \Gamma \propto (M_\nu)_{ij} \end{array}}$$

<u>Seesaw</u> $\qquad M_D = Y_D v$

$\swarrow$ Type I

$$M_\nu \sim y_D^2 \frac{v^2}{m_N}$$

$\downarrow$ Type II

$$M_\nu \propto y_\Delta \mu \frac{v^2}{m_\Delta^2}$$

$$\left(\mu \sim m_\Delta \;\Downarrow\; y_\Delta \frac{v^2}{m_\Delta} \right)$$

<u>Common factor</u>

$$\sim \frac{v^2}{M_{new}} \simeq \frac{M_W^2}{M_{new}}$$

$\Uparrow$ $d = 5$ Weinberg

$$\frac{(l^T i\sigma_2 \phi)\, C\, (\phi^T i\sigma_2 l)}{M_{new}}$$

$$\Rightarrow \quad M_\nu \simeq \frac{v^2}{M_{new}} \simeq \frac{M_W^2}{M_{new}}$$

Heeck 2022

$\Leftarrow$

Type II seesaw

$\longrightarrow$ CDF W-mass deviation

CDF — W-mass deviation

$$\rho_{SM} = \left(\frac{M_Z \cos\theta_W}{M_W}\right)^2 = 1 \quad (\text{tree})$$

$$+ \frac{\alpha}{\pi} \frac{m_t^2 - m_b^2}{M_W^2} + (O(\alpha))$$

CDF $\Rightarrow \rho \neq \rho_{SM}$

Type II : $\langle \Delta \rangle$

$$\downarrow$$

$$\left(\frac{M_Z \cos\theta_W}{M_W}\right)^2 = 2 \quad \Delta$$

$$\Downarrow$$

$$\rho = \rho_{SM} + O\left(\frac{\langle\Delta\rangle}{v}\right)^2 + \dots \text{loops}$$

if this was all

$$\Rightarrow \langle\Delta\rangle \simeq 5 \text{ GeV}$$

but

$$\delta\rho_{loop} = \frac{\alpha}{\pi} \frac{m_{\delta^{++}}^2 - m_{\delta^+}^2}{M_W^2} + \dots$$

both loops and $\langle\Delta\rangle$

$$\Downarrow$$

?

$\Downarrow$? Heech

$$\langle \Delta \rangle \xrightarrow{?} eV \quad (< MeV)$$

$$M_\nu = y_\Delta \langle \Delta \rangle$$

- if $\langle \Delta \rangle \simeq GeV \Rightarrow y_\Delta \simeq 10^{-9}$
- if $\langle \Delta \rangle \ll MeV \Rightarrow y_\Delta = \ldots$?

observable

$$\boxed{\langle \Delta \rangle \neq \text{Higgs}}$$

$$\boxed{\bullet\ \text{Higgs}: \quad m_h \simeq \langle \phi \rangle \\ \Uparrow \\ -\mu^2 \phi^2 + \dots \Rightarrow \langle \phi \rangle \neq 0}$$

$$\boxed{\bullet\ \langle \Delta \rangle = \text{anti-Higgs} \\ m_\Delta \gg M_W \Rightarrow \langle \Delta \rangle \ll v_{SM}}$$

$$D_\mu \langle \Phi \rangle = \partial_\mu - ig\, T_L W_L \langle \phi \rangle$$
$$+ ig \langle \phi \rangle T_R W_R$$

$$\Downarrow$$

$$M_{W_R}^2 = \langle \Delta_R \rangle^2 + \langle \Phi \rangle^2$$

$$\boxed{W_L - W_R \ \text{mix}}$$

$$\Downarrow$$

$$\zeta_{LR} \propto \frac{\langle \phi \rangle^2}{\langle \Delta_R \rangle^2} \simeq \frac{M_{W_L}^2}{M_{W_R}^2}$$

$$M_{W_R} \gtrsim 5 \text{ TeV}$$

$$M_{W_L} \simeq 80 \text{ GeV}$$

$$\boxed{\zeta_{LR} \lesssim 3 \times 10^{-4}.}$$

$$\begin{array}{cc} W_L & \\ W_R & \end{array} \begin{pmatrix} M_L^2 & \simeq & \langle\phi\rangle^2 \\ \langle\phi\rangle^2 & & M_R^2 \end{pmatrix}$$

$$\Rightarrow \quad \zeta_{LR} \simeq \frac{\langle\phi\rangle}{M_R^2}$$