Neutrino Physics Course

Lecture III

LMU

Spring 2022

$$G_{min} = SU(2) \qquad \boxed{\theta_W}$$

$$\Downarrow$$

$$Q_{em} = T_3$$

$$\Rightarrow \; \ell = \pm M(1/2) \qquad \text{WRONG}$$

(1) $$\boxed{\overset{g}{SU(2)_L} \times \overset{g'}{U_Y(1)}} \qquad \text{1961 Glashow}$$

$$g, \quad \tan\theta_W = g'/g$$

$$s_W = \frac{g'}{\sqrt{g^2+g'^2}}, \quad c_W = \frac{g}{\sqrt{g^2+g'^2}}$$

$$Q_{em} = T_3 + \frac{Y}{2} \qquad Y \leftarrow U(1)$$

$$[T_a, T_b] = i\varepsilon_{abc} T_c \leftrightarrow SU(2)$$

$$\Rightarrow \boxed{Y = 2(Q - T_3)}$$

(ii) matter $= f = q, \ell$

quark lepton

$$D \text{ (doublet)} \rightarrow U D$$

$$U = e^{i \vec{T}_a \cdot \vec{\theta}_a} \qquad UU^+ = 1 = U^+U$$

$$T_a = \sigma_a/2 \quad \Leftarrow \quad \det U = 1$$

$$S \rightarrow S$$

$$\boxed{T_a D = \frac{\sigma_a}{2} D, \quad T_a S = 0}$$

$W^\pm$: $\frac{g}{\sqrt{2}} \bar{u}_L \gamma^\mu d_L W_\mu^+ + h.c.$

$$D_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$$

$\cancel{P}$ max.

$$\Downarrow$$

$$q_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad u_R, \; d_R$$

$$l_L \equiv \begin{pmatrix} \nu \\ l \end{pmatrix}_L \qquad e_R$$

$$G = SU(2) \times U(1)$$

$$a = 1, 2, 3 \quad A_a \qquad B$$

$$\Downarrow$$

$$D_\mu = \partial_\mu - i g T_a A_\mu^a - i g' \frac{Y}{2} B_\mu$$

$$A_1^\mu,\; A_2^\mu,\; \boxed{A_3^\mu \quad ; \quad B_\mu}$$

$$W^\pm_\mu = \frac{(A_1 \mp A_2)_\mu}{\sqrt{2}}$$

$$\Downarrow$$

$$\mathcal{L}_f = i \bar{f} \gamma^\mu D_\mu f$$

$$\downarrow$$

$$\mathcal{L}_f^{n.c.} = \bar{f} \gamma^\mu \left(g T_3 A_3 + g' \tfrac{Y}{2} B\right)_\mu f$$

$$= \bar{f} \gamma^\mu \left(g T_3 A_3 + g' (Q - T_3) B\right)_\mu f$$

$$= \bar{f} \gamma^\mu \left(T_3 \underbrace{(g A_3 - g' B)_\mu} + g' Q B_\mu\right) f$$

$\exists$ photon $A_\mu \longleftrightarrow e \bar{f} Q \gamma_\mu f$

$$\Downarrow$$

$$\exists \boxed{Z_\mu = \frac{(g A_3 - g' B)_\mu}{\sqrt{g^2 + g'^2}}}$$

$$\boxed{\begin{aligned} Z_\mu &= \cos\theta_W A_{3\mu} - \sin\theta_W B_\mu \\ A_\mu &= \sin\theta_W A_{3\mu} + \cos\theta_W B_\mu \end{aligned}}$$

$$\Downarrow$$

$$A_3^\mu = \sin\theta_W A^\mu + \cos\theta_W Z^\mu$$

$$B^\mu = \cos\theta_W A^\mu - \sin\theta_W Z^\mu$$

$$\Downarrow$$

$$\mathcal{L}_f^{n.c.} = \bar{f}\gamma^\mu\Big(\frac{g}{\cos\theta_W} T_3 Z_\mu +$$

$$g\tan\theta_W Q\left(\cos\theta_W A^\mu - \sin\theta_W Z^\mu\right)\Big) f$$

$$= \bar{f}\gamma^\mu\Big[\frac{g}{\cos\theta_W}\left(T_3 - \sin^2\theta_W Q\right) Z_\mu$$

$$+ g\sin\theta_W Q A_\mu\Big] f$$

$$\Downarrow$$

$$\boxed{e = g\sin\theta_W}$$

$\sin\theta_W \simeq 1/2$

$\theta_W \simeq 30$

$\sin^2\theta_W = 0.23$

$$\boxed{\rightarrow j^\mu_{em} = \bar{f}\gamma^\mu Q f \quad \therefore \quad e A_\mu j^\mu_{em}}$$

$$\boxed{j_Z^\mu = \bar{f}\gamma^\mu(T_3 - Q\sin^2\theta_W)f \qquad \frac{g}{\cos\theta_W} Z^\mu j_\mu^Z \qquad \theta_W}$$

$SU(2)\times U(1)$:

$$\boxed{m_A = m_Z = 0 = m_W}$$

<u>fermions</u>

$$m_f \bar{f} f = m_f(\bar{f}_L f_R + \bar{f}_R f_L)$$

$SU(2) \times U(1)$

$\frac{m}{E} \leftarrow$ measure of symm. breaking

$Z_\mu \bar{\nu}_L \gamma^\mu \nu_L$ (?)

$\frac{g}{\cos\theta_W} \frac{1}{2} \leftarrow \begin{pmatrix} \nu \\ e \end{pmatrix}_L$

$Z_\mu \bar{e} \gamma^\mu \left(-\frac{1}{2} L + \sin^2\theta_W\right) e$

only LH e : $T_3 \neq 0$

$$Q_Z = T_3 - Q \sin^2\theta_W$$

$$Q_Z = T_{3L} - Q \sin^2\theta_W \quad ?$$

P_{max}

$$\Uparrow$$

$$T_3 f_R = 0$$

$$T_3 f_L = t_3 f_L$$

Both correct

• P good $\Rightarrow$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$\Rightarrow \boxed{m_\nu \neq 0}$$

Massive gauge boson

Proca

$$\mathcal{L}_p = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathcal{L}_p = -\frac{1}{2} (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu) + \frac{1}{2} m^2 A_\mu A^\mu$$

$$= -\frac{1}{2}(\partial_\mu A_\nu \partial^\mu A^\nu - \partial_\nu A_\mu \partial^\mu A^\nu) + $$

$= \{$ integrate by part $\}$

$$= \frac{1}{2} A_\mu \left\{ (\Box + m^2) g^{\mu\nu} - \partial^\mu \partial^\nu \right\} A_\nu$$

$$\Downarrow$$

$$\partial_\mu \left[(\Box + m^2) g^{\mu\nu} - \partial^\mu \partial^\nu \right] A_\nu = 0$$

$$\Downarrow$$

$$\left[(\cancel{\Box} + m^2) \partial^\nu - \cancel{\Box \partial^\nu} \right] A_\nu = 0$$

$m^2 \neq 0$ $\Downarrow$

$$\boxed{\partial_\mu A^\mu = 0}$$

$$\Longleftrightarrow \underbrace{3 \text{ d. o. f.}}_{\text{degrees of freedom}}$$

$$A_\mu = \varepsilon_\mu(p) e^{ip\cdot x}$$

$$\Downarrow$$

$$\mathcal{L}_p \to \varepsilon_\mu \left[(p^2 - m^2) g^{\mu\nu} - p^\mu p^\nu \right] \epsilon_\nu$$

$$\Downarrow$$

propagator = inverse of

$$\Delta_{\mu\nu} = a(p) g^{\mu\nu} + b(p)\, p^\mu p^\nu$$

$$\Downarrow$$

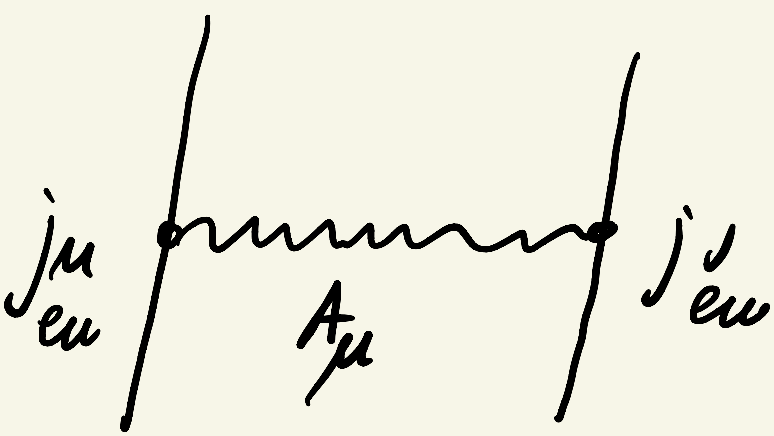
$$\boxed{\Delta_{\mu\nu} \propto \frac{g_{\mu\nu} - \frac{p_\mu p_\nu}{m_A^2}}{p^2 - m_A^2}}$$

$$\Delta_{\mu\nu} \xrightarrow[m_A \to 0]{} \text{infinity}$$

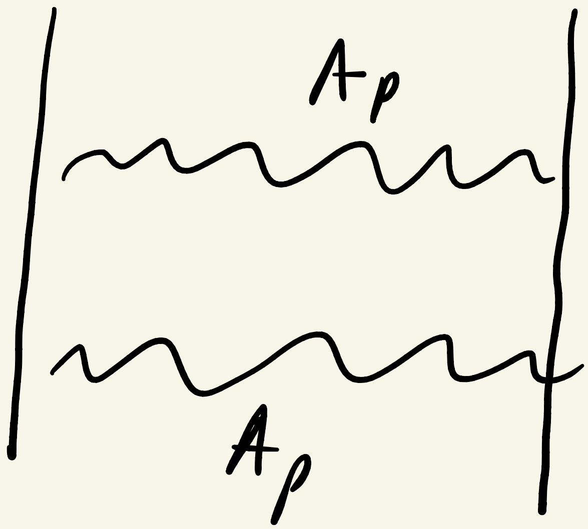
<u>QED</u> $e A_\mu j^\mu_{em}$ $\therefore$

$$\partial^\mu j_\mu^{em} = 0$$

$$\Rightarrow \boxed{p^\mu j_\mu^{em}(p) = 0}$$



$$\propto j^\mu_{em} \frac{p_\mu p_\nu}{m_A^2} j^\nu_{em} = 0$$



loop

Proca

$$\Delta_{\mu\nu} \xrightarrow[p\to\infty]{} \frac{1}{m_A^2}$$

$$\propto \int d^4p \; \frac{1}{m_A^2}\frac{1}{m_A^2}\;\frac{1}{\not{p}}\;\frac{1}{\not{p}}$$

large p

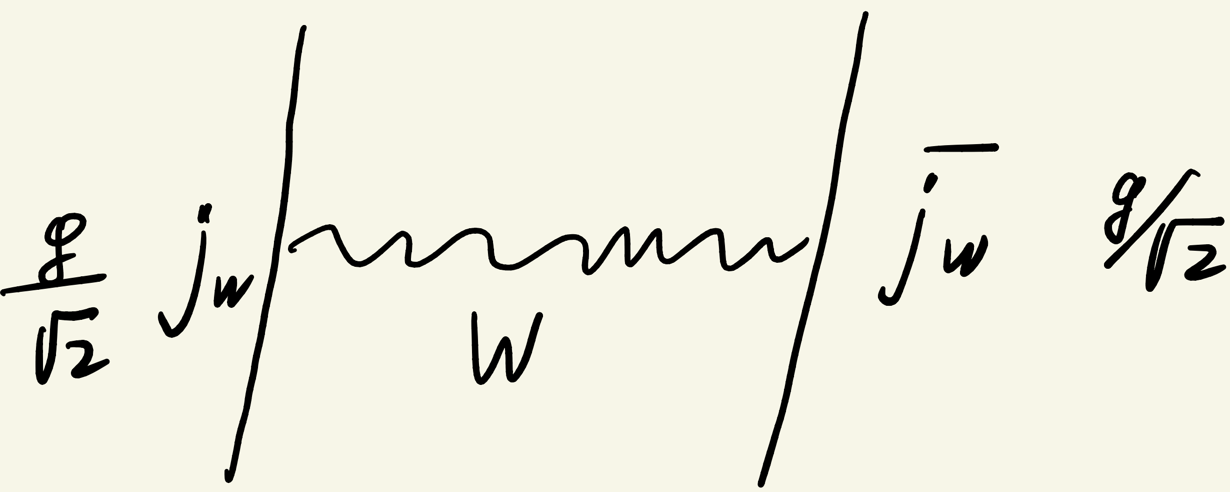
$$\propto \int^{\Lambda} \frac{\cancel{p^2}\, dp^2}{\cancel{p^2}} \frac{1}{m_A^4} = \frac{\Lambda^2}{m_A^4}$$

$$SM = SU(2) \times U(1)$$

$$\downarrow$$

$$W^{\pm} \qquad \frac{g}{\sqrt{2}}\, j_L^{W\mu} W_\mu^{+} + h.c.$$

$$j_L^{W\mu} = \left(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L\right)$$



$$\Downarrow$$

$$H_{wk} = j_{W_L}^{\mu}\ \Delta_{\mu\nu}(W)\ \overline{j}_{W_L}^{\nu}\ g^2/2$$

$$\|$$

$$\underbrace{\mathcal{H}^{W}_{\text{eff}} = j^{\mu}_{W_L} \, \frac{g_{\mu\nu} - \frac{p_\mu p_\nu}{M_W^2}}{p^2 - M_W^2} \, \bar{j}^{\nu}_{W_L} \, \frac{g^2}{2}}_{}$$

$$(p \ll M_W) \qquad \frac{p_\mu p_\nu}{M_W} \sim \frac{p^2}{M_W^2} \to 0$$

$$\Downarrow$$

$$g = e \sin\theta_W$$

$$\frac{4 G_F}{\sqrt{2}} = \frac{g^2}{2 M_W^2}$$

$$\Downarrow$$

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{e^2}{8 (M_W \sin\theta_W)^2}}$$

$\theta_W \longrightarrow M_W$

exp

$$\boxed{M_W \sin\theta_W \simeq 40\,\text{GeV}}$$

$$\boxed{M_Z = ?}$$ $\Longleftarrow$ experiment

· Higgs (Weinberg '67)

$\Downarrow$

$$\boxed{M_Z \cos\theta_W = M_W}$$

CDF $\leftarrow$ Fermilab

$\Uparrow$ modified