Neutrino Physics Course

Lecture XV

21/6/2022

LMU

Summer 2022

LRSM: Gauge Interactions

· Summary of lecture $\underline{\overline{XIV}}$

Higgs sector

a) $\Delta_L \longleftrightarrow \Delta_R$

$$\boxed{\begin{array}{c} \Delta_{L,R} \to U_{L,R}\, \Delta_{L,R}\, U^{\dagger}_{L,R} \\ (B-L)\, \Delta_{L,R} = 2\, \Delta_{L,R} \end{array}}$$

$\therefore \quad \langle \Delta_L \rangle = 0, \quad \langle \Delta_R \rangle = v_R$

(def. of LR) $\qquad\qquad \simeq M_{W_R}$

b) $\Phi \longrightarrow U_L \Phi U_R^+$

$\langle \Phi \rangle \simeq M_{W_L}$ (SM)

$$\bar{V} = \bar{V}_\Phi + \bar{V}_\Delta +$$

$$+ \quad \bar{V}_{\Phi\Delta}$$

$$\parallel$$

$$\alpha_1 (\mathrm{Tr}\, \Delta_L^+ \Delta_L + L \leftrightarrow R)\ \mathrm{Tr}\, \Phi^+ \Phi +$$

$$\alpha_2 \quad -\,\|\,- \qquad (\det \Phi + h.c.)$$

$$+$$

$$\underbrace{\alpha_3 \left(\mathrm{Tr}\, \Phi \Phi^+ \Delta_L^+ \Delta_L + L \leftrightarrow R \right)}_{\not\!/}$$

$$\Downarrow$$

$$\Phi = (\phi_1 \ \tilde{\phi}_2) \quad \therefore \langle \phi_i \rangle = v_i$$

$$\phi = (v_1 \phi_1 + v_2 \phi_2) N \quad \therefore \quad \langle \phi \rangle \neq 0$$

$$\phi' \perp v \quad \therefore \quad \langle \phi' \rangle = 0$$

$$\Downarrow$$

$$\boxed{\underbrace{m_{\phi'}^2 = m_H^2}_{\text{heavy scalar doublet}} = \alpha_3 v_R^2}$$

$$\therefore \phi = \text{SM doublet} = \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

SM Higgs

- $m_{\Delta_L}^2 \simeq v_R^2 \leftarrow$ heavy LH Triplet

- $(m_{\delta_R^{++}}, \; m_{\delta_R^0}) \propto v_R$

LR Higgs

$$\boxed{\text{What about } \langle\phi\rangle \text{ in } \Delta_L, \Delta_R, H \text{ sector?}}$$

$\Downarrow$

there can be a small split ($\sim M_W$) in the Δ_L, H multiplets

Flavour Violation

H: $\widetilde{FV}$ in neutral current

(NCFV)

$\Longrightarrow$ $\boxed{m_H > 10\ \text{TeV}}$

small split NOT relevant

Δ_L: $m_{\Delta_L} \gtrsim$ few hundred GeV a small split?

yes, it may matter

$$\langle \Delta_R \rangle \gg \langle \Phi \rangle$$

$$\wr \qquad\qquad \wr$$

$$M_{W_R} \gg M_{W_L}$$

$(> 5\,\mathrm{TeV})$ $\qquad$ $(\sim 80\,\mathrm{GeV})$

Treat $\langle \Phi \rangle$ as a perturbation

→ back to gauge interactions

$$G_{LR} = SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$W_L^{\pm} \longleftrightarrow W_R^{\pm}$$

$$Z\ (Z_L) \longleftrightarrow Z'\ (Z_R)$$

A (photon)

LR symmetric ?

SM digression

$$SU(2)_L \times U_Y(1) \longrightarrow U(1)_{em}$$

$$\Uparrow$$

whatever Higgs used

$$\Downarrow$$

$\exists\ A_\mu$ (photon) $\longleftrightarrow$ Q_{em}
$(m_A = 0)$

$$\Downarrow$$

$\exists\ Z\ (M_Z \neq 0) \longleftrightarrow Q_Z$

$$\Uparrow$$

$$\boxed{\begin{aligned} A &= \sin\theta_W\, A_3 + \cos\theta_W\, B \\ Z \perp A &= \cos\theta_W\, A_3 - \sin\theta_W\, B \end{aligned}} \qquad (0)$$

(g) $SU(2)$: A_i $(i = 1, 2, 3)$

(g') $U_Y(1)$: B

$$\Downarrow$$

$$\boxed{\begin{aligned} \tan\theta_W &= g'/g \\ Q_{em} &= T_3 + Y/2 \end{aligned}}$$

$$\therefore \quad A_\mu e \bar{f} \gamma^\mu Q_{em} f \qquad \boxed{e = g \sin\theta_W}$$

$$Z_\mu \frac{g}{\cos\theta_W} \bar{f} \gamma^\mu \underbrace{(T_3 - Q \sin^2\theta_W)}_{Q_Z} f$$

$$\tan\theta_W = g'/g \Rightarrow g \sin\theta_W = g' \cos\theta_W$$

$$\Downarrow$$

$$\left(\frac{e}{g}\right)^2 = \sin^2\theta_W \qquad \left(\frac{e}{g'}\right)^2 = \cos^2\theta_W$$

$$\Downarrow$$

$$\boxed{\frac{e^2}{g^2} + \frac{e^2}{g'^2} = 1}$$

$$\boxed{\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}} \quad (1)$$

$\Updownarrow$

$$\begin{array}{ccccc} SU(2)_L & \times & U_Y(1) & \longrightarrow & U_{em}(1) \\ g & & g' & & e \end{array}$$

$$\boxed{e = \frac{g g'}{\sqrt{g^2 + g'^2}}}$$

$\Downarrow$

$$\boxed{g_L = g_R \equiv g \quad \text{in LR}}$$

$$\underbrace{SU(2)_R \times U(1)_{B-L}}_{g \qquad g_{BL}} \xrightarrow{M_R} \underbrace{U(1)_Y}_{g'}$$

Independent of the Higgs

$$\Updownarrow$$

$$G_{LR} \xrightarrow{M_R} G_{SM}$$

$\Downarrow$ analogy with (1)

$$\boxed{\frac{1}{g'^2} = \frac{1}{g^2} + \frac{1}{g_{BL}^2}} \qquad (2)$$

· Notation :

SM $\underbrace{A_i \longrightarrow A_{iL}}_{SU(2)_L}$ $(i = 1, 2, 3)$

$+ \ A_{iR}$ from $SU(2)_R$

$+ \ B_{BL} \longleftarrow U(1)_{BL}$

$\Downarrow$ at M_R (analogy with (0))

$$B = \sin\theta_R \, A_{3R} + \cos\theta_R \, B_{BL}$$

$(m_B = 0)$

$$\boxed{Z' = \cos\theta_R \; A_{3R} - \sin\theta_R \; B_{BL}}$$

$\therefore$

$$\boxed{\tan\theta_R = \frac{g_{BL}}{g}}$$

$$\boxed{\frac{1}{g'^2} = \frac{1}{g^2} + \frac{1}{g_{BL}^2}} \qquad (2)$$

$\Downarrow$

$$\sin\theta_R = \frac{g_{BL}}{\sqrt{g^2 + g_{BL}^2}}, \quad \cos\theta_R = \frac{g}{\sqrt{g^2 + g_{BL}^2}}$$

from (2):

$$g' = \frac{g\, g_{BL}}{\sqrt{g^2 + g_{BL}^2}}$$

$$\Rightarrow \boxed{\underbrace{g'/g}_{\tan\theta_W} = \underbrace{\frac{g_{BL}}{\sqrt{g^2 + g_{BL}^2}}}_{\sin\theta_R}}$$

$$\Downarrow$$

★★★ $$\boxed{\sin\theta_R = \tan\theta_W} \quad (3)$$

$$- \theta_W \simeq 30^\circ \Rightarrow \sin\theta_W \simeq 1/2$$

$$\cos\theta_W \simeq \sqrt{3}/2$$

$$\boxed{\tan\theta_W \simeq 1/\sqrt{3} < 1}$$

From (3)

$$\boxed{Z'(Z_R) = \sqrt{1-\tan^2\theta_W}\, A_{3R} - \tan\theta_W\, B_{BL}'}$$

$$B = \tan\theta_W\, A_{3R} + \sqrt{1-\tan^2\theta_W}\, B_{BL}$$

||

$$\Downarrow \quad Q = T_{3L} + T_{3R} + \frac{B-L}{2}$$

$$A = \sin\theta_W\, A_{3L} + \cos\theta_W \left(\tan\theta_W\, A_{3R} + \sqrt{1-\tan^2\theta_W}\; B_{BL} \right)$$

$$\boxed{A = \sin\theta_W \left(A_{3L} + A_{3R} \right) + \sqrt{\cos 2\theta_W}\, B_{BL}}$$

LR symmetric (Photon)

$$Z = \cos\theta_W\, A_{3L} - \sin\theta_W \left(\tan\theta_W\, A_{3R} + \sqrt{1-\tan^2\theta_W}\; B_{BL} \right)$$

$$\Downarrow$$

$$Z = \frac{\cos^2\theta_W A_{3L} - \sin^2\theta_W A_{3R}}{\cos\theta_W} - \sin\theta_W \sqrt{1-\tan^2\theta_W}\, B_{BL}$$

(Z_L)

NO $Z \leftrightarrow Z'$ symmetry

SM Z boson

$\Downarrow$ interactions

NC

$$\bar{f}\gamma^\mu \left[g T_{3L} A_{3L} + g T_{3R} A_{3R} + g_{BL} \frac{B-L}{2} B_{BL} \right]_\mu f$$

$$\Uparrow$$

$$D_\mu f = \left(\partial_\mu - i g\, T_{3L} A_{3L\mu} - i g\, T_{3R} A_{3R\mu} - i g_{BL} \frac{B-L}{2} B_{BL\mu}\right) f$$

$$\boxed{\begin{aligned} A, Z, Z' &= f(A_{3L}, A_{3R}, B_{BL}) \\ A_{3L}, A_{3R}, B_{BL} &= f^{-1}(A, Z, Z') \end{aligned}}$$

$$\Updownarrow$$

$$\boxed{V' = O\, V} \qquad (O O^T = O^T O = 1)$$

$$\Updownarrow$$

$$V_i' = O_{ij} \, V_j$$

$$\Rightarrow \quad V = O^T V'$$

$$\boxed{\begin{aligned} V_i &= (O^T)_{ik} \, V_k' \\ &= O_{ki} \, V_k' \end{aligned}}$$

$$V_i' = O_{ij} \, V_j , \qquad V_i = O_{ji} \, V_j'$$

$$\downarrow \qquad \Downarrow$$

$$V_1' = O_{11} \, V_1 + O_{12} \, V_2 + O_{13} V_3$$

$$V_2' = O_{21} \, V_1 + O_{22} \, V_2 + O_{23} \, V_3$$

$$V_3' = O_{31} V_1 + O_{32} V_2 + O_{33} V_3$$

$$\bar{V}_1 = O_{11} \bar{V}_1' + O_{21} \bar{V}_2' + O_{31} \bar{V}_3'$$

$$\bar{V}_2 = O_{12} \bar{V}_1' + O_{22} \bar{V}_2' + O_{32} \bar{V}_3'$$

$$\bar{V}_3 = O_{13} \bar{V}_1' + O_{23} \bar{V}_2' + O_{33} \bar{V}_3'$$

$$\Downarrow$$

$$A_{3L} = \sin\theta_W A + \cos\theta_W Z + 0 \cdot Z'$$

$$A_{3R} = \sin\theta_W A - \sin\theta_W \tan\theta_W Z + \sqrt{1-\tan^2\theta_W}\, Z'$$

$$B_{BL} = \sqrt{\cos 2\theta_W}\, A - \sin\theta_W \sqrt{1-\tan^2\theta_W}\, Z - \tan\theta_W\, Z'$$

$$g\, T_{3L} \left(\sin\theta_W\, A + \cos\theta_W\, Z \right) +$$

$$+ g\, T_{3R} \left(\sin\theta_W\, A - \sin\theta_W \tan\theta_W\, Z + \sqrt{1-\tan^2\theta_W}\, Z' \right)$$

$$+ g_{BL} \frac{B-L}{2} \left(\sqrt{\cos 2\theta_W}\, A - \sin\theta_W \sqrt{1-\tan^2\theta_W}\, Z - \tan\theta_W\, Z' \right)$$

$$g_{BL} = g \tan\theta_R = g \frac{\sin\theta_R}{\cos\theta_R}$$

$$\Downarrow$$

$$\boxed{g_{BL} = g \frac{\tan\theta_W}{\sqrt{1-\tan\theta_W}}}$$

$$\boxed{\frac{B-L}{2} = Q - T_{3L} - T_{3R}}$$

$$\Downarrow$$

$$A\left[g \sin\theta_W \left(T_{3L} + T_{3R}\right) + \right.$$

$$\left. + g \frac{\tan\theta_W}{\sqrt{1-\tan\theta_W}} \sqrt{\cos 2\theta_W} \; \frac{B-L}{2}\right]$$

$$\parallel$$

$$\left(\frac{\sin\theta_W}{\sqrt{1-\tan^2\theta_W}} \frac{\sqrt{\cos^2\theta_W - \sin^2\theta_W}}{\cos\theta_W} \right.$$

$$= \frac{\sin\theta_W}{\cancel{\sqrt{1-\tan^2\theta_W}}} \cancel{\sqrt{1-\tan^2\theta_W}}$$

$$\left. = \sin\theta_W \right)$$

$$\Downarrow$$

$$A_\mu \bar{f} \gamma^\mu \, g \sin\theta_W \underbrace{\left(T_{3L} + T_{3R} + \frac{B-L}{2} \right)}_{Q_{em}} f$$

$$= e A_\mu \bar{f} \gamma^\mu Q_{em} f \quad \text{※}$$

$$\boxed{e = g \sin\theta_W}$$

$$\bullet \; Z_\mu \Big(g\, T_{3L} \cos\theta_W - g\, T_{3R} \sin\theta_W \tan\theta_W$$

$$- g_{BL} \frac{B-L}{2} \sin\theta_W \sqrt{1-\tan^2\theta_W} \Big)$$

$$= Z_\mu \Big(g\, T_{3L} \cos\theta_W \; -$$

$$- g\, T_{3R} \sin\theta_W \tan\theta_W \; -$$

$$- \frac{B-L}{2} g \sin\theta_W \tan\theta_W \frac{1}{\cancel{\sqrt{1-\tan^2\theta_W}}} \cancel{\sqrt{1-\tan^2\theta_W}} \Big)$$

$$= Z_\mu \Big(g\, T_{3L} \cos\theta_W - g \sin\theta_W \tan\theta_W$$

$$\left(T_{3R} + \frac{B-L}{2}\right)\Big)$$

$$= Z_\mu \Big(g\, T_{3L} \cos\theta_W - g \sin\theta_W \tan\theta_W \, (Q - T_{3L}) \Big)$$

$$= Z_\mu\, g \left(T_{3L} \left(\cos\theta_W + \frac{\sin^2\theta_W}{\cos\theta_W} \right) - \frac{\sin^2\theta_W}{\cos\theta_W}\, Q \right)$$

$$\Downarrow$$

$$\boxed{Z_\mu \frac{g}{\cos\theta_W} \bar{f} \gamma^\mu \left(T_{3L} - \sin^2\theta_W\, Q \right) f}$$

$$\Downarrow$$

no T_{3R}

$$Z'_\mu \frac{g}{\sqrt{1-\tan^2\theta_W}} \bar{f}\gamma^\mu \left[\tan^2\theta_W (T_{3L} - Q) + T_{3R}\right] f$$

Check!

HW

$\boxed{\text{use } \langle \Delta_R \rangle}$

$$\frac{M_{Z_R}^2}{M_{W_R}^2} = \,?$$

$Z_R \equiv Z'$

$\boxed{A, Z, Z' = \text{mass eigenstates}}$

$\Uparrow$

$A, Z \Leftarrow$ SM knowledge

$\oplus$ $Z' \perp A \perp Z \perp Z'$

SM $\quad \exists\, A \longleftrightarrow Q_{em}$ Glashow 1961

$\Leftrightarrow \quad e A_\mu \bar{f} \gamma^\mu Q_{em} f$ (def.)

(def. of photon)

$\Downarrow$

$$A = \sin\theta_W A_3 + \cos\theta_W B$$

$$\Downarrow$$

$$\exists\ Z \perp A \Rightarrow$$

$$Z = \cos\theta_W A_3 - \sin\theta_W B$$

$$\Downarrow$$

rest is history!

Weinberg 1967

ϕ

$$\Rightarrow \boxed{M_W = M_Z \cos\theta_W}$$