

Neutrino Physics Course

Lecture XXIV

22/7 / 2022

LMU

Summer 2022



$e - \mu$ oscillations?

oscillations \Leftrightarrow

$$\Delta m^2 \ll \sigma_m^2$$

$$\sigma_m^2 \approx E \sigma_E \approx E \Gamma$$

in order to have $e - \mu$

\Rightarrow large E

• $\bar{W}_R \rightarrow N l \quad l = e, \mu$

$$N = N_1, N_2$$

$$\begin{pmatrix} N_1 \\ l_1(\theta) \end{pmatrix}_R, \begin{pmatrix} N_2 \\ l_2(\theta) \end{pmatrix}_R \longleftrightarrow W_R$$

$N_1, N_2 =$ mass eigenstates

$$l_1(\theta) = \cos\theta e + \sin\theta \mu$$

$$l_2(\theta) = -\sin\theta e + \cos\theta \mu$$

$$\Delta m_N^2 > \sigma_m^2 = E \Gamma \simeq M_R \Gamma_R$$

$$\Gamma_R \simeq \alpha M_R$$

(a) $\Delta m_N^2 > \alpha M_R^2$

$$M_N \propto \mathcal{V}_R$$

$$M_{WR} \equiv M_R \propto \mathcal{V}_R$$

(b) $l_{\text{coh}}(N) \equiv \frac{E^2}{\Delta m_N^2} \frac{1}{\Gamma}$

$$\simeq \frac{M_R^2 \alpha^{-1}}{\Delta m_N^2 M_R} \simeq 100 \frac{M_R}{\Delta m_N^2}$$

$$\Delta m_{\nu}^2 \approx M_R^2 \Rightarrow$$

$$l_{\text{coh}}(N) \approx 10^2 / M_R \approx \underbrace{10^{-2} 10^{-14}} \text{ cm}$$

$$M_R \approx 10^4 \text{ GeV}$$

\Rightarrow $N_{1,2} \neq \text{coherent}$

• Q. Can $l_1(\nu) \rightarrow l_2(\nu)$?

$$(a) \Delta m_{e\mu}^2 \approx m_{\mu}^2 \ll \Delta m^2$$

$$m_{\mu}^2 \ll \alpha M_R^2$$

W

$$l_{\text{coh}} \approx \frac{E^2}{\Delta m_{\nu\mu}^2} \frac{1}{\sqrt{R}} \rightarrow \frac{M_R \alpha^{-1}}{m_{\mu}^2}$$

$$\gtrsim \underline{\underline{100 \text{ m}}} \Rightarrow \underline{\underline{\text{oscillations}}}$$

$$\frac{M_R \alpha^{-1}}{m_{\mu}^2} \gtrsim 10^4 \text{ cm}$$

$$M_R \gtrsim 10^2 10^{-2} \text{ GeV}^2 \text{ cm}$$

$$\left(m_{\mu} \approx 10^{-1} \text{ GeV} \right)$$

$$\text{GeV cm} \approx 10^{14}$$



$$M_R \gtrsim 10^{14} \text{ GeV}$$

$$\cdot \tau_{\mu} \approx 10^{-6} \text{ sec} \approx 10^4 \text{ au} \approx 100 \text{ m}$$

$$\tau_{\mu}(E) \approx \frac{E}{m_{\mu}} \tau_{\mu}(\text{rest})$$

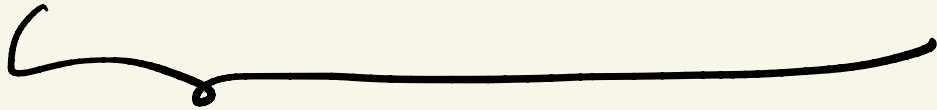
$$E \approx M_{\text{P}} \approx 10^{19} \text{ GeV}$$

$$\Rightarrow \frac{E}{m_{\mu}} \approx 10^{15}$$

$$\tau_{\mu}(E = M_{\text{P}}) \approx 10^{19} \text{ au}$$

Untangling seesaw

$$M_\nu = - M_D^T \frac{1}{M_N} M_D$$



not complete in LRSM

$$\rightarrow M_N = Y_{\Delta_R}^* \nu_R \quad M_{\nu_R} = g \nu_R$$

$$M_D = Y_D \nu_{SM} \quad M_{\nu_L} = \frac{g}{2} \nu_{SM}$$

$$\rightarrow \nu_L \approx \frac{\nu_{SM}^2}{\nu_R}$$

induced

$$\mathcal{L}_Y(\Delta) = l_L^T C Y_{\Delta_L} \Delta_L l_L + \\ + l_R^T C Y_{\Delta_R} \Delta_R l_R + h.c.$$

$$\langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ \varrho_L & 0 \end{pmatrix} \Rightarrow$$

$$\delta M_\nu = Y_{\Delta_L} \varrho_L$$

(LR)

• P: $\Delta_L \leftrightarrow \Delta_R$
 $\Phi \leftrightarrow \Phi^+ \Leftrightarrow \boxed{f_L \leftrightarrow f_R}$

\Downarrow

$$Y_{\Delta_L} = Y_{\Delta_R}$$

$$Y_D = Y_D^+$$

• C: $f_L \rightarrow C \overline{f_R}^{-T} = i\gamma_2 f_R^*$



$$\left. \begin{aligned} Y_{\Delta L} &= Y_{\Delta R}^* \\ Y_D &= Y_D^T \end{aligned} \right\} \underline{\underline{\text{verify!}}}$$



neutrino mass

$$M_\nu = Y_{\Delta L} \nu_L - M_D^T \frac{1}{M_N} M_D$$

complete neutrino mass
matrix



$$LR = C$$

$$M_V = Y_{\Delta R}^* \vartheta_L - M_D \frac{1}{M_N} M_D$$

$$M_N = M_{V_R}^* = Y_{\Delta R}^* \vartheta_R$$

$$\Downarrow LR = C$$

$$M_V = \varepsilon M_N - M_D \frac{1}{M_N} M_D$$

$$\varepsilon \equiv \frac{\vartheta_L}{\vartheta_R}$$

Nemevšek, Tello, G. S.

2012



$$\frac{1}{M_N} M_\nu = \varepsilon - \left(\frac{1}{M_N} M_D \right)^2$$

$$\left(\frac{1}{M_N} M_D \right)^2 = \varepsilon - \frac{1}{M_N} M_\nu$$

⇓

$$M_D = M_N \sqrt{\varepsilon - \frac{1}{M_N} M_\nu}$$

(I) (I)

$$\Rightarrow \Theta = \frac{1}{M_N} M_D = \sqrt{\varepsilon - \frac{1}{M_N} M_\nu}$$

LUC

oscill.

$$\underline{M}_N = V_R M_N V_R^T$$

$$\underline{M}_D = V_L^* M_D V_L^+$$

Message (LRSM)

$S_M + ?$ (minimized)

(a) $S_M + J_R$ (Type I
see law)

semantics

Why?

$$\underline{M}_v = -M_0^T \frac{1}{M_v} \underline{M}_D$$

\Downarrow

$$\underline{M}_D = i \sqrt{M_v} \circ \sqrt{M_v}$$

$$O O^T = I, O \in \mathbb{C}$$

(6) $SM + \Delta_c \equiv \Delta$

$\underbrace{\hspace{10em}}_{\Downarrow} \Delta \rightarrow U D U^T$

predictive

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_\Delta + \mathcal{L}_{\Delta, SM}$$

}

$$\mathcal{L}_0 = \text{Tr} (D_\mu \Delta)^\dagger (D^\mu \Delta) - \bar{V}(\Delta)$$

$$\rightarrow \mathcal{L}_{\Delta, \text{SM}} = \underbrace{l_L^\dagger C \gamma_\Delta \Delta l_L}_{(1)} + \text{h.c.}$$

$$\gamma(\Delta) = -2 \gamma(l_L) = +2$$

$$\gamma(\phi) = +1$$

~~$$+ \bar{\Phi}^\dagger \Delta \Phi$$~~

$$+ \mu \phi^\dagger i \sigma_2 \Delta^* \phi + \text{h.c.}$$

$$\gamma: \quad 1 \quad -2 \quad 1 \quad = 0$$

\Uparrow

$$\text{Tr} \Delta_L^\dagger \Phi (\Delta_R) \Phi^\dagger$$

$$V(\Delta) = \textcircled{+} m_{\Delta}^2 T_{\Delta} \Delta^{\dagger} \Delta + \dots$$

$$+ \mu \phi^{\dagger} i \sigma_2 \Delta^* \phi + \text{h.c.}$$

$$\Delta = \begin{pmatrix} \delta^+ & \boxed{\delta^{++}} \\ \delta^0 & -\delta^+ \end{pmatrix}$$

$$\Downarrow \quad \phi = \begin{pmatrix} 0 \\ \nu \end{pmatrix}$$

$$V(\delta^0) = + m_{\Delta}^2 \delta^0 \delta_0^* +$$

$$+ \mu \nu^2 \delta_0^* + \text{h.c.}$$

$$\frac{\partial V}{\partial \delta_0^*} = \mu \nu^2 + m_{\Delta}^2 \delta_0 + \dots = 0$$

\Downarrow

$$\langle \delta_0 \rangle = \mu \frac{v^2}{\mu_0^2}$$

$$\mu_0 \rightarrow \infty \Rightarrow \langle \delta_0 \rangle \rightarrow 0$$

$$\text{From (1)} \Rightarrow M_\nu = Y_\Delta \langle \delta_0 \rangle$$

Type II seesaw

Maeg, Wittenich
Lazarides, Shati,
Moliseptro, G.S.



$$\delta^{++} (e_i)^T C (Y_0)_{ij} e_j =$$

$$= \delta^{++} e_i^T C \frac{(M_\nu)_{ij}}{\langle \Delta \rangle} e_j$$



$$\delta^{--} \longrightarrow e_i e_j$$

$$\Gamma \propto (M_\nu)_{ij}$$



↓ Type I

$$M_\nu \sim \gamma_D^2 \frac{v^2}{M_N}$$

↓ Type II

$$M_\nu \propto \gamma_D \mu \frac{v^2}{M_\Delta^2}$$

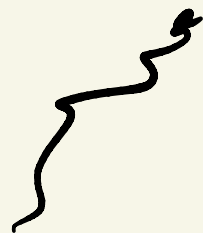
$$\left(\begin{array}{c} \mu \sim m_\Delta \\ \Downarrow \\ \gamma_\Delta \frac{v^2}{M_\Delta} \end{array} \right)$$

Common factor

$$\sim \frac{v^2}{M_{\text{new}}} \sim \frac{M_W^2}{M_{\text{new}}}$$



$d=5$ Weinberg



$$\frac{(\ell^\top i \sigma_2 \phi) C (\phi^\top i \sigma_2 \ell)}{M_{\text{new}}}$$

$$\Rightarrow \underline{M}_\nu \simeq \frac{v^2}{M_{\text{new}}} \simeq \frac{M_W^2}{M_{\text{new}}}$$

Heeds 2022



Type II seesaw

→ CDF W-mass derivation

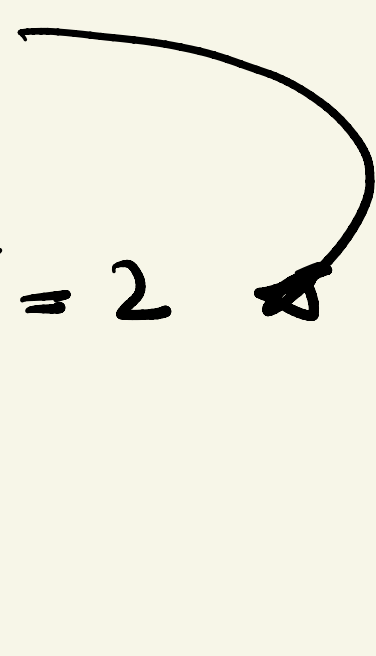
CDF = W-mass derivation

$$\rho_{SM} = \left(\frac{M_Z \cos \theta_W}{M_W} \right)^2 = 1 \quad (\text{tree})$$

$$+ \frac{\alpha}{\pi} \frac{m_t^2 - m_b^2}{M_W^2} + (\text{O}(1))$$

CDF $\Rightarrow \rho \neq \rho_{SM}$

Type II : $\langle \Delta \rangle$

$$\left(\frac{M_Z \cos \theta_W}{M_W} \right)^2 = 2$$




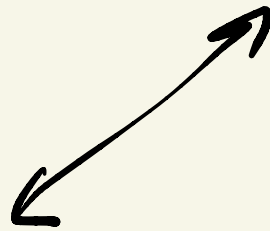
$$P = P_{SM} + O\left(\frac{\langle \Delta \rangle}{v}\right)^2 + \dots$$

(loops)

if this was all

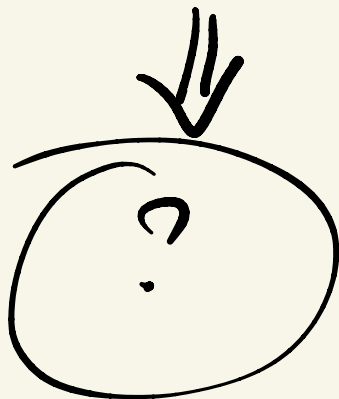
$$\Rightarrow \langle \Delta \rangle \simeq 5 \text{ GeV}$$

but



$$\delta P_{\text{loop}} = \frac{\alpha}{\pi} \frac{M_{S^{++}}^2 - M_{S^+}^2}{M_W^2} + \dots$$

both loops need $\langle \Delta \rangle$



↓? Heech

$$\langle \Delta \rangle \xrightarrow{\sim} e\bar{\nu} \quad (< \text{MeV})$$

$$M_\nu = Y_\Delta \langle \Delta \rangle$$

• if $\langle \Delta \rangle \simeq \text{GeV} \Rightarrow Y_\Delta \simeq 10^{-9}$

• if $\langle \Delta \rangle \ll \text{MeV} \Rightarrow Y_\Delta = \dots ?$

↗
observable

$$\langle \Delta \rangle \neq \text{Higgs}$$

• Higgs : $M_h \simeq \langle \phi \rangle$

$$-\mu^2 \phi^2 + \dots \xrightarrow{\uparrow} \Rightarrow \langle \phi \rangle \neq 0$$

• $\langle \Delta \rangle = \text{ewb} - \text{Higgs}$

$$M_\Delta \gg M_W \Rightarrow \langle \Delta \rangle \ll v_{SM}$$

$$D_\mu \langle \Phi \rangle = \partial_\mu - ig T_L W_L \langle \Phi \rangle + ig \langle \Phi \rangle T_R W_R$$



$$M_{WR}^2 = \langle \Delta_R \rangle^2 + \langle \Phi \rangle^2$$

$W_L - W_R$ mix



$$\zeta_{LR} \propto \frac{\langle \Phi \rangle^2}{\langle \Delta_R \rangle^2} \approx \frac{M_{WL}^2}{M_{WR}^2}$$

$$M_{WR} \geq 5 \text{ TeV}$$

$$M_{WL} \approx 80 \text{ GeV}$$

$$\zeta_{LR} \leq 3 \times 10^{-4}$$



$$\begin{matrix} W_L \\ W_R \end{matrix} \begin{pmatrix} \mu_L^2 \approx \langle \phi \rangle^2 \\ \langle \phi \rangle^2 & \mu_R^2 \end{pmatrix}$$

$$\Rightarrow \zeta_{LR} \approx \frac{\langle \phi \rangle}{\mu_R^2}$$