Neutrino Physics Course

Lecture XIII

14/6/2022

LMU

Summer 2022

LRSM: SSB (2)

$$G_{LR} \xrightarrow[\langle \Delta_R \rangle]{} G_{SM}$$

$$\Delta_L \longleftrightarrow \Delta_R$$

$$V = -\frac{\mu_\Delta^2}{2}\left(\mathrm{Tr}\,\Delta_L^\dagger \Delta_L + \mathrm{Tr}\,\Delta_R^\dagger \Delta_R\right)$$

$$+ \frac{\rho_1}{4}\left(\left(\mathrm{Tr}\,\Delta_L^\dagger \Delta_L\right)^2 + L \leftrightarrow R\right)$$

$$+ \frac{\rho_2}{2}\left(\mathrm{Tr}\,\Delta_L^2\, \mathrm{Tr}\,\Delta_L^{\dagger 2} + L \leftrightarrow R\right)$$

$$+ \frac{\rho_3}{2}\, \mathrm{Tr}\,\Delta_L^\dagger \Delta_L\, \mathrm{Tr}\,\Delta_R^\dagger \Delta_R \leftarrow$$

$$\Big\{ + \frac{\rho_4}{2}\left(\mathrm{Tr}\,\Delta_L^2\,\mathrm{Tr}\,\Delta_R^{+2} + L \leftrightarrow R\right)$$

(a) If $\langle \Delta_L \rangle = 0$, $\Rightarrow \langle \Delta_R \rangle \neq 0$

$$\Rightarrow \boxed{\rho_2 > 0} \quad \text{i.e.}$$

$$\boxed{\langle \Delta_R \rangle = v_R \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} \frac{1}{2}}$$

$$\Uparrow$$

$$\Delta_R = \Delta_1 + i\,\Delta_2\,, \quad \Delta_i^+ = \Delta_i$$

$(i = 1, 2)$

$$\Downarrow$$

$$\langle D_R \rangle = \begin{pmatrix} z & ir \\ ir & -z \end{pmatrix}, \quad r \in R$$

Then : $\rho_2 > 0 \Rightarrow z = r$

$$\frac{1}{2}\begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} = U_R^+ \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} U_R$$

$$U_R = \begin{pmatrix} \cos\theta\, e^{i\alpha} & \sin\theta\, e^{i\beta} \\ -\sin\theta\, e^{-i\beta} & \cos\theta\, e^{-i\alpha} \end{pmatrix}$$

$$\Uparrow$$

$\boxed{\text{SM}}$

$$\left.\begin{aligned} &\langle \phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \\ \Leftrightarrow\ &\langle \phi' \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix} \end{aligned}\right\}$$

$\boxed{\text{back to LR}}$

Q'_{em}

$$\langle \Delta_R' \rangle = \frac{v_R}{2} \begin{pmatrix} 1 & i \\ i & -i \end{pmatrix}$$

$=$ global minimum for $\rho_2 > 0$

<u>now:</u> $\Leftrightarrow$ $$\boxed{\langle \Delta_R \rangle = v_R \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}}$$

Q_{em}

$$\boxed{(B-L)\,\Delta_R = 2\,\Delta_R}$$

$$Q_{ew} = T_{3_L} + T_{3_R} + \frac{B-L}{2}$$

$$\Downarrow$$

$$\Delta_{L,R} = \begin{pmatrix} \frac{1}{\sqrt{2}}\delta^+ & \delta^{++} \\ \delta^0 & -\frac{1}{\sqrt{2}}\delta^+ \end{pmatrix}$$

$$\left\{ \begin{array}{l} \langle \Delta \rangle = \begin{pmatrix} 0 & 0 \\ v & 0 \end{pmatrix} \\ \Rightarrow \langle \Delta^2 \rangle = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow \mathrm{Tr}\langle \Delta \rangle^2 = 0 \\ \qquad\qquad\qquad\qquad \rho_2 \text{ term} \end{array} \right\}$$

$$\Downarrow$$

$$V \to -\frac{\mu_\Delta^2}{2}\left(\mathrm{Tr}\,\Delta_L^\dagger \Delta_L + L \leftrightarrow R\right)$$

$$+ \frac{\rho_1}{4}\left[\left(\mathrm{Tr}\,\Delta_L^\dagger \Delta_L\right)^2 + L \leftrightarrow R\right]$$

$$+ \frac{\rho_3}{2}\,\mathrm{Tr}\,\Delta_L^\dagger \Delta_L\,\mathrm{Tr}\,\Delta_R^\dagger \Delta_R$$

$$= -\frac{\mu_\Delta^2}{2} \cdots \quad +$$

$$+ \frac{\rho_1}{4}\left(\mathrm{Tr}\,\Delta_L^\dagger \Delta_L + \mathrm{Tr}\,\Delta_R^\dagger \Delta_R\right)^2$$

$$+ \frac{\rho_3 - \rho_1}{2}\,\mathrm{Tr}\,\Delta_L^\dagger \Delta_L\,\mathrm{Tr}\,\Delta_R^\dagger \Delta_R$$

$$\boxed{\rho_3 - \rho_1 > 0 \Rightarrow \langle \Delta_L \rangle = 0}$$

$\Rightarrow$ Check!

$$\boxed{\text{Mass spectrum}}$$

Minimum:

$$\langle \Delta_L \rangle = 0, \quad \langle \Delta_R \rangle = v_R \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\mathrm{Tr}\, \Delta^+ \Delta = \ ?$$

$$\Delta = \begin{pmatrix} \frac{1}{\sqrt{2}}\delta^+ & \delta^{++} \\ \delta^0 & -\frac{1}{\sqrt{2}}\delta^+ \end{pmatrix}$$

$$\Delta^+ = \begin{pmatrix} \frac{1}{\sqrt{2}}\delta^- & \delta^{0*} \\ \delta^{--} & -\frac{1}{\sqrt{2}}\delta^- \end{pmatrix}$$

$$\Downarrow \qquad \delta_{0R} = v_R + h_R + i\, G_R$$

$$\mathrm{Tr}\, \Delta^\dagger \Delta = \frac{1}{2}\delta^+\delta^- + \delta^{++}\delta^{--} + |\delta_0|^2 + \frac{1}{2}\delta^+\delta^-$$

$$= \underbrace{|\delta_0|^2 + \delta^+\delta^- + \delta^{++}\delta^{--}}$$

$$\Rightarrow \boxed{\mathrm{Tr}\, \Delta_L^\dagger \Delta_L = (\qquad)_L}$$

$$\mathrm{Tr}\, \Delta_R^\dagger \Delta_R = (v_R + h_R)^2 + G_R^2 + \delta_R^+\delta_R^- + \delta_R^{++}\delta_R^{--}$$

$$= \underline{\underline{2 v_R h_R}} + v_R^2 + \underbrace{h_R^2 + \cdot \, \text{''} -}_{\text{squares of fields}}$$

$$\Downarrow$$

$$V \longrightarrow -\frac{\mu_\Delta^2}{2}\left(\underline{T_\nu \Delta_L^\dagger \Delta_L} + h_R^2 + \sigma_R^2 + |\delta_R^+|^2 + |\delta_R^{++}|^2 + \vartheta_R^2\right)$$

$$+ \frac{\rho_1}{4}\left\{2\vartheta_R^2\left(h_R^2 + \sigma_R^2 + \delta_R^+\delta_R^- + \delta_R^{++}\delta_R^{--}\right) + \vartheta_R^4 + \boxed{\frac{\rho_1}{4}\, 4\vartheta_R^2 h_R^2}\right\} + \frac{\rho_3}{2}\,\underline{T_\nu \Delta_L^\dagger \Delta_L\, \vartheta_R^2}$$

$$\frac{\partial V}{\partial \vartheta_R} = \left(-\mu_\Delta^2 + \rho_1 \vartheta_R^2\right)\vartheta_R = 0$$

$$\Downarrow$$

$$\boxed{\vartheta_R^2 = \mu_\Delta^2/\rho_1} \qquad (1)$$

$$\Downarrow$$

$$\boxed{m_{\Delta_L}^2 = \left(-\mu_\Delta^2 + \rho_3 \vartheta_R^2\right) = (\rho_3 - \rho_1)\,\vartheta_R^2}$$

$$\Rightarrow \boxed{\rho_3 - \rho_1 > 0}$$

$$\boxed{\Delta_L = \text{degenerate}}$$

$$\mathrm{Tr}\, \Delta_L^\dagger \Delta_L = |\delta_L^0|^2 + \delta_L^+ \delta_L^- + \delta_L^{++} \delta_L^{--}$$

$v_L = 0 \Rightarrow SU(2)_L$ unbroken

$\Downarrow$

all $SU(2)_L$ multiplets are degenerate

1) $h_R^2 + \sigma_R^2 + \delta_R^+ \delta_R^- + \delta_R^{++} \delta_R^{--}$!

$$\left(-\mu_\Delta^2 + \rho_1 v_R^2\right) = 0$$

2) $\rho_1 v_R^2 h_R^2 \Rightarrow \boxed{m_{h_R}^2 = 2 \rho_1 v_R^2}$

Higgs of $SU(2)_R$ breaking

σ_R = massless

δ_R^+ = — " —

eaten by Z_R and W_R^+

Forgot:

$$\Delta_R = \begin{pmatrix} \delta_R^+ \frac{1}{\sqrt{2}} & \delta_R^{++} \\ \delta_R^0 & \delta_R^+ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\Downarrow$$

$$\mathrm{Tr}\, \Delta_R^2 = \delta_R^0 \delta_R^{++} + \cdots$$

$$\mathrm{Tr}\, \Delta_R^2\, \mathrm{Tr}\, \Delta_R^{+2} = |\delta_R^0|^2 \delta_R^{++} \delta_R^{--}$$

$$\longrightarrow v_R^2 \delta_R^{++} \delta_R^{--}$$

$$\Downarrow$$

$$\boxed{m_{\delta_R^{++}}^2 = \rho_2 v_R^2} \quad (\rho_2 > 0)$$

Summary

$$\langle \Delta_R \rangle = v_R \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \langle \Delta_L \rangle = 0$$

$$Q_{em}\langle \Delta_R \rangle = 0$$

but $$Q_{em} = T_{3L} + \underbrace{T_{3R} + \frac{B-L}{2}}_{Y/2}$$

$$T_{3L}\langle \Delta_R \rangle = 0$$

$$\Rightarrow \underbrace{\left(T_{3R} + \frac{B-L}{2}\right)}_{Y/2}\langle \Delta_R \rangle = 0$$

$$\boxed{SU(2)_R \times U(1)_{B-L} \longrightarrow U(1)_Y}$$

$$\Downarrow$$

$$(A_{1R},\, A_{2R},\, A_{3R};\; B_{B-L})$$

$$W_R^{\pm} = \frac{A_{1R} \mp i A_{2R}}{\sqrt{2}} \quad \therefore \quad M_{W_R} = g v_R$$

$W_R^{\pm}$ must eat $\delta_R^{\pm}$

analogy

$$W_L^{\pm} = \frac{A_{1L} \mp i A_{2L}}{\sqrt{2}} \quad \therefore \quad M_{W_L} = g v_L$$

$\boxed{SM}$

$$A = \sin\theta_W A_{3L} + \cos\theta_W B$$

photon

$$Z = \cos\theta_W A_{3L} - \sin\theta_W B$$

$$\tan\theta_W = g'/g$$

$$\Updownarrow$$

$$A = \frac{g'}{\sqrt{g^2+g'^2}} A_{3L} + \frac{g}{\sqrt{g^2+g'^2}} B$$

$$Z = \frac{g}{\sqrt{\cdots}} A_{3L} - \frac{g'}{\sqrt{\cdots}} B$$

$\Updownarrow$ analogy

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$
$$g \qquad g \qquad \bar{g}$$

$$\Downarrow$$

$$B = \frac{\bar{g}}{\sqrt{g^2+\bar{g}^2}} A_{3R} + \frac{g}{\sqrt{g^2+\bar{g}^2}} \bar{B}$$

$$Z' = \frac{g}{\sqrt{g^2+\bar{g}^2}} A_{3R} - \frac{\bar{g}}{\sqrt{g^2+\bar{g}^2}} \bar{B}$$

$M_B = 0$ ($\Leftarrow$ $M_A = 0$ in SM)

$M_{Z'} \propto v_R \neq 0$ $\boxed{\text{COMPUTE}}$

$Z' \equiv Z_R$ (must eat G_R)

$$\Downarrow$$

$$\boxed{m_{G_R} = m_{\delta_R^+} = m_{\delta_R^-} = 0}$$

$$m_{h_R}^2 = 2\rho_1 v_R^2$$

$$m_{\delta_R^{++}}^2 = \rho_2 v_R^2$$

$$m_{\Delta_L}^2 = (\rho_3 - \rho_1) v_R^2$$

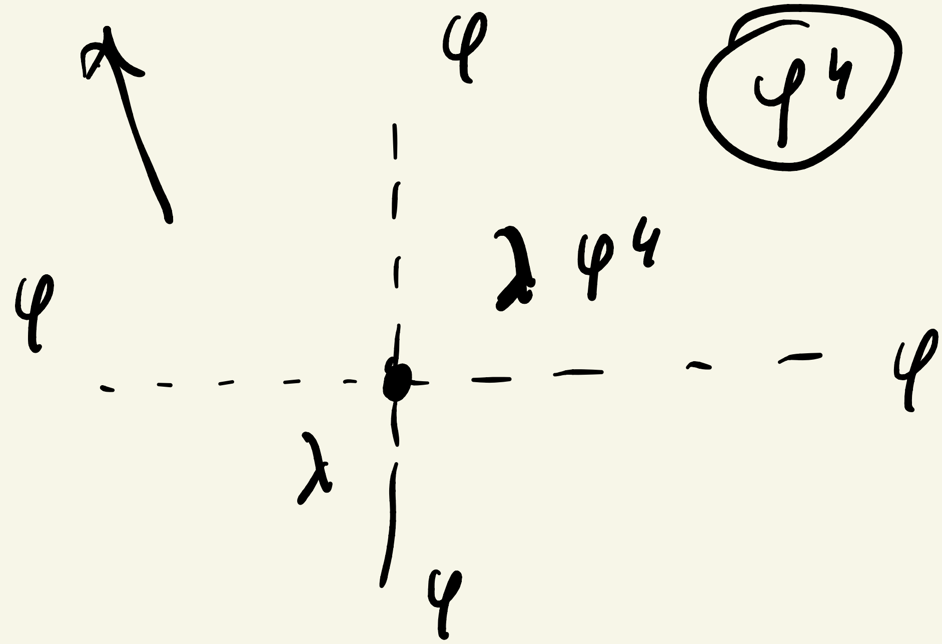
+ loop corrections

Coleman-Weinberg

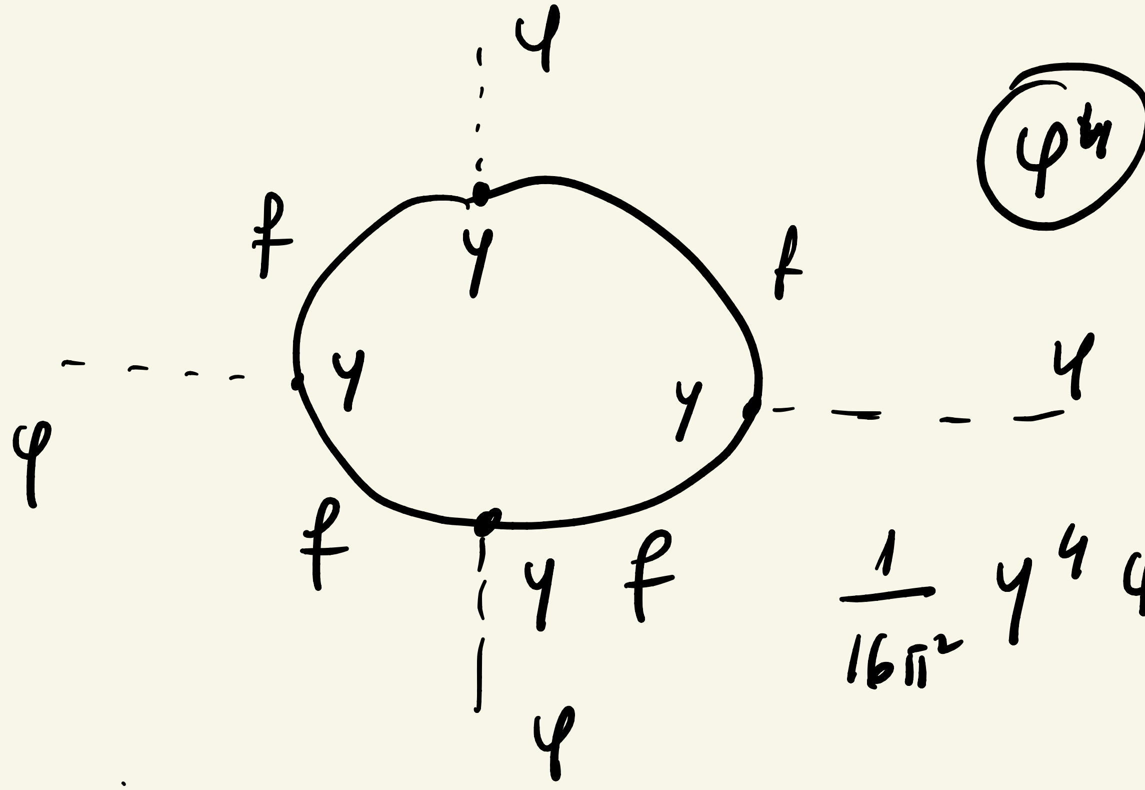
1972

SSB

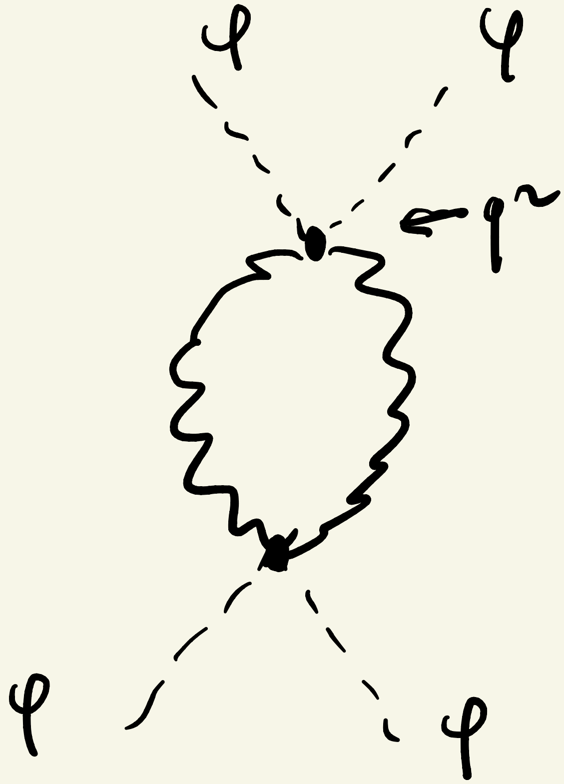
$\boxed{\text{tree}}$



$\Downarrow$ $\boxed{\text{loop}}$



$\frac{1}{16\pi^2} y^4 \varphi^4$



$$\frac{1}{16\pi^2} g^4 \varphi^4$$

relevant if: $\frac{g^4}{16\pi^2} \gtrsim \lambda$

must include loops

SM: $m_h^2 = 2\lambda v^2 > m_W^2$

$m_H \simeq 125$ GeV

$$\boxed{\text{in SM} \Rightarrow \text{tree level suffices}}$$

- even if $\lambda \to 0 \Rightarrow$

still pert. theory is good

but

starts at 1-loop!

- $G_{LR} \xrightarrow[\langle \Delta_R \rangle]{} G_{SM}$ ✓

- $G_{SM} \xrightarrow[\langle \Phi \rangle]{} U(1)_{em}$

$$\tilde{\phi} \equiv i\sigma_2 \phi^*$$

$$\Phi = \left(\phi_1 \quad \tilde{\phi}_2 \right)$$

$$= \begin{pmatrix} \varphi_1^0 & \varphi_2^+ \\ \varphi_1^- & -\varphi_2^{0*} \end{pmatrix}$$

- $\langle \Phi \rangle = ?$

Q. Can I choose by rotation:

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & -v_2^* \end{pmatrix} ?$$

$$G_{SM} = SU(2)_L \times U(1)_Y$$

$\Downarrow$ rotation

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & v_2 \sin\theta \\ 0 & v_2 \cos\theta \end{pmatrix}$$

doublet $\rightarrow \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$

$\Rightarrow$ $\boxed{\theta = 0 \text{ in order to conserve } Q_{em}}$

$\downarrow$

$$\langle \phi_i \rangle = v_i$$

$$\Downarrow$$

$$\phi = (v_1 \phi_1 + v_2 \phi_2) N$$

$$\phi' = (v_2 \phi_1 - v_1 \phi_2) N$$

$$\langle \phi' \rangle = 0, \quad \langle \phi \rangle = (v_1^2 + v_2^2) N$$

but: $$N = \frac{1}{\sqrt{v_1^2 + v_2^2}}$$

$$\Rightarrow \langle \phi \rangle = \sqrt{v_1^2 + v_2^2} \equiv v^2$$

~ SM doublet

· SU(2) Adjoint

$$A \longrightarrow U A U^{+} \quad \therefore$$

$$\mathrm{Tr}\, A = 0\ , \quad A = A^{+}$$

$$\Downarrow$$

$\mathrm{Tr}\, A^2 =$ only invariant

but

$$A_R \longrightarrow U_R A_R U_R^{+} \sim A_R$$

$$\boxed{A_R = A_{1R} + i A_{2R}}$$

$$= A_{1R} + i A_{2R}$$

(two Adjoints)

$$\Leftrightarrow \quad z = x + iy$$