Neutrino Physics Course

Lecture $\overline{\underline{XVII}}$

28/6/2022

LMU

Summer 2022

LRSM: Seesaw Mechanism

- $\langle \Delta_R \rangle \Rightarrow \frac{1}{2} \nu_R^T C \underline{M}_R \nu_R^{\cancel{*}} + h.c.$

$$\therefore \quad M_R = Y_\Delta \langle \Delta_R \rangle$$

- $\langle \Phi \rangle \Rightarrow \bar{\nu}_R \underline{M}_D \nu_L^{\cancel{*}} + h.c.$

$$\underline{M}_D = Y^\dagger \langle \Phi \rangle^\dagger + \tilde{Y}^\dagger \langle \tilde{\Phi} \rangle^\dagger$$

$$\Downarrow$$

$$N_L = C \bar{\nu}_R^T$$

$$\Downarrow$$

$$M_N = M_R^*$$

$$\Downarrow$$

$$\left\{ \frac{1}{2} N_L^T\, C\, \underline{M}_N\, N_L + h.c. \right.$$

$$\left. + N_L^T C M_D \nu_L = \frac{1}{2} N_L^T C M_D \nu_L + \frac{1}{2} \nu_L^T C M_D^T N_L \right\}$$

$$\Downarrow$$

$$\boxed{\text{up to } \tfrac{1}{2}}$$

$$\left[\ \underline{M}_{\nu N} = \begin{array}{c} \\ \nu_L \\ N_L \end{array} \begin{pmatrix} 0 & M_D^T \\ M_D & \underline{M}_N \end{pmatrix} \ \right]$$

Explanation of Majorana $\frac{1}{2}$!

$$\mathcal{L}(\nu_R) = i\,\bar{\nu}_R \gamma^\mu \partial_\mu \nu_R -$$

$$- \frac{1}{2}\, \underbrace{\nu_R{}^T C\, m_R\, \nu_R}\, + h.c.$$

why $1/2$?

$$\nu_R \equiv \begin{pmatrix} 0 \\ n_R \end{pmatrix} \leftarrow \text{2 component}$$

---

what is fermion mass?

$\Downarrow$

$$\mathcal{L}_D = i\,\bar{\psi}_D \gamma^\mu \partial_\mu \psi_D - m\,\bar{\psi}_D \psi_D$$

$$\psi_D = \psi_L + \psi_R = \begin{pmatrix} u_L \\ u_R \end{pmatrix}$$

$$\Downarrow$$

$$i\gamma^\mu \partial_\mu \psi_D = m\,\psi_D$$

$$\Downarrow$$

$$p_\mu \gamma^\mu \tilde{\psi}_D(p) = m\,\tilde{\psi}_D(p) \;/\; p_\nu \gamma^\nu$$

$$\Downarrow$$

$$\boxed{p^2 = m^2} \quad \boxed{E^2 = \vec{p}^{\,2} + m^2}$$

mass !!

$$\Uparrow$$

I cannot walk with ν_R!

I need 4-comp. spinor:

$$\nu_M = \nu_R + C\,\bar{\nu}_R^{\,T}$$

$$= \nu_R + C\gamma_0\,\nu_R^*$$

$$= \begin{pmatrix} 0 \\ u_R \end{pmatrix} + \begin{pmatrix} i\sigma_2 u_R^* \\ 0 \end{pmatrix}$$

$$\Downarrow$$

$$\boxed{\nu_M = \begin{pmatrix} i\sigma_2 u_R^* \\ u_R \end{pmatrix}}$$

$$\cdot\ \underline{\bar{\nu}_M \nu_M} = \left(\bar{\nu}_R + C\,\overline{\bar{\nu}_R}^T\right)\left(\nu_R + C\,\bar{\nu}_R^T\right)$$

$$= \left(\underline{\nu_R}^{+}\gamma^0 + \cdots\right)\left(\underline{\nu_R} + C\,\bar{\nu}_R^T\right)$$

$$\bar{\nu}_R \nu_R = \nu_R^{+}\gamma_0 \nu_R = \nu^{+} R\, \gamma_0 R\, \nu$$

$$= \nu^{+}\gamma_0 L R\, \nu = 0$$

$$\Downarrow$$

$$\bar{\nu}_M \nu_M = \bar{\nu}_R\, C\, \bar{\nu}_R^T + \overline{C\,\bar{\nu}_R^T}\,\nu_R$$

$$\bar{\nu}_R\, C\, \bar{\nu}_R^T = \nu_R^{+}\gamma^0 C \gamma_0 \nu_R^{*} = \nu_R^{+}(-C)\gamma_0^2 \nu_R^{*} = \nu_R^{+} C^{+} \nu_R^{*} \ (h.c.)$$

$$\overline{C\,\bar{\nu}_R^T}\,\nu_R = \boxed{\nu_R^T C\, \nu_R}$$

$$\Downarrow$$

1) $$\boxed{\overline{\nu_M}\,\nu_M = \nu_R^T C \nu_R + h.c.}$$

2) $$\overline{\nu_M}\,\gamma^\mu \partial_\mu \nu_M \qquad\qquad \nu_M = \nu_R + C\overline{\nu_R}^T$$

$$\parallel$$

$$\overline{\nu_R}\,\gamma^\mu \partial_\mu \nu_R + \underbrace{\overline{C\overline{\nu_R}^T}\,\gamma^\mu \partial_\mu C\,\overline{\nu_R}^T}_{\parallel}$$

$$\overline{\nu_R}\,\gamma^\mu \partial_\mu \nu_R \qquad \boxed{\text{PROVE!}}$$

$$\Downarrow$$

$$\boxed{\overline{\nu_M}\,\gamma^\mu \partial_\mu \nu_M = 2\,\overline{\nu_R}\,\gamma^\mu \partial_\mu \nu_R}$$

$$\Downarrow$$

$$\mathcal{L}(\nu_R) = i\bar{\nu}_R \gamma^\mu \partial_\mu \nu_R - \frac{1}{2} m \nu_R^T C \nu_R + h.c.$$

$$= \cancel{\frac{1}{2}} \left[i\bar{\nu}_M \gamma^\mu \partial_\mu \nu_M - m_R \bar{\nu}_M \nu_M \right]$$

irrelevant

$$\Downarrow \quad \mathcal{L}_D$$

$$\boxed{\begin{gathered} i\gamma^\mu \partial_\mu \nu_M = m_R \nu_M \\ \Downarrow \\ E^2 = \bar{p}^2 + m_R^2 \end{gathered}}$$

mass

Dirac

$$M_{\nu M} = \begin{array}{cc} \nu_L & \\ N_L & \end{array} \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}$$

Dirac

$\boxed{\text{messy}}$

$\Downarrow$ $\boxed{\text{seesaw limit: } M_N \gg M_D}$

$\boxed{\text{Simple}}$

1 generation

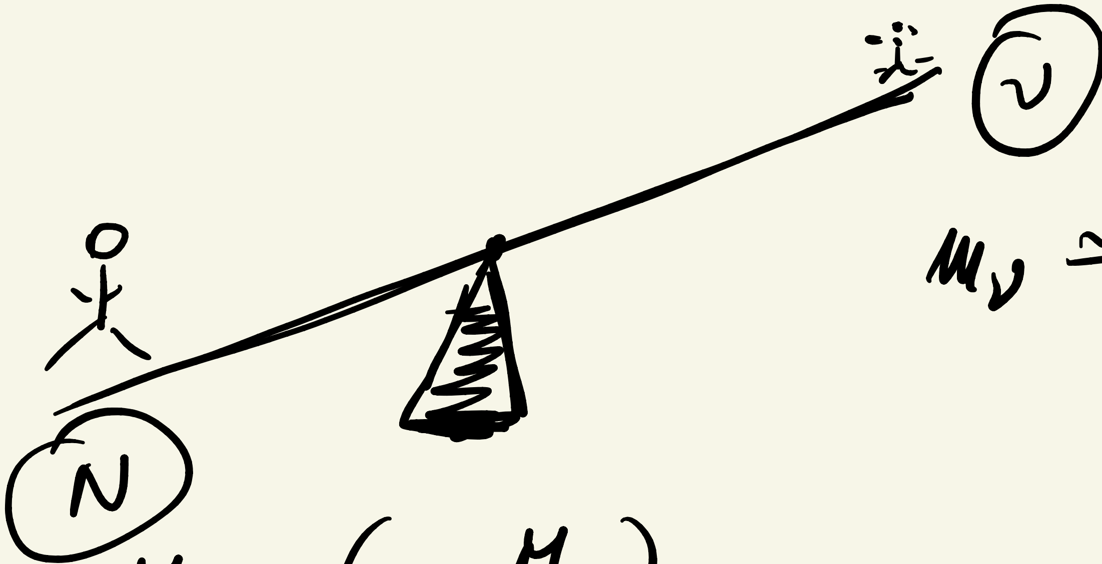
$$M_{\nu N} = \begin{array}{c} \nu \\ \\ N \end{array} \begin{pmatrix} 0 & m_D \\ & \\ m_D & m_N \end{pmatrix} \quad (m_N \gg m_D)$$

$$m_H \simeq m_N + \frac{m_D^2}{m_N} \quad (\text{heavy})$$

$$m_L \simeq -\frac{m_D^2}{m_N} \quad (\text{light})$$

$$\mathrm{Tr}\, M_{\nu N} = m_N = m_L + m_H$$

$$\det M_{\nu N} = -m_D^2 = m_L \cdot m_H$$



$$m_\nu \simeq \frac{m_D^2}{m_N}$$

M_N $(\gg M_D)$

<u>LRSM</u>

$$\begin{pmatrix}\nu\\ e\end{pmatrix}_L \qquad \begin{pmatrix}\nu\\ e\end{pmatrix}_R$$

"natural"

$$m_\nu = -\frac{m_D^2}{m_N} \simeq -\frac{m_e^2}{m_N}$$

$(m_D \sim m_e)$

$$\Downarrow$$

$$|m_N| \simeq \frac{m_e^2}{|m_\nu|} \gtrsim \frac{10^{-6}\,\mathrm{GeV}^2}{10^{-9}\,\mathrm{GeV}}$$

$$\boxed{m_N \gtrsim 10^3\,\mathrm{GeV}} \quad (\simeq \mathrm{TeV})$$

$$m_N = Y_\Delta v_R \simeq Y_\Delta\, 10\,\mathrm{TeV}$$

$$\boxed{\begin{aligned} & v_R \to \infty \Rightarrow m_N \to \infty \\ & \Rightarrow m_\nu \to 0 \\ & \text{(SM limit)} \end{aligned}}$$

$$\Downarrow$$

fine in $\boxed{\text{LR SM}}$ v_R = finite,

but $v_R \gg M_W$

$\Downarrow$

neutrino is light since

$\underline{P}$ is $\boxed{\text{(almost)}}$ maximally

broken

$\Downarrow$

$$\begin{array}{c} \\ \nu_L \\ N_L \end{array}\begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix} \xrightarrow{\text{diag}} \begin{array}{c} \\ \nu_L \\ N_L \end{array}\begin{pmatrix} -m_D^2/m_N & 0 \\ 0 & m_N \end{pmatrix}$$

Majorana! ~~Dirac?~~

~~Both~~?

$$\underbrace{N^T C\, m_N\, N}_{\text{Majorana!}}$$

$\Downarrow$ diagonalise

$$\begin{pmatrix} \nu \\ N \end{pmatrix} \longrightarrow O \begin{pmatrix} \nu \\ N \end{pmatrix}_L = \begin{pmatrix} \nu' \\ N' \end{pmatrix}_L$$

$$O = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \qquad \begin{aligned} c &= \cos\theta \\ s &= \sin\theta \end{aligned}$$

$$\nu_L' = \cos\theta\, \nu_L + \sin\theta\, N_L$$

$$N_L' = -\sin\theta\, \nu_L + \cos\theta\, N_L$$

$$\boxed{\frac{1}{2}\tan 2\theta = \frac{b}{c-a}}$$

$$\begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

PROVE!

$\Downarrow$ seesaw

$$\frac{1}{2}\tan 2\theta \simeq \frac{m_D}{m_N} \ll 1$$

$$\Downarrow$$

$$\theta \simeq m_D / m_N$$

$$\Downarrow$$

$$\boxed{\begin{aligned} \nu' &\simeq \nu + \theta N \\ N' &\simeq -\theta \nu + N \end{aligned}}$$

$$\nu = \nu' - \theta N'$$

$$\Downarrow$$

$$\frac{g}{\sqrt{2}} \bar{\nu}_L \gamma^\mu e_L W_\mu^+ =$$

$$= \frac{g}{\sqrt{2}} \bar{\nu}'_L \gamma^\mu e_L W_\mu^+ - \theta \bar{N}' \gamma^\mu e_L W_\mu^+$$

neutrino

if N' is produced

$$\boxed{\begin{aligned} N' &\to e + W^+ \\ &\to e^c + W^- \end{aligned}}$$

$$\underline{M} = \text{symmetric} = O^T m \, O$$

$\uparrow$ diagonal

$$O = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}$$

$$\Downarrow$$

$$O^T m \, O = \begin{pmatrix} c & -s \\ +s & c \end{pmatrix} \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \begin{pmatrix} c & s \\ -s & c \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ b & c \end{pmatrix} \quad = \dots$$

$\Downarrow$ Check!

$$\boxed{\frac{1}{2} \tan 2\theta = \frac{b}{c-a}}$$

3(n) gen.

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & M_D^T \\ M_D & \mathcal{M}_N \end{pmatrix}$$

$(\mathcal{M}_N \gg M_D)$ seesaw

$$\boxed{\mathcal{M}_{\nu N}^T = \mathcal{M}_{\nu N}} \quad \boxed{M_N^T = M_N}$$

$$U^T \mathcal{M}_{\nu N} U = \begin{pmatrix} \mathcal{M}_\nu & \\ & \mathcal{M}_N \end{pmatrix}$$

Block - diagonalization

$$\mathcal{M}_\nu = ?$$

$$U = \begin{pmatrix} 1 & \theta' \\ \theta & 1 \end{pmatrix}$$

$\theta, \theta' =$ matrices

$$U^+ U = \begin{pmatrix} 1 & \theta^+ \\ \theta'^+ & 1 \end{pmatrix} \begin{pmatrix} 1 & \theta' \\ \theta & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 + \theta^+ \theta & \theta' + \theta^+ \\ \theta'^+ + \theta & 1 + \theta'^+ \theta' \end{pmatrix} = \mathbb{1}$$

$$\boxed{\theta' = -\theta^+}$$

$$\Downarrow$$

$$\bar{U} \simeq \begin{pmatrix} 1 & \Theta^{+} \\ -\Theta & 1 \end{pmatrix}$$

$$U^{T} \simeq \begin{pmatrix} 1 & -\Theta^{T} \\ \Theta^{*} & 1 \end{pmatrix}$$

$$\Downarrow$$

$$\bar{U}^{T} M_{\nu N}\, U = \begin{pmatrix} 1 & -\Theta^{T} \\ \Theta^{*} & 1 \end{pmatrix} \begin{pmatrix} 0 & M_D^{T} \\ M_D & M_N \end{pmatrix}$$

$$\simeq \begin{pmatrix} -\Theta^{T} M_D & \underbrace{M_D^{T} - \Theta^{T} M_N}_{0} \\ M_D & M_N \end{pmatrix} \begin{pmatrix} 1 & \Theta^{+} \\ -\Theta & 1 \end{pmatrix}$$

$$\simeq \begin{pmatrix} -\Theta^{T} M_D & 0 \\ \underbrace{M_D - M_N \Theta}_{} & M_N \end{pmatrix}$$

must vanish

$$\Rightarrow \boxed{M_D = M_N \theta} \quad (1)$$

$$\Downarrow$$

$$\theta^T \underline{M}_N = M_D^T \quad (M_N = M_N^T)$$

$$\Downarrow \boxed{\theta^T = M_D^T \frac{1}{M_N}}$$

$$\boxed{\underline{M}_\nu = - \underline{M}_D^T \frac{1}{M_N} M_D}$$

seesaw neutrino mass

From (1):

$$\boxed{\theta = \frac{1}{M_N} M_D}$$

$$\begin{pmatrix} \nu \\ N \end{pmatrix} \longrightarrow U \begin{pmatrix} \nu \\ N \end{pmatrix} = \begin{pmatrix} \nu' \\ N' \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & \theta^+ \\ -\theta & 1 \end{pmatrix}$$

$$D$$

I will drop '

$$\boxed{\bar{\nu}\, e \to \bar{N}\, \Theta(?)\, e}$$

$$\mathcal{M}_N = V_R\, m_N\, V_R^T$$

$(m_N > 0)$ $\qquad V_R^\dagger V_R = 1$

$$\left(\mathcal{M}_{\nu_R} = \mathcal{M}_R = V_R^*\, m_N^*\, V_R^\dagger \right)$$

$$\mathcal{M}_\nu = V_L^*\, m_\nu\, V_L^\dagger \qquad \left(\mathcal{M}_\nu^T = \mathcal{M}_\nu \right)$$

diagonal $\quad (m_\nu > 0)$

In seesaw:

M_ν = input (from neutrino oscillations)

$M_N \overset{\downarrow}{=}$ input ($\Leftarrow$ LHC)

$\Downarrow$

seesaw formula determines M_D

$\Downarrow$

decay rates!

$$\begin{array}{c} \nu \\ N \end{array}\begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix} \xrightarrow[U]{} \begin{pmatrix} M_\nu & 0 \\ 0 & M_N + \ldots \end{pmatrix}$$

$$\nu'^T C \, M_\nu \, \nu'$$

Majorana mass matrix

$$\Downarrow$$

$$\boxed{\nu, N = \text{Majorana particles}}$$

...

$$M_\nu = - M_D^T \frac{1}{M_N} M_D$$

low E

high E (LHC)

$$\Downarrow$$

$$\boxed{M_D = f(M_\nu, M_N)}$$

CHALLENGE!

$\Downarrow$ diagonalize M_ν, M_N

$$\nu_L' \longrightarrow \nu_M' = \nu_L' + C\bar{\nu}_L'^T$$

Majorana 4-dim.