# Neutrino Physics Course

## Lecture XI

7/6/2022

LMU

Summer 2022

# LRSM: Spontaneous $\not{P}$

$$g \equiv g_L = g_R \qquad \bar{g}$$

- $$G_{LR} = SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$P$$

Pati, Salam '74

Mohapatra, G.S. '75

- $$f_L \left\{ \begin{array}{c} \begin{pmatrix} u \\ d \end{pmatrix}_L \\ \\ \begin{pmatrix} \nu \\ e \end{pmatrix}_L \end{array} \right. \longleftrightarrow \left. \begin{array}{c} \begin{pmatrix} u \\ d \end{pmatrix}_R \\ \\ \begin{pmatrix} \nu \\ e \end{pmatrix}_R \end{array} \right\} f_R$$

$$\Downarrow$$

$$\boxed{\exists\, \nu_R \Rightarrow m_\nu \neq 0}$$

before Higgsing

$$\Downarrow$$

$$m_f = 0$$

$$\Downarrow$$

$$\exists \; \Phi \; \because$$

$$\mathcal{L}_Y = \bar{f}_L \left( Y_\Phi \, \Phi + \tilde{Y}_\Phi \, \tilde{\Phi} \right) f_R + h.c.$$

$$\Downarrow$$

$$\Phi \to U_L \, \Phi \, U_R^\dagger$$ bi-doublet

$$\tilde{\Phi} \to \sigma_2 \, \Phi^* \sigma_2$$

$$\therefore \quad \tilde{\Phi} \to U_L \, \tilde{\Phi} \, U_R^\dagger$$

$$\boxed{\underset{Y}{SU(2)_L \times U(1)} \longrightarrow \underset{em}{U(1)} \qquad M_W = \langle \Phi \rangle}$$

but

$$SU(2)_L \times \underbrace{SU(2)_R \times U(1)}_{B-L}$$

$$\Big\downarrow \langle \Delta_R \rangle = M_R$$

$$U(1)_Y$$

$$\Downarrow$$

if $\Delta_R \Rightarrow \Delta_L$

$\Delta_R = SU(2)_L$ singlet

$\Delta_L = SU(2)_R$ —〃—

$$\Downarrow$$

at $\mu_R$ : $\left.\begin{array}{l} \langle \Delta_L \rangle = 0 \\ \langle \Delta_R \rangle \neq 0 \end{array}\right\}$ $\cancel{P}$ SSB

$\Uparrow$ repr. of $SU(2)$

$$\underbrace{\Delta^+ \Delta , \quad \mathrm{Tr}\, \Delta^+ \Delta , \text{ or } ---}_{\Delta^2 \text{ symbolically}}$$

$$\Downarrow$$

$$V(\Delta) = -\mu^2 \left( \Delta_L^2 + \Delta_R^2 \right) +$$

$$+ \frac{\lambda}{4} \left( \Delta_L^4 + \Delta_R^4 \right) + \frac{\lambda'}{2} \Delta_L^2 \Delta_R^2$$

$$= -\mu^2 (\Delta_L^2 + \Delta_R^2) + \frac{\lambda}{4} (\Delta_L^2 + \Delta_R^2)^2$$

$$\Rightarrow \quad + \frac{\lambda' - \lambda}{2} \Delta_L^2 \Delta_R^2 \leftarrow$$

(a) $\lambda' - \lambda = 0$

$$\Downarrow$$

$$\langle \Delta_L^2 + \Delta_R^2 \rangle = \mu^2 / \lambda$$

to see : $\Delta^2 = \Delta_L^2 + \Delta_R^2$

$$\Downarrow$$

$$V (\lambda' = \lambda) = -\mu^2 \Delta^2 + \frac{\lambda}{4} \Delta^4$$

$$\frac{\partial V}{\partial \Delta} = (-\mu^2 + \lambda \Delta^2) \Delta = 0$$

$$\Downarrow$$

$\langle\Delta\rangle = 0 \Rightarrow$ maximum

$\langle\Delta\rangle^2 = \mu^2/\lambda \Rightarrow$ (local) minimum

$\Downarrow$

$$\underbrace{\langle \Delta_L^2 + \Delta_R^2 \rangle}_{} \neq 0$$

flat direction

(b) $\lambda' - \lambda \neq 0$

(b1) $\boxed{\lambda' - \lambda > 0}$ ; (b2) $\lambda' - \lambda < 0$

$\Downarrow$ $\Downarrow$

$\langle \Delta_L^2 \Delta_R^2 \rangle = 0$ $\langle\Delta_L\rangle \neq 0 \neq \langle\Delta_R\rangle$

$\Downarrow$ $\Downarrow$

$\langle\Delta_L\rangle = \langle\Delta_R\rangle$

$$\left.\begin{array}{l}\langle \Delta_L \rangle = 0 \\ \langle \Delta_R \rangle = v_R \neq 0\end{array}\right\} \underline{\underline{\text{definition of } L}}$$

# OUR WORLD

- $\boxed{\text{boundedness of } V}$

$\Rightarrow \lambda > 0 \leftarrow$ to ensure no abyss in large $\Delta_L$ or $\Delta_R$

but: $\lambda' > \lambda \Rightarrow$ $\underline{\underline{\text{bounded}}}$ !

masses at $\Delta_L, \Delta_R$

$$\propto \lambda v_R^2 ; (\lambda' - \lambda) v_R^2$$

$\circledast$ | PROVE! | $\circledast$

compute the masses $\Leftrightarrow$

$$\text{compute } \frac{\partial^2 V}{\partial \Delta_i \partial \Delta_j}, \quad i,j = L, R$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{\text{diagonalize}}$$

$$\left( \frac{\partial^2 V}{\partial \Delta_L^2}\Bigg| \; , \; \frac{\partial^2 V}{\partial \Delta_R}\Bigg| \right.$$

$$\langle \Delta_L \rangle = 0$$

$$\langle \Delta_R \rangle = v_R$$

$$\left. \frac{\partial^2 V}{\partial \Delta_L \partial \Delta_R}\Bigg| \right)$$

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Next step !

$\Delta_L , \Delta_R = ?$ quantum numbers?

doublets ?

triplet ? - - -

How to decide ?

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## SM

$$q_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \Big\| \quad u_R , \; d_R$$

$$\Downarrow$$

$$\left( \begin{array}{l} \phi = \text{doublet} \\ Y \phi = \phi \end{array} \right)$$

$$\mathcal{L}_Y = \overline{q_L}\, Y_d\, \phi\, d_R + \cdots$$

## LR

today SM = high precision

$$\Downarrow$$

$$G_{LR} \longrightarrow G_{SM}$$
$$\langle \Delta_R \rangle = M_R$$

$\therefore$ 1) all the SM states get mass $\propto M_W$

2) all new states get mass $\propto M_R$

1) $\underbrace{W_L, Z, e, u, d}_{\text{massive SM states}}$, $h$ ← Higgs boson

2) $W_R$, $Z'$, $\nu_R$, new Higgses

↑ crucial

$\Downarrow$

$$\boxed{\Delta_{L,R} \text{ must have } \varphi_\Delta \text{ with leptons}}$$

$$\Downarrow$$

$$\cancel{\bar{l}_L \Delta_L l_R} + LR \text{ terms}$$

$$\updownarrow$$

$\Delta_L$ — purely $L$

$\Delta_R$ — 〃 — $R$

$$\Downarrow$$

(i) $\bar{l}_L \Delta_L l_L + L \leftrightarrow R$

$l_L \to U_L l_L \Rightarrow \Delta_L \to U_L \Delta_L U_L^+$

NO

$$L = \frac{1+\gamma_5}{2}$$

$$\bar{\ell}_L \ell_L \equiv \ell_L^+ \gamma_0 \ell_L =$$

$$\equiv (L\,\ell)^+ \gamma_0 \, L\, \ell$$

$$= \ell^+ L \, \gamma_0 \, L \, \ell$$

$$= \ell^+ \gamma_0 \underbrace{R\,L}_{=\,0} \ell$$

$$\text{(ii)} \quad \overbrace{\ell_L^T \, C \, \Delta_L \, \ell_L}^{\text{Lorentz invariant}}$$

+ SU(2) properties

fix during break

$$\boxed{\Delta_L \to U_L \Delta_L U_L^\dagger}$$ Adjoint

$$l_L^T \underbrace{U_L^T U_L}_{\neq 1} \Delta_L \underbrace{U_L^\dagger U_L}_{1} \Delta l$$

NOT good

$\Downarrow$

(ii) SU(2) Invariant $\Leftrightarrow$

$\Downarrow$

$$\mathcal{L}_y(\Delta) = l_L^T Y_\Delta\, i\sigma_2 C\, \Delta_L\, l_L +$$

$$+\, l_R^T Y_\Delta\, i\sigma_2 C\, \Delta_R\, l_R + h.c.$$

$$l_L^T U_L^T i\sigma_2 U_L \Delta_L U_L^\dagger U_L l_L =$$

$$= l_L^T i\sigma_2 U_L^\dagger U_L \Delta_L U_L^\dagger U_L l_L = \text{inv.}$$

$$(B-L)\,\Phi = 0 \qquad (\bar{f}_L\, \Phi\, f_R)$$

$$(B-L)\,\Delta_{L,R} = 2\,\Delta_{L,R}$$

$$\Uparrow$$

$$(B-L)\,l_{L,R} = -\,l_{L,R}$$

$$\left( \Delta_{L,R} = \text{complex adjoints} \right)$$

$$\Delta \to U \Delta U^{+} \quad \Leftrightarrow$$

$$\hat{T} \Delta = [\, T, \Delta \,]$$

$\uparrow$ fund. $\quad T_a = \frac{\sigma_a}{2}$

use: $\quad Q_{em} = T_{3L} + T_{3R} + \frac{B-L}{2}$

$$\hat{T}_{3L} \Delta_L = \left[ \frac{\sigma_3}{2}, \Delta_L \right]$$

$$\Delta_L = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\Downarrow$$

$$\hat{T}_{3L}\,\Delta_L = \begin{pmatrix} 0\cdot a & 1\cdot b \\ -1\cdot c & 0\cdot d \end{pmatrix}$$

$$\Downarrow$$

$$Q_{em}\,\Delta_L = \left(\hat{T}_{3L} + \hat{T}_{3R} + 1\right)\Delta_L$$

$$= \begin{pmatrix} 1\cdot a & 2\cdot b \\ 0\cdot c & 1\cdot d \end{pmatrix}$$

$$\hat{T}_{3R}\,\Delta_L = 0$$

$$\Rightarrow \boxed{\Delta_L = \begin{pmatrix} \delta_L^{+} & \delta_L^{++} \\ \delta_L^{0} & -\delta_L^{+} \end{pmatrix}}$$

$$\Delta_R = (\text{same} \ --)$$

$$\Downarrow$$

$$\boxed{\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}}$$

only neutral

$$\Downarrow \text{ vacuum}$$

$$\mathcal{L}(\Delta_R) \to l_R^T Y_\Delta C\, i\sigma_2 \langle \Delta_R \rangle l_R + h.c.$$

$$= (\nu_R^T\ e_R^T) Y_\Delta C \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} + h.c.$$

$$= (\nu_R^T\ e_R^T)\, C Y_\Delta \begin{pmatrix} v_R & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_R \quad - \text{''} -$$

$$= \nu_R^T y_\Delta v_R C \nu_R \quad + h.c.$$

$$= M_R \, \nu_R^T C \, \nu_R \quad + h.c.$$

Majorana mass term

$$M_R = y_\Delta v_R$$

$M_R \to \infty \Rightarrow \nu_R$ decouples

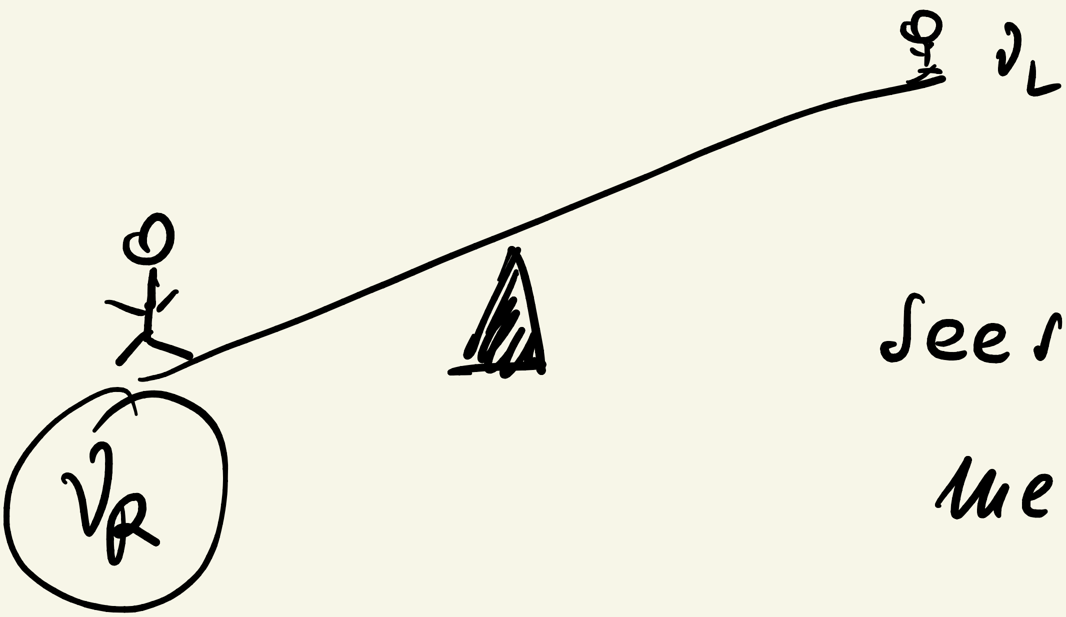
$\Rightarrow m_\nu = 0$ (SM)

$\Downarrow$

dimensional

- $m_\nu \propto \dfrac{M_W^{\,n+1}}{M_R^{\,n}} \longrightarrow 0$

$M_R \to \infty$




Seesaw mechanism

new heavy neutral lepton

$$\nu_R \longrightarrow C\,\bar{\nu}_R^{\,T} = LH$$

$N$

Imagine:

$\nu_R \quad // \quad \nu_L$

no coupling

$\Rightarrow \boxed{m_\nu = 0 \text{ (as in SM)}}$

$\Downarrow$

$\exists$ Dirac mass term

$\overline{\nu_L}\, m_D\, \nu_R \not\ni$

$$\mathcal{L}_Y(\Phi) = \bar{f}_L \, \bar{\Phi} \, f_R$$

$$\longrightarrow (\bar{\nu}\ \bar{e})_L \ \bar{\Phi} \ \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$\longrightarrow \boxed{\begin{array}{l} \bar{\nu}_L \langle \phi \rangle \nu_R + \\ \bar{e}_L \langle \Phi \rangle e_R \end{array}}$$

---

$$\nu_R \longrightarrow C \bar{\nu}_R^{\,T} \equiv N_L$$

$$\shortparallel$$

$$C(\nu_R^+ \gamma_0)^T = C\gamma_0 \nu_R^*$$

$$\checkmark$$

$$= i\gamma_2 \begin{pmatrix} 0 \\ u_R^* \end{pmatrix} = \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ u_R^* \end{pmatrix}$$

$$= \begin{pmatrix} i\sigma_2 u_R^* \\ 0 \end{pmatrix} \equiv N_L$$

$$\Downarrow$$

$$\boxed{(\nu_L, N_L) \text{ system}}$$

$$\Downarrow$$

$\cancel{\nu_L^T C \nu_L}$ ( <u>no</u> direct term )

$\Updownarrow$

$\nu_R^T C \nu_R \; M_R$ ou

$\rightarrow$ singlet | ( SM symmet

$\Downarrow$

$N_L^T C N_L \mu_R^*$ Q. why?

A. $\nu_R^T C \nu_R \mu_R + \underbrace{\nu_R^\dagger C^\dagger \nu_R^*}_{\parallel \atop N_L^T C N_L} \mu_R^*$

$$\Rightarrow N_L^T C N_L \mu_N \ , \quad \mu_N = \mu_R^*$$

<u>but</u>

$$\underbrace{M_D N_L^T C \nu_L + h.c.}_{\text{Dirac}}$$

$\Downarrow$

$$
\underbrace{
\begin{array}{c|cc}
 & \nu_L & N_L \\
\hline
\nu_L & 0\ (?) & M_D^T \\
N_L & M_D & M_N
\end{array}
}_{\boxed{\text{mass matrix}}}
$$

$$\Downarrow$$

$$m_{\nu_L} \neq 0$$

• $\underbrace{\psi_L^T C \psi_L}_{\text{breaks any quantum number of } \psi_L}$ = Majorana mass

• $\psi_{1L}^T C \psi_{2L} \stackrel{?}{=}$ Majorana

NOT Majorana in general

after all, $\psi_1$ and $\psi_2$ can have opposite charges

• $\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L$ = Dirac

$$(\psi^c)_L \equiv C \, \bar{\psi}_R^{\,T}$$

$$\Downarrow$$

$$\bar{\psi}_R \psi_L = \underbrace{(\psi^c)_L^T \, C \, \psi_L}_{\text{(looks like Majorana)}}$$

$$\Downarrow$$

$$\psi_{1L}^T \, C \, \psi_{2L} = \text{Majorana like}$$

$$\psi_L^T \, C \, \psi_L = \text{Majorana}$$

and

$$\left( \begin{array}{l} \psi_{iL}^T \, C \, \psi_{jL} \quad i,j = 1, \dots, n \\ = \text{Majorana if all} \\ \quad \psi_i \text{ have same charges} \end{array} \right)$$

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$$\psi_L^T \, C \, \psi_L \, m_M/2$$

$$\therefore \; \psi_M = \psi_L + C\bar{\psi}_L^T$$

$$\Rightarrow \underbrace{\bar{\psi}_M \psi_M}_{\text{Majorana}} = \psi_L^T C \psi_L + h.c.$$

$$\psi_M = \begin{pmatrix} u_L \\ i\sigma_2 u_L^* \end{pmatrix}$$

(Dirac-like)

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$$\underbrace{N_L^T C N_L}\leftarrow \langle \Delta_R \rangle$$

$$\Delta(B-L) = 2$$