# Neutrino Physics Course

## Lecture IX

31/5/2022

LMU

Spring 2022

# Strong CP violation

$g \equiv g_s$

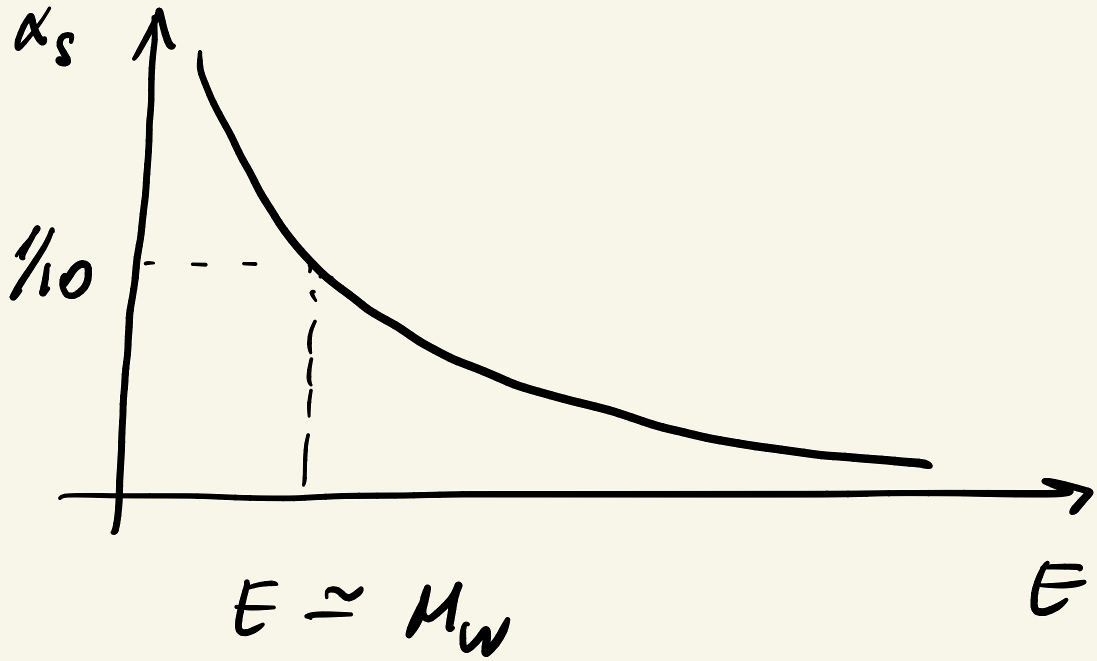
$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} + i\bar{q}\gamma^\mu D_\mu q - m_q \bar{q} q$$

$a = 1, \dots, 8$

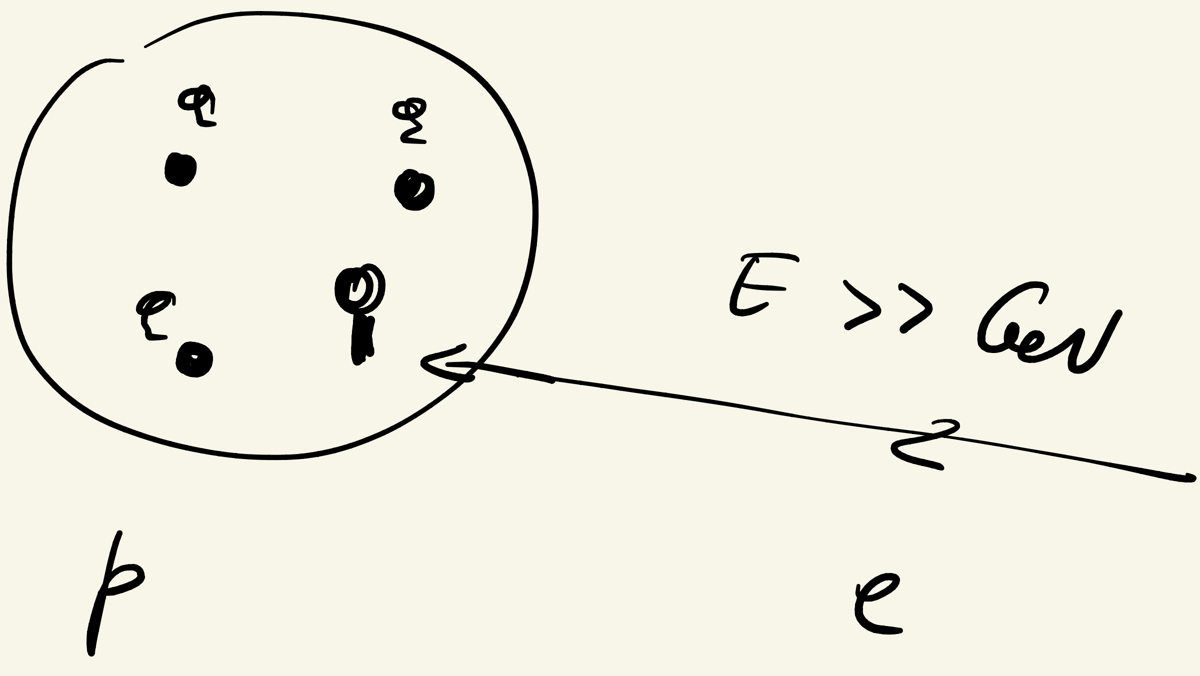
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu$$

$$[T_a, T_b] = i f_{abc} T_c$$

$E \simeq \text{GeV} \Rightarrow \alpha_s / 4\pi \gg 1$




$$\alpha_s(M_W) \simeq 1/10$$




$$D_\mu q = \left(\partial_\mu - i g\, T_a A_\mu{}^a\right) q$$

$$q = \begin{pmatrix} q^r \\ q^y \\ q^b \end{pmatrix}$$

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$$G G \equiv G_{\mu\nu}^{a} G^{\mu\nu a}$$

$$\updownarrow$$

$$F F = F_{\mu\nu} F^{\mu\nu} \quad \left( = \vec{E}^2 - \vec{B}^2 \right)$$

$$E^i = F^{0i}$$

$$2 B^i = \varepsilon^{ijk} F^{jk}$$

$$\Rightarrow \boxed{P,\ T = CP \text{ are good symmetries}}$$

$$\boxed{d(\mathcal{L}) \le 4}$$

$$\boxed{\Leftrightarrow \text{ renormalizability}}$$

$$\Delta \mathcal{L}_{QCD} = \underbrace{\frac{\theta g^2}{32\pi^2} \, G^a_{\mu\nu} \, G^a_{\alpha\beta} \, \epsilon^{\mu\nu\alpha\beta}}$$

$$\Updownarrow$$

$$F_{\mu\nu} F_{\alpha\beta} \epsilon^{\mu\nu\alpha\beta} \equiv F\tilde{F}$$

$$\equiv F F^d$$

$$= \vec{E} \cdot \vec{B} \Rightarrow \cancel{P}, \cancel{T}$$

$$\vec{F} = e\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$F\tilde{F} = \partial_\mu k^\mu$$

$$k^\mu \propto \epsilon^{\mu\nu\alpha\beta} \underline{\underline{F_{\nu\alpha}}} A_\beta$$

$$\boxed{\int F\tilde{F} d^4x = \int ds^\mu (k_\mu \to 0) = 0}$$

$$\int F^2 d^4x = \text{finite}$$

$$\Rightarrow \quad F \xrightarrow[\infty]{} 0$$

$$\Rightarrow \quad \text{FORGET } F\tilde{F}!$$

QCD

$$G\tilde{G} = \partial_\mu k^\mu$$

$$k^\mu \propto \epsilon^{\mu\nu\alpha\beta}\left(\cancel{\sim G^a_{\nu\alpha} A^a_\beta} \xrightarrow{} 0 + \right.$$

$$\left. + \boxed{A^a_\nu A^b_\alpha A^c_\gamma \epsilon^{abc}}\right)$$

$$\boxed{G \to 0 \;\not\Rightarrow\; A \to 0}$$

$$\int k^\mu \, dS_\mu \neq 0 \text{ in general}$$

$$k^\mu \underset{\infty}{\not\to} 0$$

$$\mathcal{L}_{QCD} = -\frac{1}{4} G G + \frac{\theta g^2}{32\pi^2} G \tilde{G}$$

$$+ \; - - - -$$

$$\Rightarrow \boxed{QCD \longrightarrow \cancel{P}, \cancel{T}}$$

[ STRONG CP VIOLATION ]

- <u>Axial Anomaly</u>

$$\mathcal{L}_D = i \bar{f} \gamma^\mu \partial_\mu f - m \bar{f} f$$

(i) $f \rightarrow e^{i\alpha} f \qquad \bar{f} = f^+ \gamma_0$

$$\Rightarrow \partial_\mu j^\mu = 0$$

$$\therefore \quad j^\mu = \bar f \gamma^\mu f$$

(ii) $f \to e^{i\beta\gamma_5} f$

$$\Rightarrow \begin{cases} \partial^\mu j_\mu^5 = 2im\, \bar f \gamma_5 f \\ j_\mu^5 = \bar f \gamma_\mu \gamma_5 f \end{cases}$$

- $m = 0 \Rightarrow \partial_\mu j^{\mu 5} = 0$

$$\delta S = \beta \int d^4x\, \partial^\mu j_\mu^5 = 0$$

$$\Downarrow$$

$\boxed{\text{Noether 1918}}$

$$\boxed{\begin{array}{l}\text{However, QFT} \\ \partial_\mu j^\mu_5 = \dfrac{g^2}{32\pi^2} G\tilde{G} \\ \therefore f = Q\end{array}}$$

Adler '64

Bell, Jackiw '64

$$\Downarrow$$

$$\delta S = \beta \frac{g^2}{32\pi^2} \int G\tilde{G}$$

$$\Rightarrow \boxed{\theta \to \theta + \beta}$$

if $m_q = 0 \Rightarrow \theta + \beta = 0$

$\Downarrow$

$$\boxed{\text{NO effect from } \theta}$$

- electric dipole moment of neutron

"Derivation" $\qquad \Lambda_{QCD} = \text{GeV}$

$$d_n^e = \theta \frac{1}{\Lambda_{QCD}} \left( \frac{m_u}{\Lambda_{QCD}} \simeq 10^{-2} \right)$$

$$= \theta \, 10^{-14} \text{cm} \cdot 10^{-2} \leq 10^{-26} \text{cm}$$

$$\Downarrow$$

$$\boxed{\theta \leq 10^{-10}}$$

$$\boxed{\text{STRONG CP ``problem''}}$$

what about $m_d$

$$\Rightarrow \quad \frac{m_u m_d}{(m_u + m_d)\Lambda_{QCD}} \simeq \frac{m_u}{\Lambda_{QCD}}$$

## Summary

- $\mathcal{L}_{QCD} = \cdots + \frac{\theta g^2}{32\pi^2} G\tilde{G}$

- $\partial_\mu j^\mu_5 = 2im\,\bar{q}\,\gamma_5 q +$

$$+ \frac{g^2}{32\pi^2} G\tilde{G}$$

$$j^\mu_5 = \bar{q}\,\gamma^\mu \gamma_5 q$$

- $m_q = 0 \Rightarrow \theta = 0$

- $\theta \frac{m_q}{\Lambda_{QCD}} \leq 10^{-12} \Rightarrow$

$$\boxed{\theta \leq 10^{-10}}$$

+ ew interactions

Higgs

$$m_q \longrightarrow M_q = Y_q \langle \phi \rangle$$

$$M_q \longrightarrow U_L M_q U_R^{+} = m_q$$

$$\| $$

$$\mathrm{diag}(m_1, m_2, \dots)_q$$

$$\boxed{U_{L,R}^{+} U_{L,R} = \mathbb{1}}$$

$m_q \in \mathbb{R}$

$\Downarrow$

$$V_{CKM} = ?$$

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} W^+_\mu \left( \bar{u}_L \; \bar{c}_L \; \bar{t}_L \right) V_{CKM} \gamma^\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

$$\boxed{V_{CKM} = U^+_{Lu} U_{Ld}}$$

$$\boxed{V_{CKM} \in \mathbb{C}}$$

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- $q \to e^{i\beta\gamma_5} q \Rightarrow \theta \to \theta + \beta$

$$\boxed{\beta = \arg \det U_L U_R^+}$$

$$\Downarrow$$

$$\boxed{\theta \to \theta + \arg\det M_q}$$

$$m_q = V_L M_q V_R^\dagger$$

$$\arg\det m_q = 0$$

$$\Downarrow$$

$$\boxed{\arg\det V_L V_R^\dagger = \arg\det M_q}$$

$$\Downarrow$$

$$\boxed{\bar{\theta} = \theta + \arg\det M_q}$$

physical strong CP

parameter

$\Rightarrow \boxed{\bar{\Theta} \leq 10^{-10}}$

$\Uparrow$

why? $\quad d_n^{\ell} \propto \bar{\Theta}$ - - -

$\arg\det M_\ell \sim \epsilon_{cp}$ (weak)

should be

$\epsilon_{cp}$ = Jarlskog invariant

$\simeq \theta_1\,\theta_2\,\theta_3\,\sin\delta$

$\theta_1 \simeq 1/5, \quad \theta_2 \simeq 4\times10^{-2}, \quad \theta_3 \simeq 4\times10^{-3}$

$$\left\{ \begin{array}{l} \theta_i = 3 \text{ Euler angles} \\ \quad\quad \uparrow \\ V_{CKM} = 3\times 3 \text{ unitary} \\ \quad\quad \downarrow \\ \delta = CP \text{ phase} \end{array} \right\}$$

bottom line: $\boxed{\varepsilon_{cp} \simeq 10^{-5}}$

$\Downarrow$

<u>Claim</u>: $\bar{\theta} \simeq \varepsilon_{cp} \simeq 10^{-5}$

but exp $\Rightarrow \bar{\theta} \leq 10^{-10}$

$\Rightarrow$ (STRONG CP PROBLEM)

WRONG

Ellis, Gaillard '79

SM

$$\bar{\theta}^{\,loop} \simeq f(\varepsilon_{\not{CP}})$$

$$\lesssim \underbrace{\left(\frac{\alpha}{\pi}\right)^3}_{10^{-6}} \underbrace{\left(\frac{m_b}{M_W}\right)^6}_{10^{-8}} \underbrace{\epsilon_{\not{CP}}}_{10^{-5}}$$

$$\boxed{\begin{array}{c} \bar{\theta}^{\,loop} \leq 10^{-19} \quad !!! \\ \bar{\theta}_{exp} \leq 10^{-10} \end{array}}$$

$\Downarrow$

$\boxed{\text{NO Strong CP problem}}$

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(1) PQ $\longrightarrow$ axion

$m_\ell = 0 \; (\Leftrightarrow y_\ell = 0) \Rightarrow$

chiral symmetry $\Rightarrow \bar{\theta} = 0$

$$\mathcal{L}_y^{PQ} = \overline{q_L}\, Y_d\, \Phi_d\, d_R \; \cancel{\left( + Y_d' \Phi_u^* \right)} +$$

$$+ \overline{q_L}\, Y_u\, i\sigma_2 \Phi_u^* u_R + h.c.$$

$\int d_R \to e^{i\alpha} d_R, \; u_R \to e^{i\alpha} u_R$

$\wr \quad q_1 \to q_2 , \quad \Phi_d \to e^{-i\alpha}\Phi_d, \; \Phi_u \to e^{i\alpha}\Phi_u$

(PQ) Chiral symm!

$$\langle \Phi_u, \Phi_d \rangle \neq 0$$

$\Longrightarrow$ ~~PQ~~

$\Rightarrow \exists$ NG boson!

$\quad \parallel$

$\boxed{\text{axion}}$ Wilczek '77

Weinberg '77

$$\therefore \quad \bar{\Theta} = \frac{\langle a \rangle}{M_{PQ}} = 0$$

• $\cancel{P}$ spont. Mohapatra, G.S. '78

$P$ is good : $\theta = 0$

$$\bar{q}_L \, M_q \, q_R + \bar{q}_R \, M_q^{+} \, q_L$$

$$\Rightarrow \boxed{M_q = M_q^{+} \qquad \arg\det M = 0}$$

$\Leftrightarrow$ $\bar{\theta} = 0$ before $\cancel{P}$ spont.

- $\underline{\Theta}$ ($\bar{\Theta}$) $\neq$ <u>perturbative</u>

effects $\simeq$ $e^{-\frac{4\pi}{\alpha}}$

non-perturbative

- QCD: $4\pi/\alpha = O(1)$

- QFD: $\frac{4\pi}{\alpha} = 300$
(ew)

$$\boxed{e^{-300} = 0 \; !}$$