Neutrino Physics Course

Lecture XVIII

1/7/2022

LMU

Summer 2022

Origin and nature of ν mass

$$LR: \quad \nu_L \longleftrightarrow \nu_R$$

$$\Downarrow \ SSB$$

$$\begin{array}{c} \nu_L \\ N_L \end{array} \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}$$

$$\begin{array}{cc} \nu_L & N_L \end{array}$$

$$\boxed{N_L = C \bar{\nu}_R^T}$$

$$\Downarrow \quad \boxed{M_N \gg M_D}$$

$$\begin{pmatrix} \nu \\ N \end{pmatrix} \longrightarrow \begin{pmatrix} \nu' \\ N' \end{pmatrix} = U \begin{pmatrix} \nu \\ N \end{pmatrix}$$

$$\therefore \quad U^T M_{\nu N} U = D_{\nu N}$$

$$D_{\nu N} = \begin{pmatrix} M_\nu & 0 \\ 0 & M_N + x \end{pmatrix}$$

$$M_\nu = - M_D^T \frac{1}{M_N} M_D$$

$$U \cong \begin{pmatrix} 1 & \theta^+ \\ -\theta & 1 \end{pmatrix}, \quad (\theta \ll 1)$$

$$\therefore \quad \theta = \frac{1}{M_N} M_D$$

$$\Downarrow$$

$$\nu_L'^T C M_\nu \nu_L' + N_L'^T C M_N N_L'$$

$\Uparrow$

$\boxed{\text{Majorana masses}}$

M_ν = matrix , M_N = matrix

(generations)

$\boxed{\text{Convention: we work in diagonal basis of charged lepton masses}}$

$\Uparrow$

Reminder: u, d quarks

$$\begin{cases} M_u = U_L^+ \, m_u \, U_R \\ M_d = D_L^+ \, m_d \, D_R \end{cases} \qquad U^+U = 1 \qquad D^+D = 1$$

$$\Downarrow \qquad \boxed{m_u, \, m_d > 0}$$

$$\boxed{V_{CKM} = U_L^+ D_L}$$

$$\frac{g}{\sqrt{2}} \, \bar{u}_L \gamma^\mu V_{CKM} \, d_L \, W_\mu^+$$

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix}_L \qquad \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

$$M'_{\nu_L} \equiv V_L^* \, m'_\nu \, V_L^+ \quad (1) \qquad M_\nu^T = M_\nu$$

$$M'_{N_L} \equiv V_R \, m'_N \, V_R^T \quad (2) \qquad M_N^T = M_N$$

$$N_L = C \bar{\nu}_R^T = C \gamma_0 \nu_R^*$$

$$\Longrightarrow \boxed{M_N = M_{\nu_R}^*}$$

$$(2) \Longrightarrow \boxed{M_{\nu_R} = V_R^* \, m_N \, V_R^+}$$

m'_ν, $m_N' =$ diagonal matrices

$$\Downarrow$$

$$\nu_L' \longrightarrow V_L \, \nu_L'$$

$$N_L' \longrightarrow V_R^* \, N_L' \quad (\nu_R' \longrightarrow V_R \, \nu_R')$$

$$\Downarrow$$

$$\frac{g}{\sqrt{2}}\Big[\bar{\nu}_L{}' \gamma^\mu V_L{}^+ e_L\, W_{\mu L}{}^+ +$$

$$\bar{\nu}_R{}' \gamma^\mu V_R{}^+ e_R\, W_\mu{}^+\Big] + h.c.$$

$$\boxed{\begin{aligned} V^{(L)}_{PMNS} &= V_{leptonic}(L) \equiv V_L{}^+ \\ V^{(R)}_{PMN} &= V_{leptonic}(R) \equiv V_R{}^+ \end{aligned}}$$

$$\Downarrow$$

physical states : $\nu'' = V_L \nu'$

$N'' = V_R^* N'$

$$\Downarrow$$

$\boxed{\nu,\ N}$ $\boxed{m_\nu,\ m_N > 0}$

bottom line: $V_{CKM} \to 1$ phase

3gen. $\left\{ V_{PMNS} \to \underset{\text{Dirac}}{1} + \underset{\text{Majorana}}{2} = 3 \text{ phases} \right.$

$$
2\text{gen} \begin{cases} V_c \to 0 \text{ phases} \\ V_p \to 1 \text{ phase (Majorana)} \end{cases}
$$

$$\Downarrow$$

$$\left.\begin{array}{ll} \nu_i & (i = 1, 2, 3) \\ N_i & (\;\text{—''—}\;) \end{array}\right\} \underline{\text{mass eigenstates}}$$

$$\boxed{\frac{1}{2}\underbrace{\nu_{iL}^T C\, m_{\nu_i}\, \nu_{iL}}_{\Delta L = 2} + \frac{1}{2}\underbrace{N_{iL}^T C\, m_{N_i}\, N_{iL}}_{\Delta L = -2} + h.c.}$$

$$\boxed{\text{Well defined mass}}$$

(Majorana)

$$\Downarrow$$

$$\boxed{\Delta L = 2 \quad \text{processes}}$$

ν sector

$\begin{pmatrix}\nu \\ e\end{pmatrix}_L \Rightarrow \nu_L = \text{lepton}$

N sector

$\begin{pmatrix}\nu \\ e\end{pmatrix}_R$

$\Downarrow$

$\nu_R = \text{lepton} \Rightarrow$

$N_L = \text{anti-lepton}$

$\downarrow$ go to 4-comp. notation

$$\nu_M = \nu_L + C\bar{\nu}_L^T = \underline{\nu_L} + \underline{(\nu^c)}_R$$

$$N_M = N_L + C\bar{N}_L^T = \underline{N_L} + \underline{(N^c)}_R$$

$$\Downarrow$$

$$\mathcal{L}_{(\nu,N)} = i\,\overline{\nu_M}\,\gamma^\mu \partial_\mu \nu_M - m_\nu^M\,\bar{\nu}_M \nu_M$$

$$+ i\,\overline{N_M}\,\gamma^\mu \partial_\mu N_M - m_N^M\,\bar{N}_M N_M$$

Q. What does it mean to be Majorana?

ν_L = lepton

N_L = anti - lepton

$$\boxed{\begin{array}{l}\nu_M = \underbrace{\nu_L}_{\text{lepton}} + \underbrace{C\bar{\nu}_L^T}_{\text{anti-lepton}} = \text{hybrid} \\ \\ \Leftarrow \text{ having Majorana mass}\end{array}}$$

$$\nu_L^T C \nu_L \equiv \overline{\nu^c_R}\, \nu_L \longleftarrow$$

$$\nu^c_R = C \bar{\nu}_L^T = C\gamma_0 \nu_L^*$$

$$\Downarrow$$

$$\overline{\nu^c_R}\, \nu_L = (C\gamma_0 \nu_L^*)^+ \gamma^0 \nu_L$$

$$= \nu_L^T \gamma_0 C^+ \gamma^0 \nu_L$$

$$= \nu_L^T (-C^+)\nu_L = \nu_L^T C \nu_L$$

$\boxed{\text{Q.E.D.}}$

$m_M \Longrightarrow$ takes p into anti $-p$

$\Downarrow$

$$\boxed{(p)_M = p + \bar{p}}$$

<u>Dirac:</u> $\psi = \psi_L + \psi_R$

$$m_D \overline{\psi_R} \psi_L + h.c.$$

<u>Majorana:</u> $\psi_L = \psi_L + (\psi^c)_R$

$$(\psi^c)_R \equiv C \overline{\psi_L}^T$$

$$m_M \psi_L^T C \psi_L = m_M \overline{\psi^c_R} \psi_L$$

Majorana 1937

<u>neutrino</u> = Majorana?

NO $\Rightarrow$ has moments

but Majorana spinor $\Rightarrow$

all moments $= 0$

Neutrino $=$ Majorana

$\Downarrow$

Implications? How to verify?

$\frac{m_\nu}{2} \nu_L^T C \nu_L + h.c.$

$\Leftrightarrow \quad \nu_M = \nu_L + C\bar{\nu}_L^T = \nu_L + (\nu^c)_R$

beta decay

$$n \longrightarrow p + e + \bar{\nu}$$

$$(Co \longrightarrow Ni + e + \bar{\nu})$$

Goeppert-Mayer
1935

$$Ge \not\longrightarrow Ar + e + \bar{\nu}$$

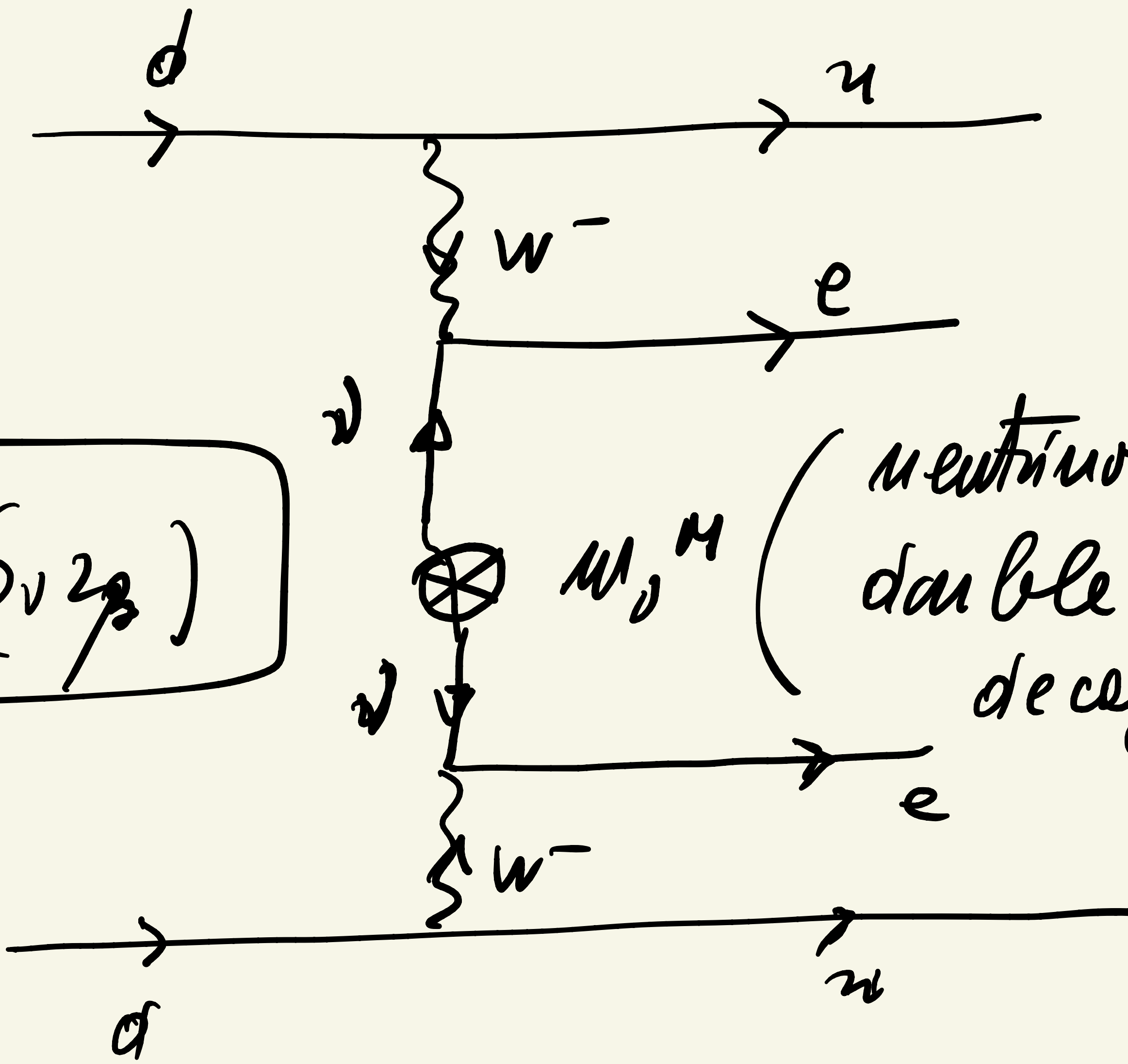
$$m_{Ar} > m_{Ge}$$

$$\Downarrow$$

$$Ge \rightarrow Se + e + e + \bar{\nu} + \bar{\nu}$$

$m_{Ge} > m_{Se}$ double beta decay

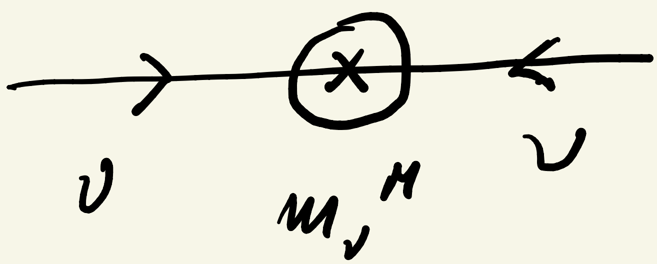
$$\boxed{\tau_{2\beta} \simeq 10^{21}\, yr}$$

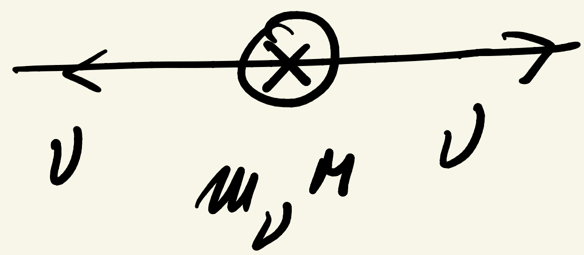


$\boxed{(0\nu 2\beta)}$

(neutrino less double beta decay)

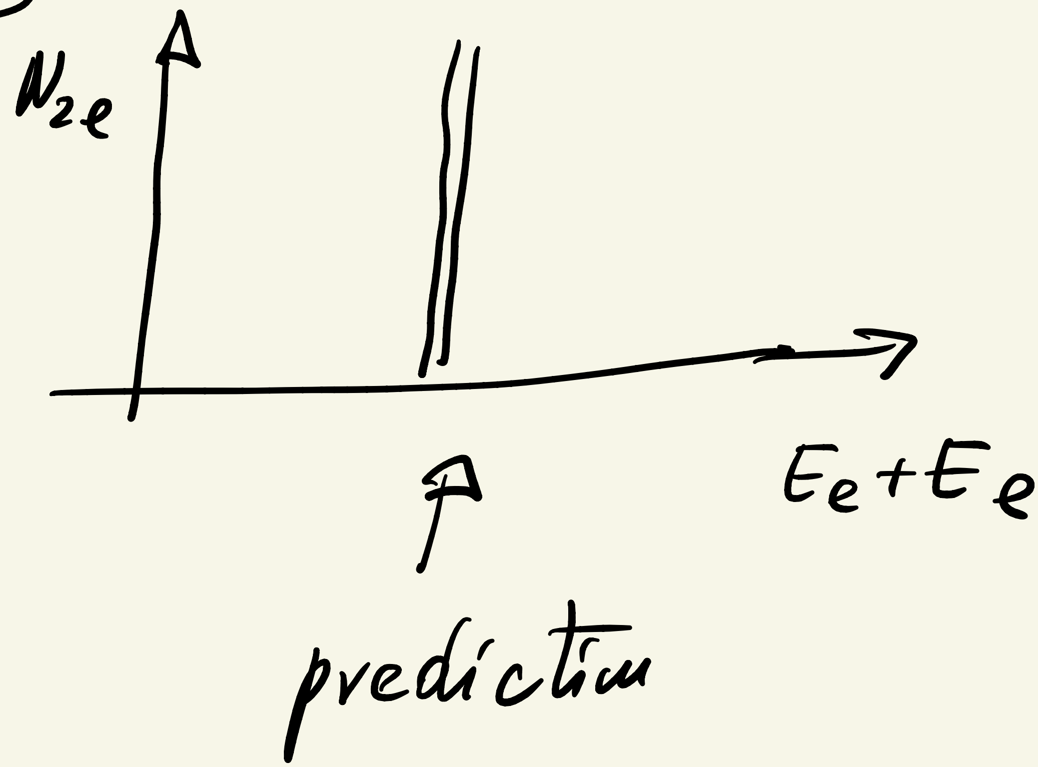
$$m_\nu^M\left(\nu_L^T C \nu_L + \nu_L^\dagger C^\dagger \nu_L^*\right)$$





ν discovery = β decay

$\boxed{0\nu 2\beta}$



$$W:\ \overline{\nu_L}\gamma^\mu e_L = \overline{e^c_R}\gamma^\mu \nu^c_R \quad (\pm)$$

$$e^c_R \equiv C\bar{e}_L^T, \quad \nu^c_R = C\bar{\nu}_L^T$$

Proof

$$\overline{e^c_R}\gamma^\mu \nu^c_R = \overline{C\bar{e}_L^T}\gamma^\mu C\bar{\nu}_L^T$$

$$= (C\gamma_0 e_L^*)^+ \gamma^0 \gamma^\mu C \bar{\nu}_L^T$$

$$= e_L^T \gamma_0 C^+ \gamma^0 \gamma^\mu C \bar{\nu}_L^T$$

$$= e_L^T \underbrace{(-C^+ \gamma^\mu C)}_{(\gamma^\mu)^T} \bar{\nu}_L^T$$

$$= \pm \bar{\nu}_L \gamma^\mu e_L \qquad \text{Q.E.D.}$$

$$\nu_M = \nu_L + C\bar{\nu}_L^T = \nu_L + \nu_R^c$$

$$\bar{\nu}_L \gamma^\mu e_L = \overline{e_R^c} \gamma^\mu \nu_R^c$$

$$\Downarrow$$

$$(\bar{\nu}_M)_L \gamma^\mu e_L = \overline{e_R^c} \gamma^\mu (\nu_M)_R$$

$$(\nu_M)_L = \nu_L$$

$$(\nu_M)_R = \nu_R^c$$

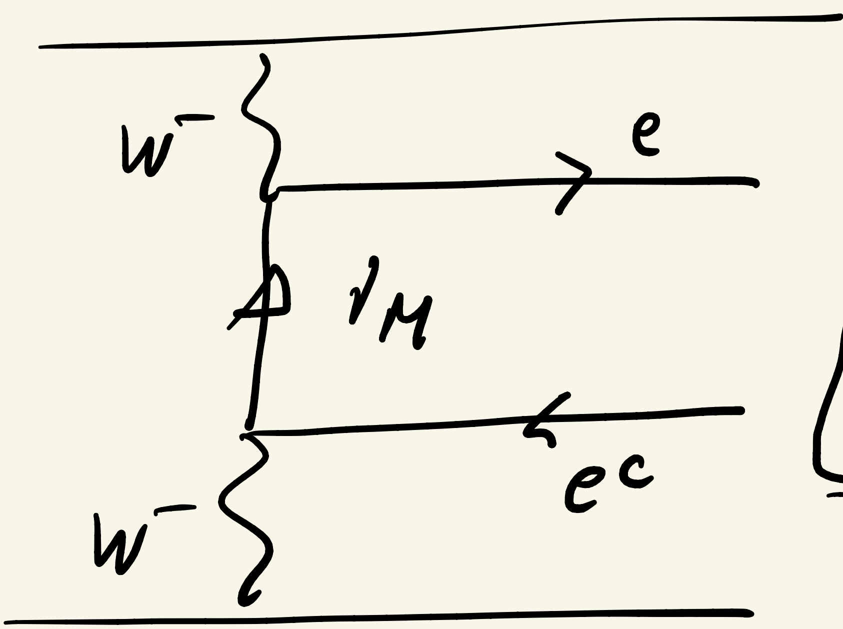
$$\Updownarrow$$

$$\bar{e}_L \gamma^\mu \nu_{ML} = \overline{\nu_{MR}} \gamma^\mu e^c_R$$

$$\Downarrow$$

$$\boxed{\bar{e}\gamma^\mu L \nu_M = \overline{\nu_M}\gamma^\mu R e^c}$$

$$\Downarrow$$



$$k \simeq 100\,\mathrm{MeV} \gg m_\nu^M$$

$$A^{(\nu)}_{0\nu 2\beta} \simeq G_F^2\, \bar{e}\, \gamma_\mu L \frac{\not{k} + m_\nu^M}{k^2 - (m_\nu^M)^2} \gamma_\nu R e^c \quad (\phi, u)$$

$$\simeq G_F^2\, \bar{e}\, \gamma^\mu \frac{\not{k} R + m_\nu^M L}{k^2} \gamma^\nu R e^c$$

$$\simeq G_F^2\, \bar{e}\, \gamma^\mu \frac{\not{k} \gamma^\nu L + m_\nu^M \gamma^\nu R}{k^2} R e^c$$

$$\simeq G_F^2\, \bar{e}\, \frac{m_\nu^M}{k^2} \gamma^\mu \gamma^\nu R e^c$$

$$\Downarrow$$

$$\boxed{A^{(\nu)}_{0\nu 2\beta} \propto G_F^2 \frac{m_\nu^M}{k^2}}$$

$$\tau_{0\nu 2\beta} \gtrsim 10^{26}\ \mathrm{yr}$$

GERDA 2021

$$\Rightarrow \boxed{m_\nu^M \leq 1\ \mathrm{eV}}$$

(a) ν = Majorana $(\Leftrightarrow m_\nu^M)$

$$\Rightarrow \quad 0\nu 2\beta \propto m_\nu^M$$

(b) if $0\nu 2\beta \Rightarrow \nu$ = Majorana?

A. We do not know.

1958 Feinberg
Goldhaber

there could be new physics

⇒ $0\nu2\beta$ (per se) is <u>not</u> a measure of m_ν^M

$$\bullet \quad \overline{e_L}\,\gamma^\mu \nu_L = \overline{\nu^c_R}\,\gamma^\mu e^c_R \quad \left.\right\} \; W_L$$

$$\Rightarrow \quad \bar{e}\,\gamma^\mu L \nu_M = \bar{\nu}_M\, \gamma^\mu R\, e^c$$

$$\bullet\ \overline{e_R}\,\gamma^\mu\,\nu_R = \overline{\nu^c_L}\,\gamma^\mu\,e^c_L \quad \Big\}\ W_R$$

$$N_M = N_L + C\,\overline{N_L}^T$$

$$\begin{array}{cc} \parallel & \parallel \\ C\,\overline{\nu_R}^T & \nu_R \end{array}$$

$$\Rightarrow \boxed{\bar{e}\,\gamma^\mu\,R\,N_M = \overline{N_M}\,\gamma^\mu\,L\,e^c}$$

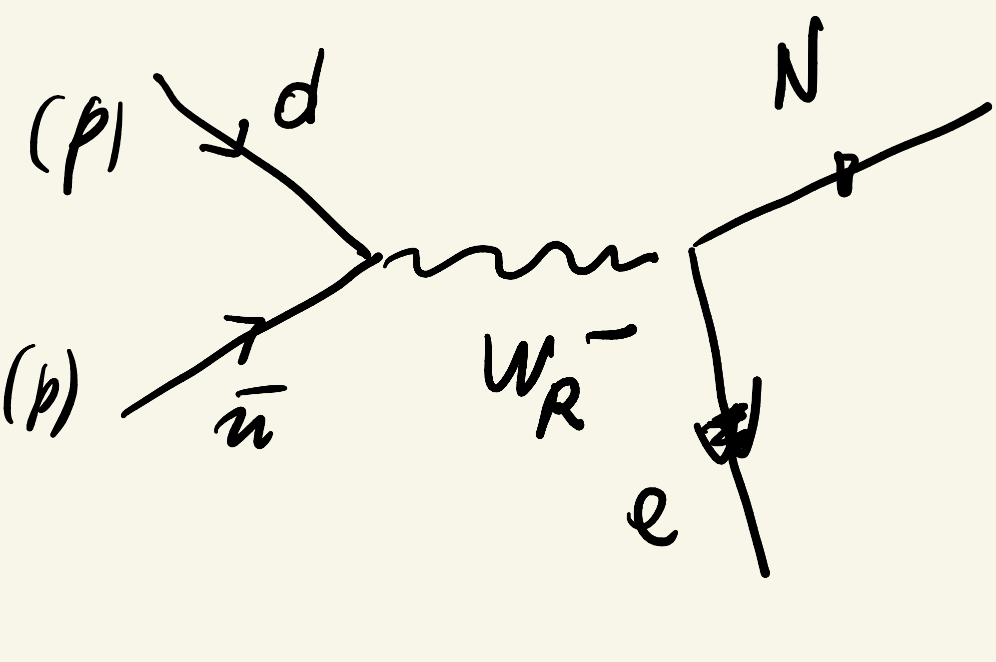
ALSO

$$\Theta\,\overline{N_L}\,\gamma^\mu\,e_L\,W^+_{\mu L} \longleftarrow \Theta \simeq \frac{M_D}{M_N}$$

$$= \Theta\,\overline{N_M}\,\gamma^\mu\,L\,e\,W^+_{\mu L} \quad ?$$

$$= \Theta\, \bar{e}\, \gamma^{\mu} R\, N_M\, W_{\mu L}^{+} \Big\}$$

$$\boxed{H\bar{W}}$$



$\Downarrow$

$$N \to e + W_L^+$$

TEST seesaw?

TEST of origin and nature of ν mass?

$$\sigma_{W_L}(N) \propto |\theta^2| \simeq \left|\frac{m_D^2}{m_N^2}\right|$$

$$m_\nu = -\frac{m_D^2}{m_N}$$

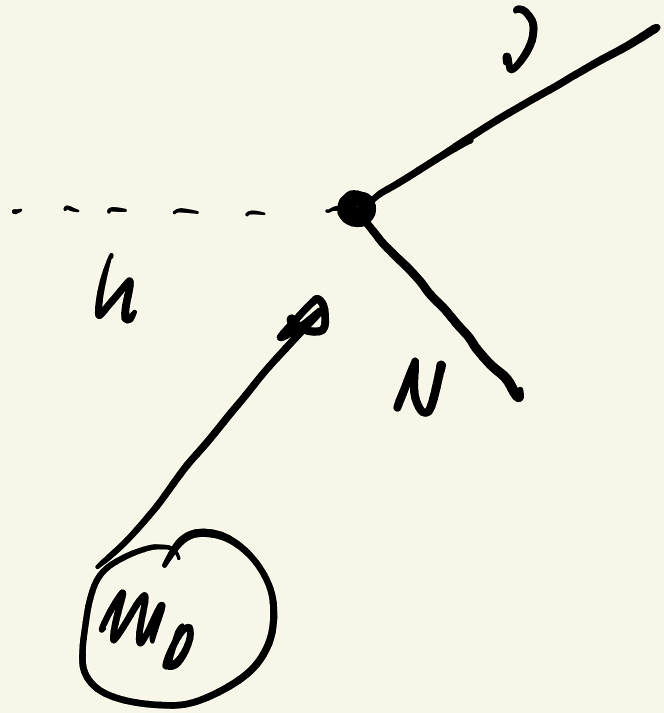
$$\Rightarrow \sigma_{W_L}(N) \propto \frac{m_\nu}{m_N} \simeq \frac{10^{-10}\,\text{GeV}}{10^{3}\,\text{GeV}}$$

$$\simeq 10^{-13}$$

$$N \longrightarrow e\,W^{+} \leftarrow \text{hopeful}$$

$$\nu\,h \leftarrow \text{hopeless}$$

$$\frac{m_d,\, m_u}{m_W} = 0$$



if ($m_N < m_h$)

$$\boxed{\text{Maiezza, G.S. } \ldots}$$

$$V = V(\Phi) + V(\Delta)$$

$$+ V(\Phi\Delta)$$

$\mathrm{Tr}\,\Phi^\dagger\Phi \quad (\mathrm{Tr}\,\Delta_L^\dagger\Delta_L + L\leftrightarrow R)$

$\mathrm{Tr}\,\tilde{\Phi}^\dagger\tilde{\Phi} \quad (\quad -\parallel- \quad)$

$\det\Phi \quad (\quad -\parallel- \quad)$

$\det\Phi^\dagger \quad (-\parallel-)$

$\alpha_3\,\mathrm{Tr}\,\Phi^\dagger\Phi\,\Delta_R\Delta_R^\dagger \; ---$

Φ

$$\boxed{\Phi^\dagger\Phi + \tilde{\Phi}^\dagger\tilde{\Phi} = \propto (\mathrm{Tr}\,\Phi^\dagger\Phi)\,\mathbb{1}}$$

$$\boxed{\underbrace{V(\Delta_L\ \Phi\ \Delta_R)}_{\text{mixed}}}$$

$$\left.\begin{aligned}
\Delta_L &\to U_L\, \Delta_L\, U_L^{\dagger} \\
\Phi &\to U_L\, \Phi\, U_R^{\dagger} \\
\Delta_R &\to U_R\, \Delta_R\, U_R^{\dagger}
\end{aligned}\right\}$$