Neutrino Physics Course

Lecture IV

10/5/2022

LMU

Spring 2022

Standard Model:

Higgs mechanism

- $G_{SM} = \overset{g}{SU(2)_L} \times \overset{g'}{U(1)_Y}$

- $\begin{pmatrix} u \\ d \end{pmatrix}_L \equiv q_L \qquad\qquad u_R,\ d_R$

  $\begin{pmatrix} \nu \\ e \end{pmatrix}_L \equiv l_L \qquad\qquad e_R$

$$\boxed{T_3 f_R = 0}$$

$$\Downarrow$$

if $\exists$ A : $e A_\mu j^\mu_{em}$

$$j^\mu_{em} = \bar{f} \gamma^\mu Q f$$

$$\Downarrow$$

$\exists\ Z \therefore \quad \frac{g}{\cos\theta_W} Z_\mu j^\mu_Z$

$$j^\mu_Z = \bar{f} \gamma^\mu (T_3 L - Q \sin^2\theta_W) f$$

$E \to \infty \Rightarrow$ small deviations

$$\sim \frac{m}{E} \to 0$$

$$\Delta_{\mu\nu}(Z) = \frac{-i}{k^2 - M_Z^2} \left[g_{\mu\nu} - \frac{k_\mu k_\nu}{M_Z^2} \right]$$

manifestly fails

$E \to \infty$, $\Delta_{\mu\nu} \to$ const!

Weinberg '67

Higgs: scalar Φ:

$$\langle \Phi \rangle \neq 0$$

$\shortparallel$

vacuum expectation value (VEV)

$$\Downarrow$$

$$M_W,\ M_Z,\ m_f \propto \langle \Phi \rangle$$

Which Φ?

What quantum numbers?

fermions

$$m_f \bar f f = (\bar f_L f_R + \bar f_R f_L)\, m_f$$

$$\bar f_L \to \text{doublet} \qquad f_R \to \text{singlet}$$

forbidden by SU(2)

$$Y: \; -1/3 \qquad +1 \qquad -2/3$$

$$\mathcal{L}_Y = \bar q_L Y_d \Phi\, d_R \qquad \underbrace{\boxed{\Phi \to U \Phi}}_{\text{doublet}}$$

$$\to \bar q_L U^\dagger U \Phi\, d_R Y_d$$

$$+ \bar q_L Y_u i \sigma_2 \Phi^* u_R \longleftarrow \boxed{Y \Phi = \Phi}$$

$$Y: \; -1/3 \qquad 4/3$$

$$Q = T_3 + Y/2 \qquad \boxed{Y\Phi^* = -\Phi^*}$$

$$\Downarrow$$

$$\Rightarrow \boxed{\begin{aligned} Y u_R &= 4/3\, u_R, \\ Y d_R &= -2/3\, d_R \end{aligned}}$$

$$Y q = \frac{1}{3} q$$

$$\Phi \to U\Phi, \quad \Phi^* \to U^* \Phi^*$$

$$\Downarrow$$

$$\bar{q}_L \Phi^* u_R \to \bar{q}_L \underbrace{U^\dagger U^*}_{\neq 1} \Phi^* u_R$$

$$D_i \to U D_i \qquad i = 1, 2$$

$$D = \begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix} \qquad s = 1/2$$

$$\Rightarrow |s=0\rangle = |\uparrow\downarrow - \downarrow\uparrow\rangle$$

$$\Updownarrow$$

$$S = D_1^T \epsilon \, D_2$$

$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\Rightarrow \boxed{\epsilon = i\sigma_2}$$

$$D_1^T \sigma_2 \, D_2 \to D_1^T U^T \sigma_2 U D_2$$

$$= D_1^T e^{i\theta_i \frac{\sigma_i^T}{2}} \sigma_2 \, e^{i\theta_i \sigma_i/2} D_2$$

$$= D_1^T \sigma_2 \underbrace{e^{-i\theta_i \sigma_i/2} e^{i\theta_i \sigma_i/2}}_{\overset{\parallel}{1}} D_2$$

$$= D_1^T \sigma_2 D_2 \checkmark$$

$$\boxed{\text{Q.E.D.}}$$

$$\Downarrow$$

$$\boxed{\mathcal{L}_Y = \bar{q}_L\, y_d\, \Phi\, d_R + \bar{q}_L\, y_u\, i\sigma_2\, \Phi^* u_R + \bar{l}_L\, y_e\, \Phi\, e_R + h.c.}$$

if $\langle \Phi \rangle \neq 0$

$\Downarrow$ SU(2) ROTATION

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \frac{1}{\sqrt{2}}$$

VEV

$$Q = T_3 + Y/2 = T_3 + \frac{1}{2}$$

$$\Rightarrow Q \langle \Phi \rangle = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 + 1/2 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \frac{1}{\sqrt{2}}$$

$$= 0$$

$$\boxed{\begin{aligned} T_3 \langle \Phi \rangle &= -\frac{1}{2} \langle \Phi \rangle \\ Y \langle \Phi \rangle &= + \langle \Phi \rangle \end{aligned}}$$

$$\Phi = \begin{pmatrix} 0 \\ v+h \end{pmatrix} \frac{1}{\sqrt{2}}$$

Higgs boson

$$\Downarrow$$

$$\mathcal{L}_Y = \bar{f}_L f_R y_f (v+h) \frac{1}{\sqrt{2}} + h.c.$$

$$(f = d, u, e)$$

$$= (v+h) \frac{1}{\sqrt{2}} \bar{f} f y_f$$

$$\Downarrow$$

$$\boxed{m_f = y_f \frac{v}{\sqrt{2}}}$$

$$\boxed{\mathcal{L}_{h\bar{f}f} = \frac{y_f}{\sqrt{2}}\, h\,\bar{f}f = \frac{m_f}{v}\, h\,\bar{f}f}$$

gauge sector

$$\partial_\mu \longrightarrow D_\mu =$$

$$= \partial_\mu - i\underbrace{g}_{SU(2)}\, T_i\, A_\mu^i - i\underbrace{g'}_{U(1)}\, \frac{Y}{2}\, B_\mu$$

$i = 1, 2, 3$

$$T_i = \frac{\sigma_i}{2}$$

$$\mathcal{L}_{kin}(\Phi) = (D_\mu \Phi)^\dagger (D^\mu \Phi)$$

$$\rightarrow (D_\mu \Phi)^\dagger U^\dagger U (D^\mu \Phi)$$

$$= (D_\mu \Phi)^\dagger (D^\mu \Phi)$$

$$D_\mu \langle \Phi \rangle = -i/2 \begin{pmatrix} gA_3 + g'B & g(A_1 - iA_2) \\ g(A_1 + iA_2) & -gA_3 + g'B \end{pmatrix}_\mu \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \begin{pmatrix} g(A_1 - iA_2) \\ (-gA_3 - g'B) \end{pmatrix} \frac{v}{\sqrt{2}} \frac{1}{2}$$

$$\Downarrow$$

$$(D_\mu \langle\Phi\rangle)^\dagger (D^\mu \langle\phi\rangle) = \frac{1}{4}\cdot\frac{1}{2}\, v^2 \quad \times$$

$$\underbrace{\left[g^2 \left(A_1^2 + A_2^2\right) + (\underbrace{g A_3 - g' B}_{Z})^2 \right]}$$

Proca masses

$$\left\{ \begin{array}{c} W^\pm = \dfrac{A_1 \mp i A_2}{\sqrt{2}} \\ \Downarrow \\ \boxed{M_W^2 = \dfrac{1}{4} g^2 v^2} \end{array} \right\}$$

$$Z = \frac{g A_3 - g' B}{\sqrt{g^2 + g'^2}} = \cos\theta_W A_3 - \sin\theta_W B$$

$$\boxed{A = \sin\theta_W A_3 + \cos\theta_W B}$$ (photon)

$$\boxed{m_A = 0}$$

$$M_Z = \frac{\sqrt{g^2 + g'^2}}{g} M_W$$

$$\boxed{M_Z \cos\theta_W = M_W}$$

$$\boxed{\underbrace{M_W = 80\ \text{GeV}, \quad M_Z = 90\ \text{GeV}}_{\Downarrow} \quad \theta_W \simeq 30^\circ}$$

$$\boxed{\text{experiment}}$$

$$Q_{em} = T_3 + \frac{Y}{2}$$

em charge $\therefore$ $q_L = q_R$

$$\boxed{Y/2 = Q - T_3}$$

$\boxed{\text{usual em charges}}$

$Y\,\Phi = 1$ (fit with Y_Q, Y_e --)

$\Downarrow$

$$\sqrt{2}\langle\Phi\rangle^{6} = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\sqrt{2}\langle\Phi\rangle^{7} = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$Q_{em}\langle\Phi\rangle^{7} = \begin{pmatrix} 1/2 + 1/2 & 0 \\ 0 & -1/2 + 1/2 \end{pmatrix}\begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} v \\ 0 \end{pmatrix} \neq 0$$

$$\boxed{\begin{aligned} Q_{em}^{6}\langle\Phi\rangle^{6} &= 0 \\ Q_{em}^{6}\langle\Phi\rangle^{7} &\neq 0 \end{aligned}}$$

???

- $Q_{em}^{7}\langle\Phi\rangle^{7} = 0$

$$Q_{em}^{7} = T_3 - Y/2$$

• $\langle\Phi\rangle^{\text{Devil}} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$\Rightarrow \boxed{Q_{em}^{\text{Devil}} = \ ?}$

$\therefore \quad Q_{em}^{\text{Devil}} \langle\Phi\rangle^{\text{Devil}} = 0$

• $M_W = \frac{g}{2}\langle\Phi\rangle \ , \quad M_Z \cos\theta_W = M_W$

• $h \frac{m_f}{\langle\Phi\rangle} \bar{f} f = \boxed{\frac{g}{2}\frac{m_f}{M_W} h \bar{f} f}$

$\downarrow$

measure m_f

$\Downarrow$

$$\boxed{\begin{array}{l}\Gamma(h \to f\bar{f}) \propto \left(\dfrac{m_f}{M_W}\right)^2 m_h \\ m_h \gg m_f\end{array}}$$

- $t, b, \tau \leftarrow$ mass from Higgs

  μ (muon) $\leftarrow$ getting there

  $W, Z \leftarrow$ mass from Higgs

$$D_\mu \Phi = \cdots\cdots \begin{pmatrix} 0 \\ v+h \end{pmatrix} \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cdots\cdots (v+h)^2 = v^2 \left(1 + h/v\right)^2$$

$$= v^2 \left(1 + 2\, h/v\right)$$

mass

coupling

$$\boxed{\text{Higgs coupling} \propto \frac{\text{mass}}{v}}$$

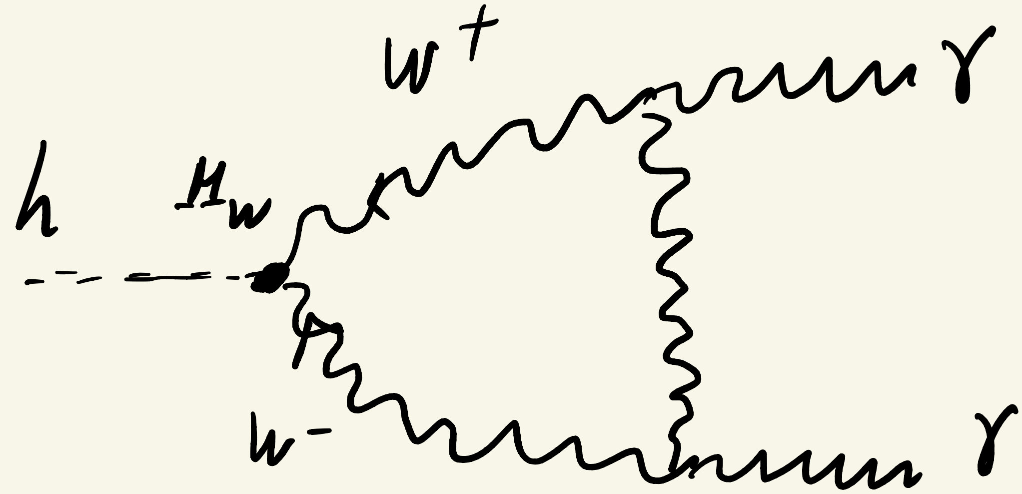
$$M_W^2\, W^+ W^- \rightarrow \frac{h}{v} M_W^2\, W^+ W^-$$

$$\propto M_W\, h\, W^+ W^-$$

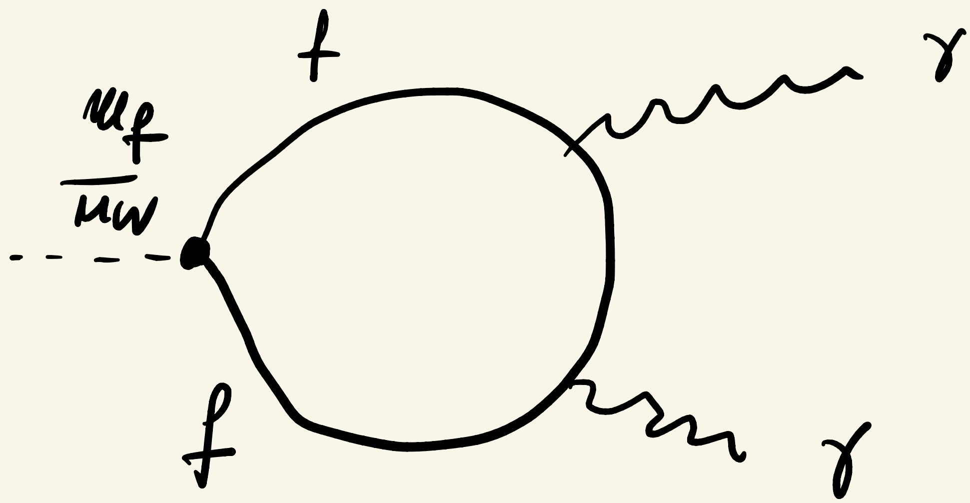
$h Z Z \leftarrow M_Z$

$\boxed{\text{Higgs SM} \Rightarrow M_A = 0 \qquad \frac{dQ_{em}}{dt} = 0}$

$\boxed{A - \text{not coupled to Higgs}}$



indirect $h\, F_{\mu\nu} F^{\mu\nu}$

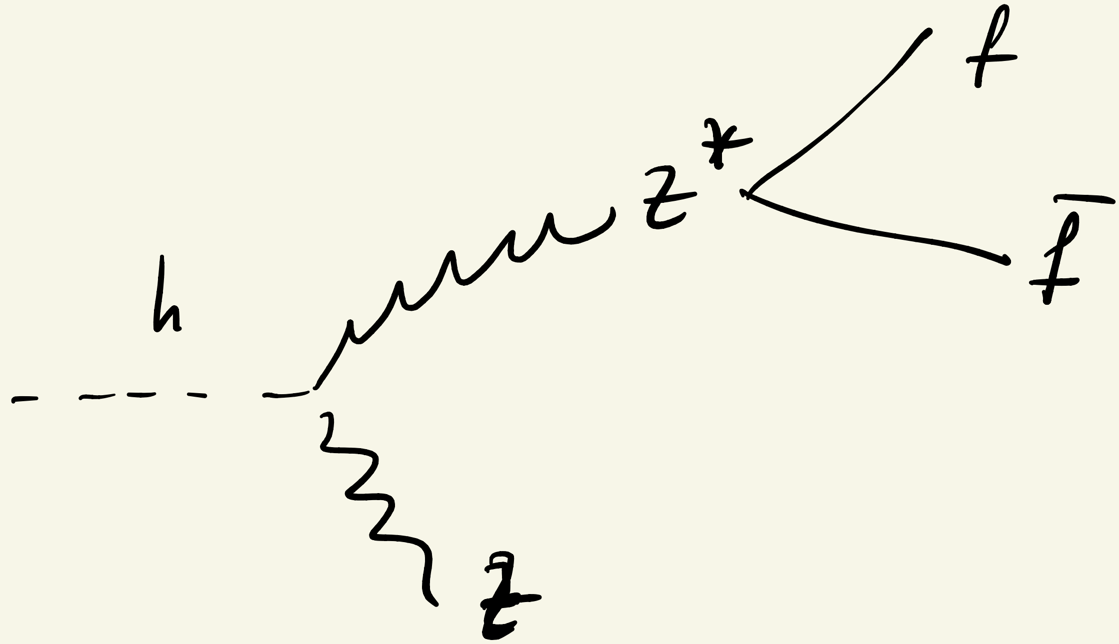


which f dominates?

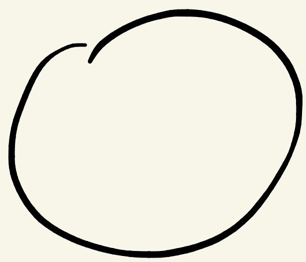
top!

$$Br(h \to \gamma\gamma) = \frac{\Gamma(h \to \gamma\gamma)}{\Gamma_{total}} \doteq 10^{-3}$$

$m_h \simeq 125\ \text{GeV} < 2M_W, 2M_Z$



· $W, Z \leftarrow 1983$

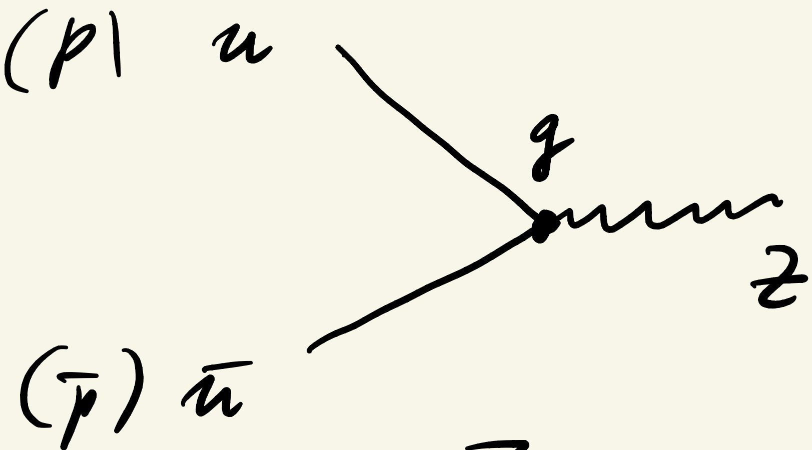
SPS  7km

$\rightarrow \boxed{\text{LEP}}$ $\quad e + \bar{e} \rightarrow$ - - -

$E_{cm} \simeq 205\ \text{GeV}$

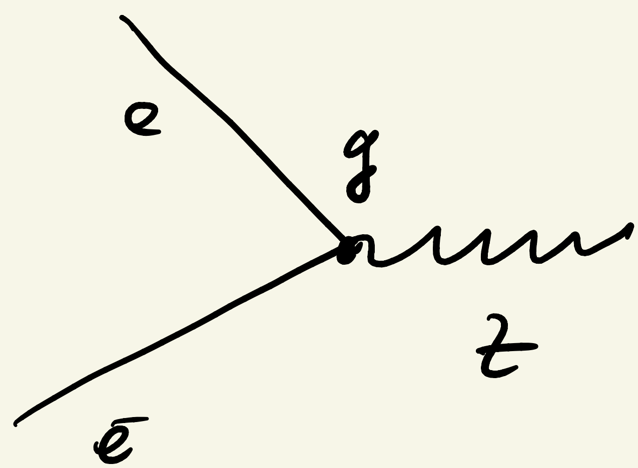
$\boxed{\text{SPS}}$ $\qquad e \simeq 1/3, \quad g \simeq 1/2$

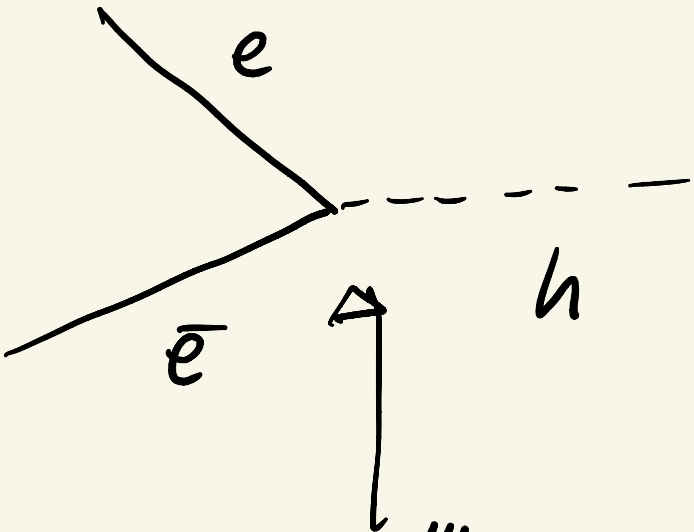
$$p + \bar{p} \longrightarrow \; ---$$



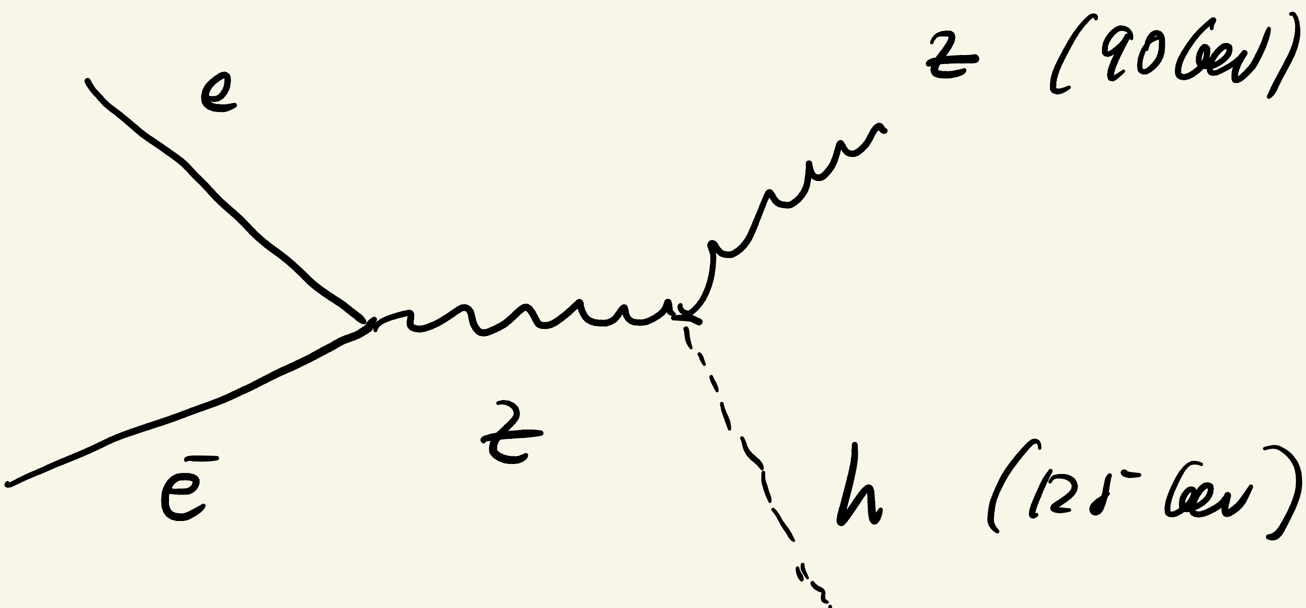
$$J_Z = T_3 - Q \sin^2\theta_W$$

$\boxed{\text{LEP}}$





$$g\,\frac{m_e}{M_W} \simeq 10^{-5} \longrightarrow 0$$



$$\Downarrow$$

$$\boxed{\begin{array}{c} 90+125 = 215\ \text{GeV} \\ > 205\ \text{GeV} \end{array}}$$

Tevatron $E \simeq \mathrm{TeV}$

SM Higgs

Halzen, Martin ?

Cheng, Li ?

Djouadi : Anatomy ----

Higgs Hunter's guide :