

# Neutrino Physics Course

## Lecture V

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13/5/2022


LMU

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Spring 2022

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# S.M Higgs mechanism

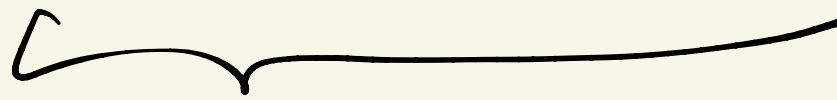
$$\langle \Phi \rangle^G = \begin{pmatrix} 0 \\ v \end{pmatrix} \quad (v \in \mathbb{R})$$

$$Q^G = T_3 + \frac{Y}{2} \quad (Y_\Phi = +1)$$

$$\langle \bar{\Phi} \rangle^D = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad v_i \in \mathbb{R}$$

• why  $\langle \bar{\Phi} \rangle =$  doublet?

$$l = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad \parallel \quad e_R$$



$e$  mass ?

$\psi_e \quad \bar{l}_L \quad \bar{\Phi} \quad e_R$

$\uparrow$   
doublet

$\uparrow$   
doublet

$$l_L \rightarrow U l_L \Rightarrow$$

$$\bar{\Phi} \rightarrow U \bar{\Phi}$$

$$\Downarrow \quad U U^\dagger = U^\dagger U = 1$$

$$\langle \bar{\Phi} \rangle^6 = U \langle \bar{\Phi} \rangle^D = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$\Downarrow \quad U = O = \begin{pmatrix} c & -s \\ s & c \end{pmatrix}$$

$$C \equiv \cos \theta, \quad S \equiv \sin \theta$$

$$\langle \bar{\Phi} \rangle^6 = \begin{pmatrix} c v_1 - s v_2 \\ s v_1 + c v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Rightarrow \tan \theta = v_1 / v_2$$



$$\langle \bar{\Phi} \rangle^6 = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = \sqrt{v_1^2 + v_2^2}$$

$$Q_{em}^6 \langle \bar{\Phi} \rangle^6 = 0$$

$$Q_{em}^6 = T_3 + 1/2$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\underline{SU(2)} \quad [T_a, T_b] = i \epsilon_{abc} T_c$$

$$U = e^{i \theta_\epsilon T_\epsilon} \quad \epsilon = 1, 2, 3$$

$$\boxed{T_a' = U T_a U^\dagger}$$

$\Downarrow$

$$[T_a', T_b'] = U [T_a, T_b] U^\dagger$$

$$= U i \epsilon_{abc} T_c U^\dagger =$$

$$= i \epsilon_{abc} U T_c U^\dagger$$

$$= i \epsilon_{abc} T_c'$$

$\Downarrow$

$$Q_{em}^G = U Q_{em}^D U^\dagger$$

$$Q_{em}^G \langle \Phi \rangle^G = 0$$

$$\langle \Phi \rangle^D = U^\dagger \langle \Phi \rangle^G$$

$\Downarrow$

$$\boxed{Q_{em}^D = U^\dagger Q_{em}^G U} \quad \therefore$$

$$\begin{aligned} Q_{em}^D \langle \Phi \rangle^D &= U^\dagger Q_{em}^G U U^\dagger \langle \Phi \rangle^G \\ &= U^\dagger Q_{em}^G \langle \Phi \rangle^G = 0 \end{aligned}$$

$\Downarrow$

$$\langle \Phi \rangle^D = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \Rightarrow$$

$$\langle \Phi \rangle^G = U \langle \Phi \rangle^D$$

$$\Downarrow \quad \langle \Phi \rangle^G = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$Q_{\text{eff}}^G = \text{diag} = T_2 + \frac{1}{2}$$

$$\Downarrow$$

$$\Phi^G = U \Phi^D$$

$$l^G = U l^D, \quad \varrho^G = U \varrho^D$$

$$\Downarrow$$

$$l^D = U^+ l^G$$

$$= \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix} =$$

$$= \begin{pmatrix} cs + se \\ -s\nu + ce \end{pmatrix} \quad \underline{\underline{\text{a mess!}}}$$

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$$V = \lambda \left( \Phi^+ \Phi - \frac{1}{2} v^2 \right)^2$$

$$V = f(\Phi^+ \Phi) \quad \text{why?}$$

$$\mathcal{O} = \Phi^T i \sigma_2 \Phi = i \nu \nu_0 \quad (11)$$



$$\underbrace{(\bar{\Phi}^+ \vec{\sigma} \Phi)}_{\vec{V}} \cdot \underbrace{(\bar{\Phi}^+ \vec{\sigma} \Phi)}_{\vec{V}} = iuv.$$

$$\vec{V} = iu.$$

$$(1) \quad \rho_i (i \sigma_2 \lambda_{ij} \varphi_j) = \rho_i \epsilon_{ij} \varphi_j$$

$$= 0$$

$$\bar{\Phi} \rightarrow \begin{pmatrix} 0 \\ \varphi_0 \end{pmatrix} \leftarrow$$

$$\Rightarrow \bar{\Phi}^+ \Phi = |\varphi_0|^2 = \text{only } iuv.$$

⇓

$$V = \lambda \left( \Phi + \bar{\Phi} - \frac{1}{2} v^2 \right)^2$$

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi)$$

$$\lambda > 0$$

ground state = vacuum manifold

$$= \mathcal{M}_0$$

$$\mathcal{M}_0 = \left\{ \Phi_0 \therefore 2\Phi_0^\dagger \Phi_0 = v^2 \right\}$$

$$\Phi = \begin{pmatrix} \varphi_u \\ \varphi_d \end{pmatrix} \in \mathbb{C}$$

$$= \begin{pmatrix} R_1 + i R_2 \\ R_3 + i R_4 \end{pmatrix}$$

$$\Phi^+ \Phi = R_1^2 + R_2^2 + R_3^2 + R_4^2$$



$$\sum_{i=1}^4 R_i^2 = \frac{1}{2} \varrho^2$$



$$\mathcal{M}_0 = S^3$$

3-dim  
sphere



$$\Phi_0^6 = \begin{pmatrix} 0 \\ \varrho \end{pmatrix}$$

$$\bar{\Phi}^G = \frac{1}{\sqrt{2}} \begin{pmatrix} G^+ \\ \nu + h + i G^0 \end{pmatrix} \begin{matrix} \leftarrow \text{charged} \\ \leftarrow \\ \text{neutral} \end{matrix}$$

$$\bar{\Phi}^W = U \bar{\Phi}^G$$

$$U = e^{i \theta_i(x) \sigma_i / 2}$$

$$\approx 1 + i \theta_i(x) \sigma_i / 2$$

$$\bar{\Phi}^W = \bar{\Phi}^G + i \theta_i(x) \frac{\sigma_i}{2} \begin{pmatrix} 0 \\ \nu \end{pmatrix} \frac{1}{\sqrt{2}} + \dots$$

$$= \bar{\Phi}^G + i \frac{1}{2\sqrt{2}} \begin{pmatrix} \theta_3 & \theta_1 - i \theta_2 \\ \theta_1 + i \theta_2 & -\theta_3 \end{pmatrix} \begin{pmatrix} 0 \\ \nu \end{pmatrix}$$

$$= \bar{\Phi}^G + \frac{1}{2\sqrt{2}} \begin{pmatrix} i(\theta_1 - i \theta_2) \nu \\ -i \theta_3 \nu \end{pmatrix}$$

$$\Phi^w = \frac{1}{\sqrt{2}} \begin{pmatrix} G^+ + (\theta_2 + i\theta_1) \frac{1}{2} \nu & \downarrow \\ \nu + h + iG_0 - \frac{1}{2} \theta_3 \nu & \downarrow \end{pmatrix}$$

$$G^+ + (\theta_2 + i\theta_1) \frac{1}{2} \nu = 0$$

$$G_0 = \frac{1}{2} \theta_3 \nu$$

↓

$$\Phi^w = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + h \end{pmatrix}$$

↑  
Higgs field

3 d.o.f. less in  $\vec{D}$

but

3 d.o.f. more in  $\vec{A}_\mu$

$(W^+, W^-, Z)$  3 massive

$$d(W, Z) = 3$$

$$d(A) = 2 \quad \uparrow \text{ massless}$$

$Z = \text{massive} = \text{Proca}$



$$\Phi^W \rightarrow \Phi^G = U^T \Phi^W$$

$\parallel$

t' Hooft

$(G^+, G^0)$

not physical



good propagator

$$\begin{array}{l} D(z) \\ \Delta(z) \end{array} \Bigg\} \xrightarrow{k \rightarrow \infty} 0$$

Unitary gauge

$$\Phi^W = \begin{pmatrix} 0 \\ v+h \end{pmatrix} \frac{1}{\sqrt{2}}$$

$$\equiv \Phi^u$$





$$h \bar{f} f : \frac{g}{2} \frac{u_f}{M_W}$$

$$h W^+ W^- : g M_W$$

$$h Z Z : \frac{1}{2} \frac{g}{\cos \theta_W} M_Z$$



$$\Gamma(h \rightarrow p p) \propto u_p^2$$

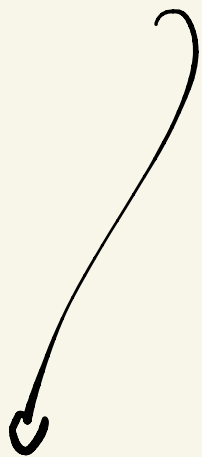
↑  
particle

$W^+, W^-, Z, t, b, \tau$

Kirzhnits '72

Weinberg '74

Dolan, Jadrin '74



$$T \rightarrow M_W$$

(high  $T$ )

$$V_T = V_0 + a T^2 \bar{\Phi}^+ \Phi$$



$$\hbar = c = k_B = 1$$

$$E_{kin} \simeq kT \simeq T$$



$$d(T) = d(m)$$

$$a = g^2 + \lambda + g_f^2 > 0$$



$(\nu = 0)$

$$V_T = \lambda (\bar{\Phi} + \Phi)^2 + aT^2 \bar{\Phi} + \Phi$$



$$\langle \Phi \rangle = 0$$

$$T_c \simeq M_w \simeq 100 \text{ GeV}$$

$$1 \text{ eV} \simeq 10^4 \text{ K}$$

$$\Rightarrow T_c \simeq 10^{13} \text{ K}$$

Halzen, Martin:

notes (Helfo, G.S.)