Neutrino Physics Course

Lecture V

13/5/2022

LMU

Spring 2022

SM Higgs mechanism

$$\langle \Phi \rangle \overset{G}{=} \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad (\text{VEV})$$

$$Q \overset{G}{=} T_3 + \frac{Y}{2} \qquad (Y_\Phi = +1)$$

$$\boxed{\langle \Phi \rangle \overset{D}{=} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}} \qquad v_i \in R$$

- why $\langle \Phi \rangle$ = doublet ?

$$l = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad \Bigg\| \qquad e_R$$

e mass ?

$$y_e \, \overline{\ell_L} \; \Phi \; e_R$$

doublet — doublet

$$\ell_L \longrightarrow U \ell_L \Rightarrow$$

$$\Phi \longrightarrow U \Phi$$

$$U U^\dagger = U^\dagger U = 1$$

$$\Downarrow$$

$$\langle \Phi \rangle^0 = U \left(\langle \Phi \rangle^0 = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \right)$$

$$\Downarrow$$

$$U = O = \begin{pmatrix} c & -s \\ s & c \end{pmatrix}$$

$c \equiv \cos\theta, \quad s \equiv \sin\theta$

$$\langle \bar{\Phi} \rangle^{G} = \begin{pmatrix} c v_1 - s v_2 \\ s v_1 + c v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Rightarrow \tan\theta = v_1 / v_2$$

$$\Downarrow$$

$$\langle \bar{\Phi} \rangle^{G} = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = \sqrt{v_1^2 + v_2^2}$$

- $Q_{em}^{G} \langle \bar{\Phi} \rangle^{G} = 0$

$$\boxed{Q_{em}^{G} = T_3 + 1/2 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}$$

SU(2)

$$[T_a, T_b] = i\epsilon_{abc} T_c$$

$$U = e^{i\theta_i T_i} \qquad i = 1, 2, 3$$

$$\boxed{T_a' = U T_a U^+}$$

$$\Downarrow$$

$$[T_a', T_b'] = U [T_a, T_b] U^+$$

$$= U i \varepsilon_{abc} T_c U^+ =$$

$$= i \varepsilon_{abc} U T_c U^+$$

$$= i \varepsilon_{abc} T_c'$$

$$\Downarrow$$

$$Q_{em}^{G} = U\, Q_{em}^{D}\, U^{+}$$

$$Q_{em}^{G} \langle \Phi \rangle^{G} = 0$$

$$\langle \Phi \rangle^{D} = U^{+} \langle \Phi \rangle^{G}$$

$$\Downarrow$$

$$\boxed{Q_{em}^{D} = U^{+}\, Q_{em}^{G}\, U}$$

!.

$$Q_{em}^{D} \langle \Phi \rangle^{D} = U^{+} Q_{em}^{G}\, U U^{+} \langle \Phi \rangle^{G}$$

$$= U^{+} Q_{em}^{G} \langle \Phi \rangle^{G} = 0$$

$$\Downarrow$$

$$\langle \Phi \rangle^D = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \Rightarrow$$

$$\langle \Phi \rangle^G = U \langle \Phi \rangle^D$$

$$\Downarrow \qquad \langle \Phi \rangle^G = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$Q_{em}^G = \text{diag} = T_3 + Y/2$$

$$\Downarrow$$

$$\Phi^G = U \; \Phi^D$$

$$l^G = U \; l^D \, , \quad q^G = U q^D$$

$$\Downarrow$$

$$\ell^D = U^+ \ell^G$$

$$= \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix} =$$

$$= \begin{pmatrix} cs + se \\ -s\nu + ce \end{pmatrix}$$ a mess!

$$\boxed{V = \lambda \left(\Phi^+ \Phi - \frac{1}{2} v^2 \right)^2}$$

$V = f(\Phi^+ \Phi)$ why?

$O = \Phi^T i \sigma_2 \Phi$ = inv. ??

$$\underbrace{(\Phi^+ \vec{\sigma}\, \Phi)}_{\vec{V}} \ \underbrace{(\Phi^+ \vec{\sigma}\, \Phi)}_{\vec{V}} = \text{inv.}$$

$$\vec{V} \cdot \vec{V} = \text{inv.}$$

(1) $\varphi_i \, (i\sigma_2)_{ij} \, \varphi_j = \varphi_i \, \varepsilon_{ij} \, \varphi_j$

$$= 0$$

$$\Phi \longrightarrow \begin{pmatrix} 0 \\ \varphi_0 \end{pmatrix} \longleftarrow$$

$\Rightarrow \Phi^+ \Phi = |\varphi_0|^2 =$ only

inv.

$$\Downarrow$$

$$V = \lambda \left(\Phi^+ \Phi - \frac{1}{2} v^2 \right)^2$$

$$\Uparrow$$

$$\mathcal{L}_\Phi = (D_\mu \Phi)^+ (D^\mu \Phi) - V(\Phi)$$

$$\boxed{\lambda > 0}$$

ground state = vacuum manifold

$= \mathcal{M}_0$

$$\mathcal{M}_0 = \{ \Phi_0 : 2 \Phi_0^+ \Phi_0 = v^2 \}$$

$$\Phi = \begin{pmatrix} \varphi_u \\ \varphi_d \end{pmatrix} \in \mathbb{C}$$

$$= \begin{pmatrix} R_1 + i R_2 \\ R_3 + i R_4 \end{pmatrix}$$

$$\Phi^{+}\Phi = R_1^2 + R_2^2 + R_3^2 + R_4^2$$

$$\Downarrow$$

$$\boxed{\sum_{i=1}^{4} R_i^2 = \frac{1}{2} v^2}$$

$$\Downarrow$$

$$\boxed{\mathcal{M}_0 = S_3}$$ $\boxed{\text{3-dim sphere}}$

$$\Downarrow$$

$$\boxed{\Phi_0^{\,6} = \begin{pmatrix} 0 \\ v \end{pmatrix}}$$

$$\Phi^G = \frac{1}{\sqrt{2}} \begin{pmatrix} G^+ \\ v + h + i G^0 \end{pmatrix} \begin{matrix} \leftarrow \text{charged} \\ \leftarrow \end{matrix}$$

neutral

$$\Phi^W = U \, \Phi^G$$

$$U = e^{i \theta_i(x) \, \sigma_i/2}$$

$$\simeq 1 + i \theta_i(x) \, \sigma_i/2$$

$$\Phi^W = \Phi^G + i \theta_i(x) \frac{\sigma_i}{2} \begin{pmatrix} 0 \\ v \end{pmatrix} \frac{1}{\sqrt{2}} + \cdots$$

$$= \Phi^G + i \frac{1}{2\sqrt{2}} \begin{pmatrix} \theta_3 & \theta_1 - i\theta_2 \\ \theta_1 + i\theta_2 & -\theta_3 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \Phi^G + \frac{1}{2\sqrt{2}} \begin{pmatrix} i(\theta_1 - i\theta_2) v \\ -i\theta_3 v \end{pmatrix}$$

$$\Downarrow$$

$$\Phi^W = \frac{1}{\sqrt{2}} \begin{pmatrix} G^+ + (\theta_2 + i\theta_1)\frac{1}{2} v \\ v + h + iG_0 - \frac{1}{2}\theta_3 v \end{pmatrix}$$

$$G^+ + (\theta_2 + i\theta_1)\frac{1}{2} v = 0$$

$$G_0 = \frac{1}{2}\theta_3 v$$

$$\Downarrow$$

$$\boxed{\Phi^W = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}}$$

Higgs field

3 d.o.f. less in Φ

but

3 d.o.f more in $\vec{A}_\mu$

(W^+, W^-, Z) 3 massive

$d(W, Z) = 3$

$d(A) = 2$

massless

$\boxed{Z = \text{massive} = \text{Proca}}$

$$\Downarrow$$

$$\Phi^W \longrightarrow \Phi^G = U^\dagger \Phi^W$$

$$\|$$

t' Hooft

$$\underbrace{(G^+, G^0)}_{\text{\underline{not} physical}}$$

$$\Downarrow$$

$$\boxed{\text{good propagator}}$$

$$\boxed{\left.\begin{matrix} D(G) \\ \Delta(Z) \end{matrix}\right\} \xrightarrow[k\to\infty]{} 0}$$

Unitary gauge

$$\boxed{\begin{aligned} \Phi^{W} &= \begin{pmatrix} 0 \\ v+h \end{pmatrix} \frac{1}{\sqrt{2}} \\ &\equiv \Phi^{un} \end{aligned}}$$

$$\Downarrow$$

$$h\bar{f}f : \quad \frac{g}{2}\frac{m_f}{M_W}$$

$$hW^+W^- : \quad g\,M_W$$

$$hZZ : \quad \frac{1}{2}\frac{g}{\cos\theta_W}\,M_Z$$

$$\Downarrow$$

$$\Gamma(h \to p\,p) \propto m_p^{\,2}$$

particle

$$W^+, W^-, Z, \; t, b, \tau$$

Kirzhnits '72

Weinberg '74

Dolan, Jackiw '74

$$\boxed{T \gg M_W}$$ (high T)

$\hbar = c = k = 1$

$$V_T = V_0 + a\,T^2\,\Phi^\dagger \Phi$$

$E_{kin} \simeq kT \simeq T$

$d(T) = d(m)$

$$\boxed{a = g^2 + \lambda + y_f^2 > 0}$$

$$\Downarrow \qquad\qquad (v=0)$$

$$V_T = \lambda\,(\Phi^\dagger\Phi)^2 + aT^2\,\Phi^\dagger\Phi$$

$$\Downarrow$$

$$\boxed{\langle \Phi \rangle = 0}$$

$$T_c \simeq M_W \simeq 100\ \text{GeV}$$

$$1\ \text{eV} \simeq 10^4\ \text{K}$$

$$\Rightarrow \boxed{T_c \simeq 10^{13}\ {}^\circ\text{K}}$$

Halzen, Martin:

$\boxed{\text{notes (Helfo, G.S.)}}$