FROM UNIVERSE

TO PLANETS

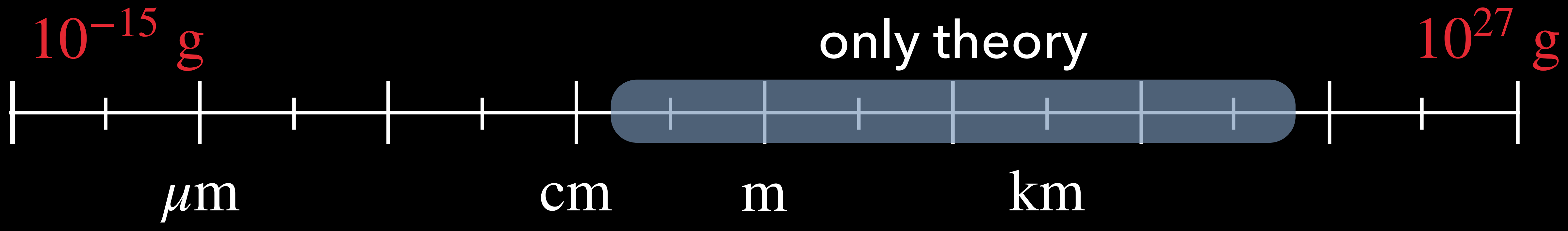
LECTURE 3: DUST TO PLANETESIMALS

REVIEW: DUST SIZES AND MASSES

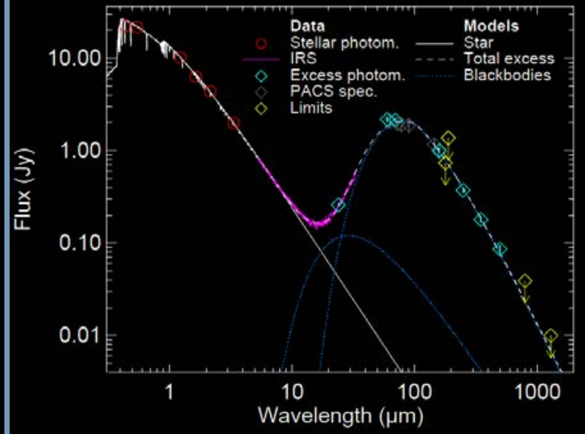
Samples

Lab & IDPs (interplanetary dust particles)

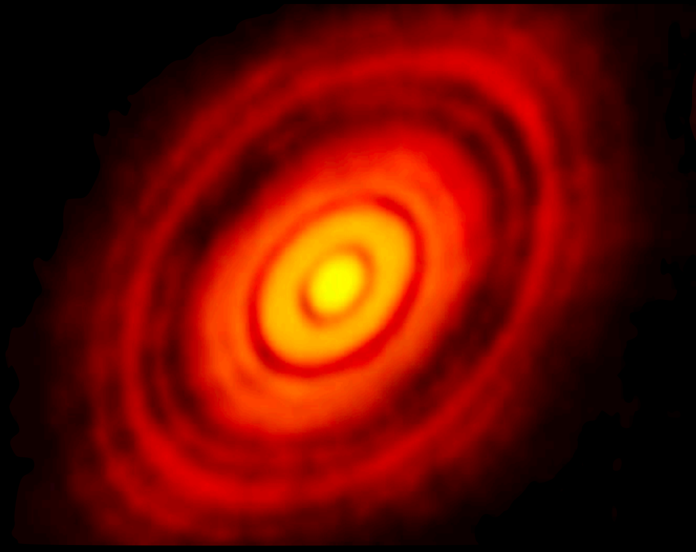
Meteorites



Observations:

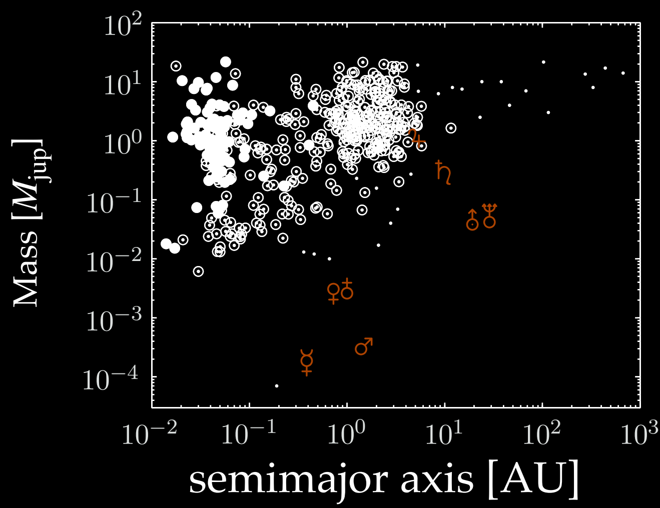


SED



Dust continuum

Direct imaging



Transit surveys

REVIEW: DUST SIZES AND MASSES

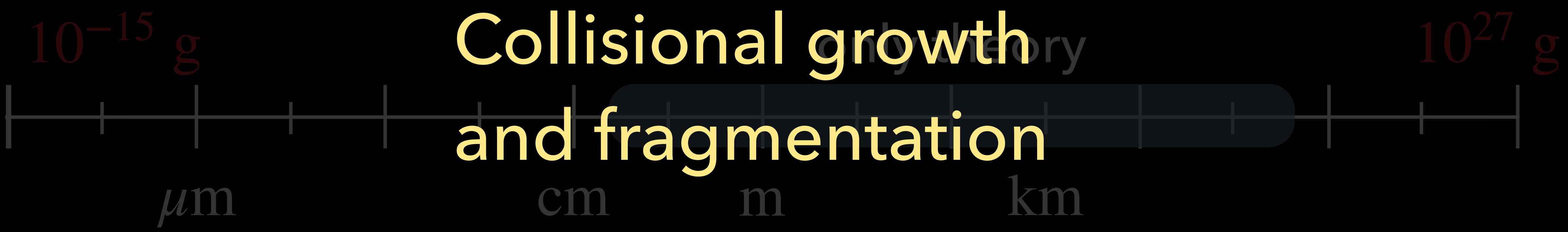
Samples

Inherited (ISM)

Condensation

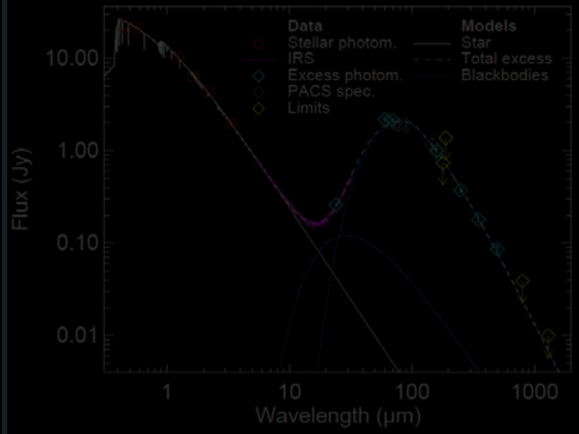
Lab & IDPs (interplanetary dust particles)

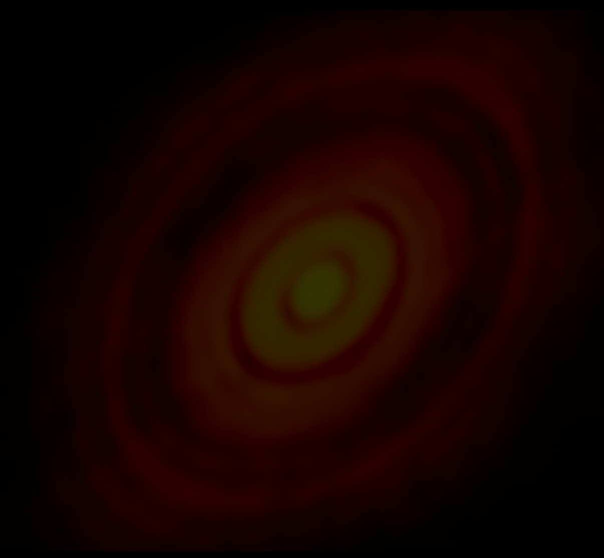
Meteorites

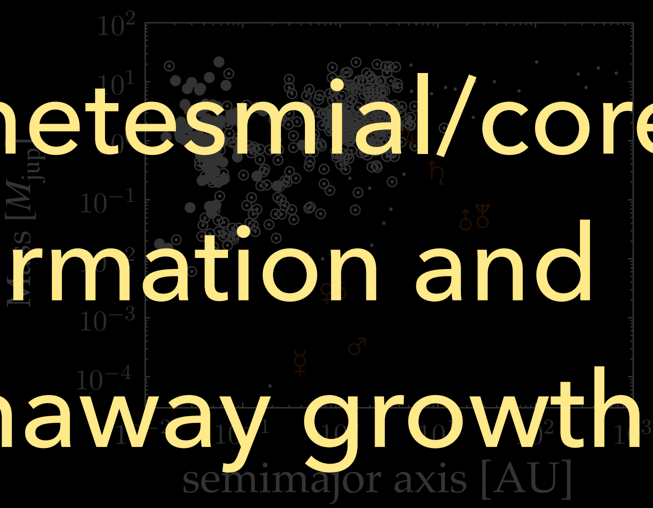


Collisional growth
and fragmentation

Observations:







Planetesmial/core
formation and
runaway growth

SED

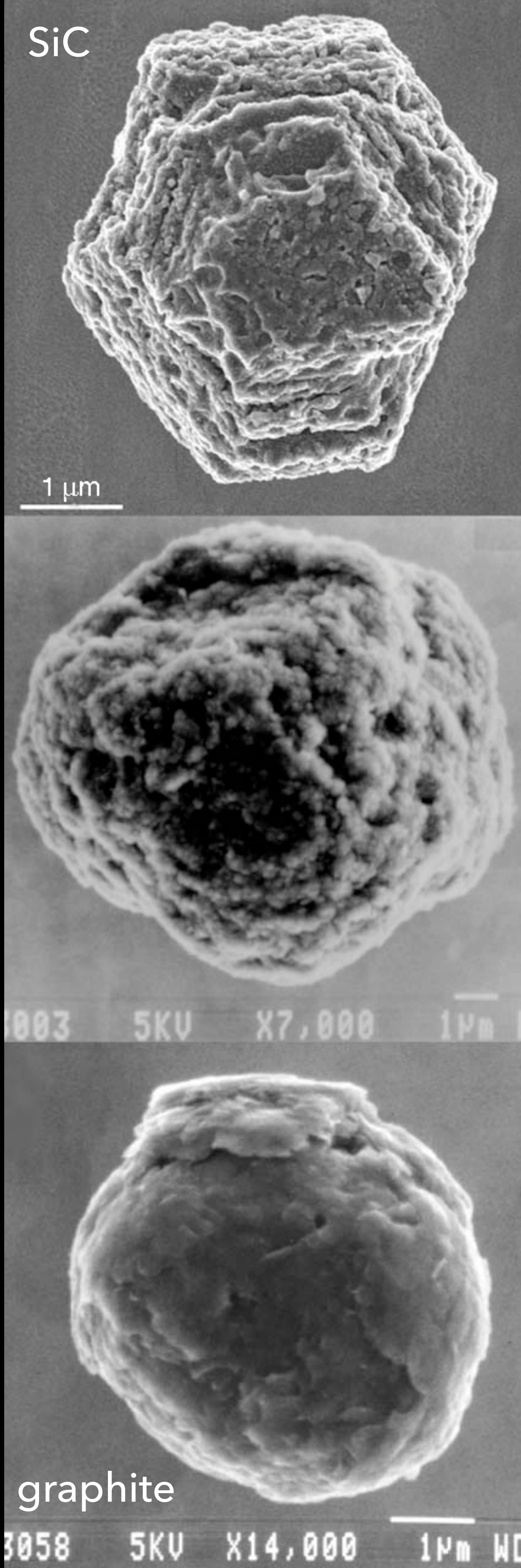
Dust continuum

Direct imaging

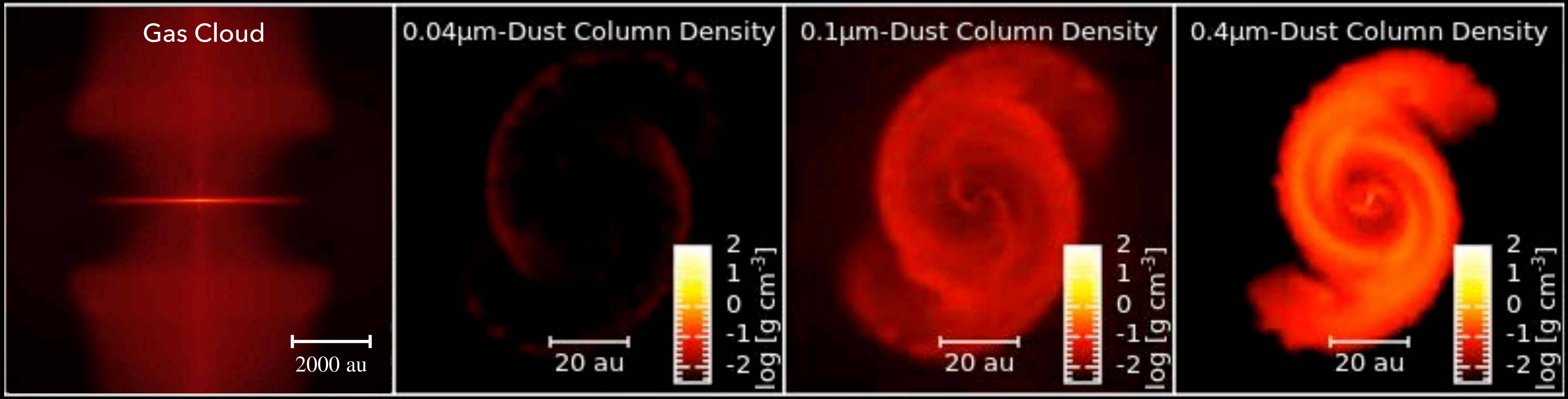
Transit surveys

INITIAL DUST DISTRIBUTION

- ISM has a dust-to-gas ratio of ~0.01 with sizes ranging from nm-μm
- Early growth may occur, but collapse of GMCs can be messy and not all grains survive
- Only presolar grains are known for sure to survive:
    - Small refractory grains like nano-diamonds, graphite particles, or silicon carbide (SiC) grains.
- Meanwhile, new dust grains condense out in the disc
    - Refractory elements in the inner disc
    - Volatile elements beyond snow lines

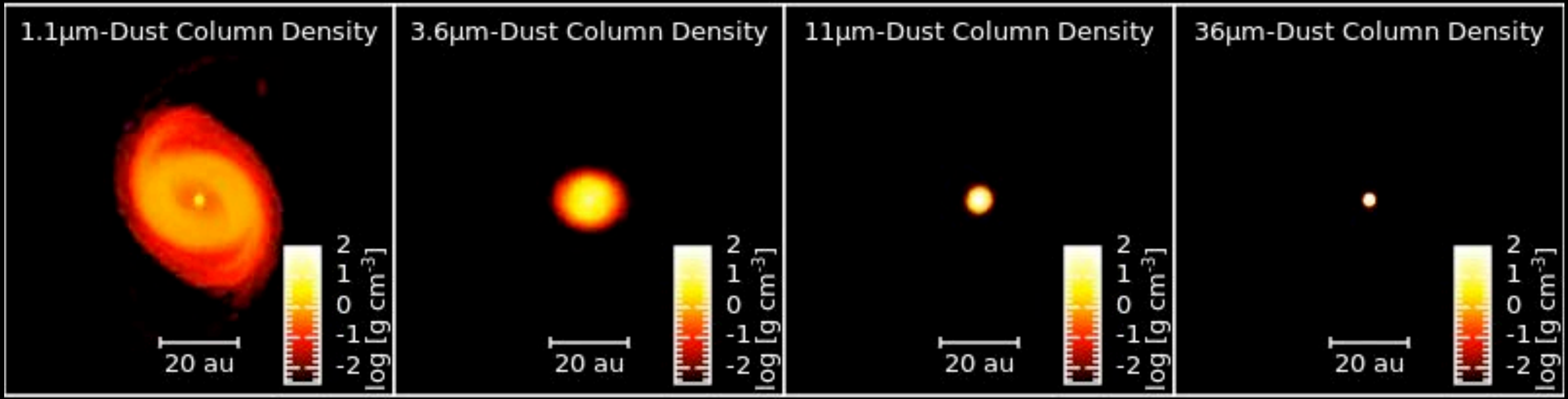


INHERITED FROM ISM



Caveat: the first hydrostatic core lasts until $T \sim 2000$ K, when H_2 dissociates. However, dust already sublimates at $T \gtrsim 1500$ K

EARLY GROWTH



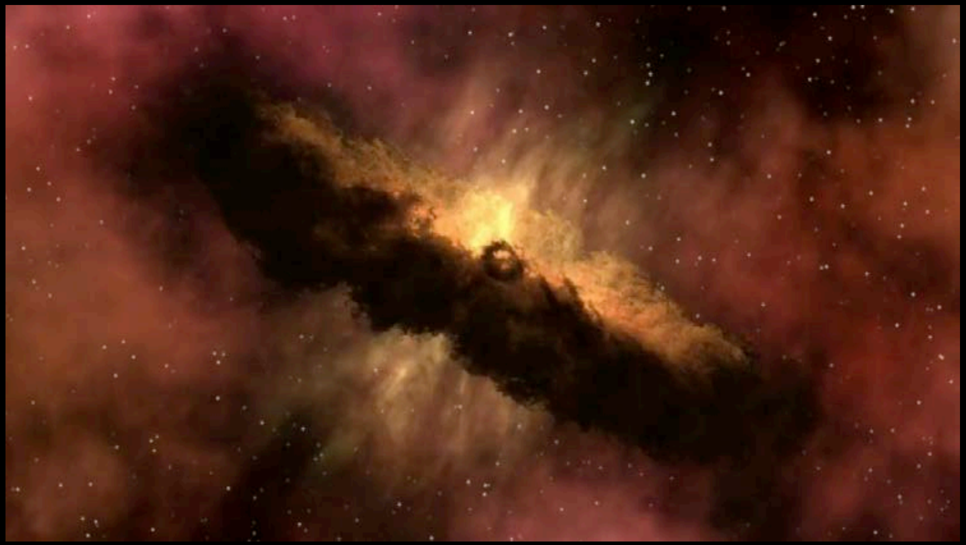
credit: Bate (2022)

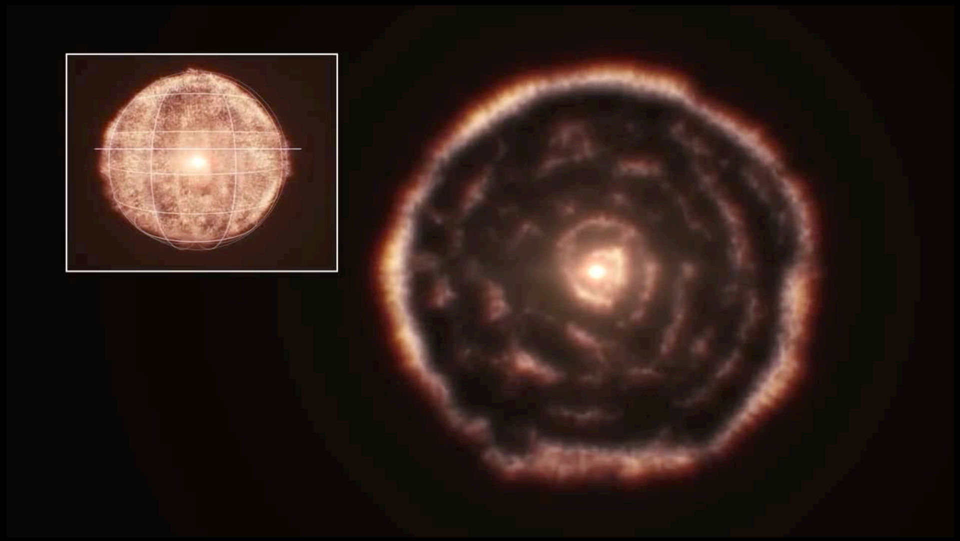
FROM UNIVERSE

TO PLANETS

LECTURE 3.1: CONDENSATION

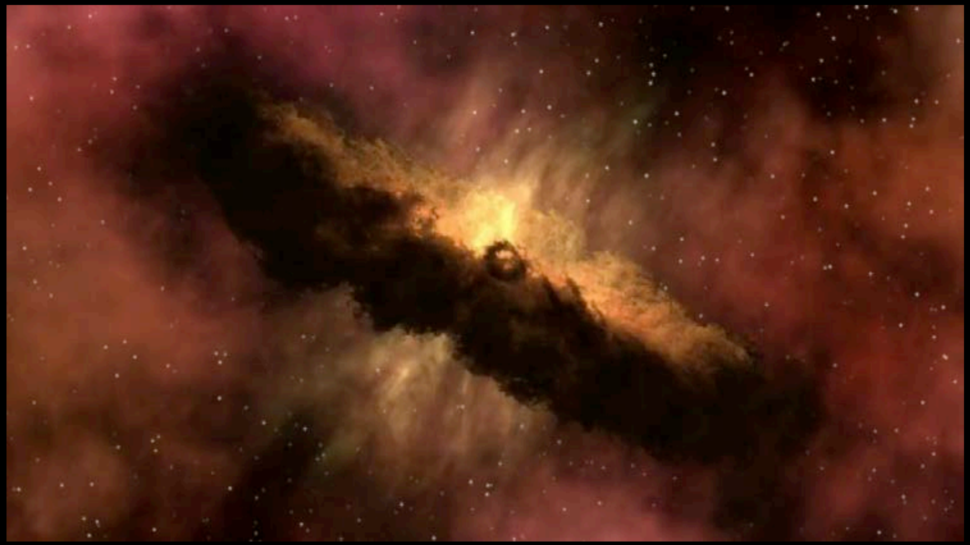
CONDENSATION





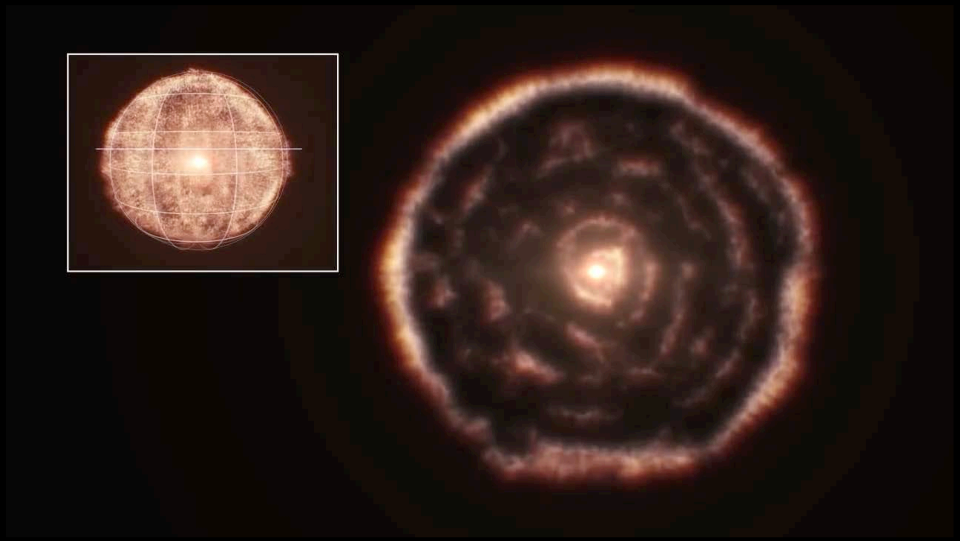
Planetary Atmospheres

CONDENSATION



Stellar Winds and Supernovae

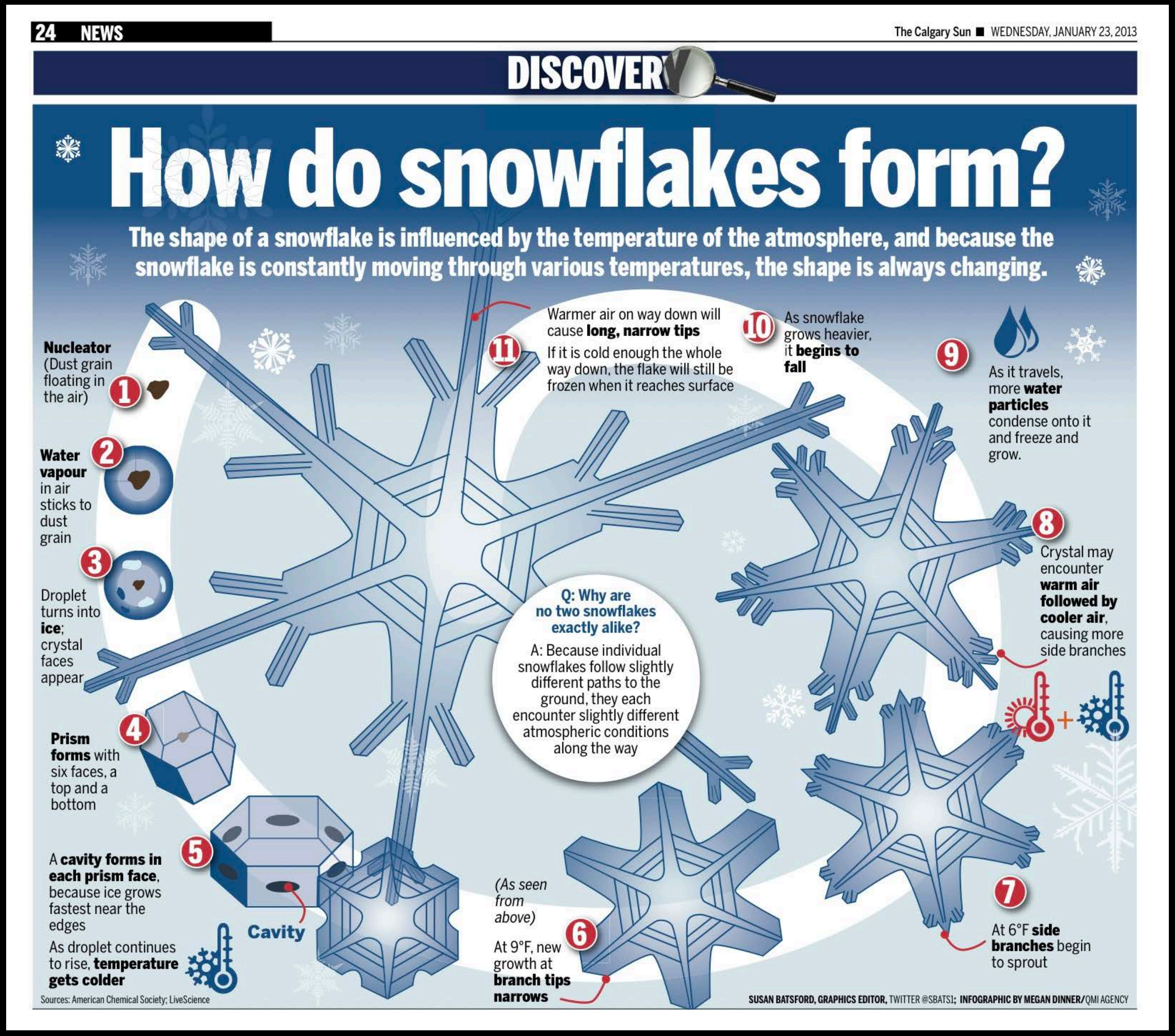
CONDENSATION



Protoplanetary disc

CONDENSATION

- Snowflakes form high in Earth's atmosphere, where the temperature and air pressure is low
- Water molecules in gas clouds bypass the liquid water stage and condense directly into ice crystals



24 NEWS

The Calgary Sun ■ WEDNESDAY, JANUARY 23, 2013

DISCOVERY

How do snowflakes form?

The shape of a snowflake is influenced by the temperature of the atmosphere, and because the snowflake is constantly moving through various temperatures, the shape is always changing.

Nucleator (Dust grain floating in the air) ①

② **Water vapour** in air sticks to dust grain

③ Droplet turns into **ice**; crystal faces appear

④ **Prism forms** with six faces, a top and a bottom

⑤ A **cavity forms in each prism face**, because ice grows fastest near the edges

As droplet continues to rise, **temperature gets colder**

(As seen from above)

⑥ At 9°F, new growth at **branch tips narrows**

⑦ At 6°F **side branches** begin to sprout

⑧ Crystal may encounter **warm air followed by cooler air**, causing more side branches

⑨ As it travels, more **water particles** condense onto it and freeze and grow.

⑩ As snowflake grows heavier, it **begins to fall**

⑪ Warmer air on way down will cause **long, narrow tips**

If it is cold enough the whole way down, the flake will still be frozen when it reaches surface

Q: Why are no two snowflakes exactly alike?

A: Because individual snowflakes follow slightly different paths to the ground, they each encounter slightly different atmospheric conditions along the way

Sources: American Chemical Society; LiveScience

SUSAN BATSFORD, GRAPHICS EDITOR, TWITTER @SBATS1; **INFOGRAPHIC BY MEGAN DINNER/**QMI AGENCY

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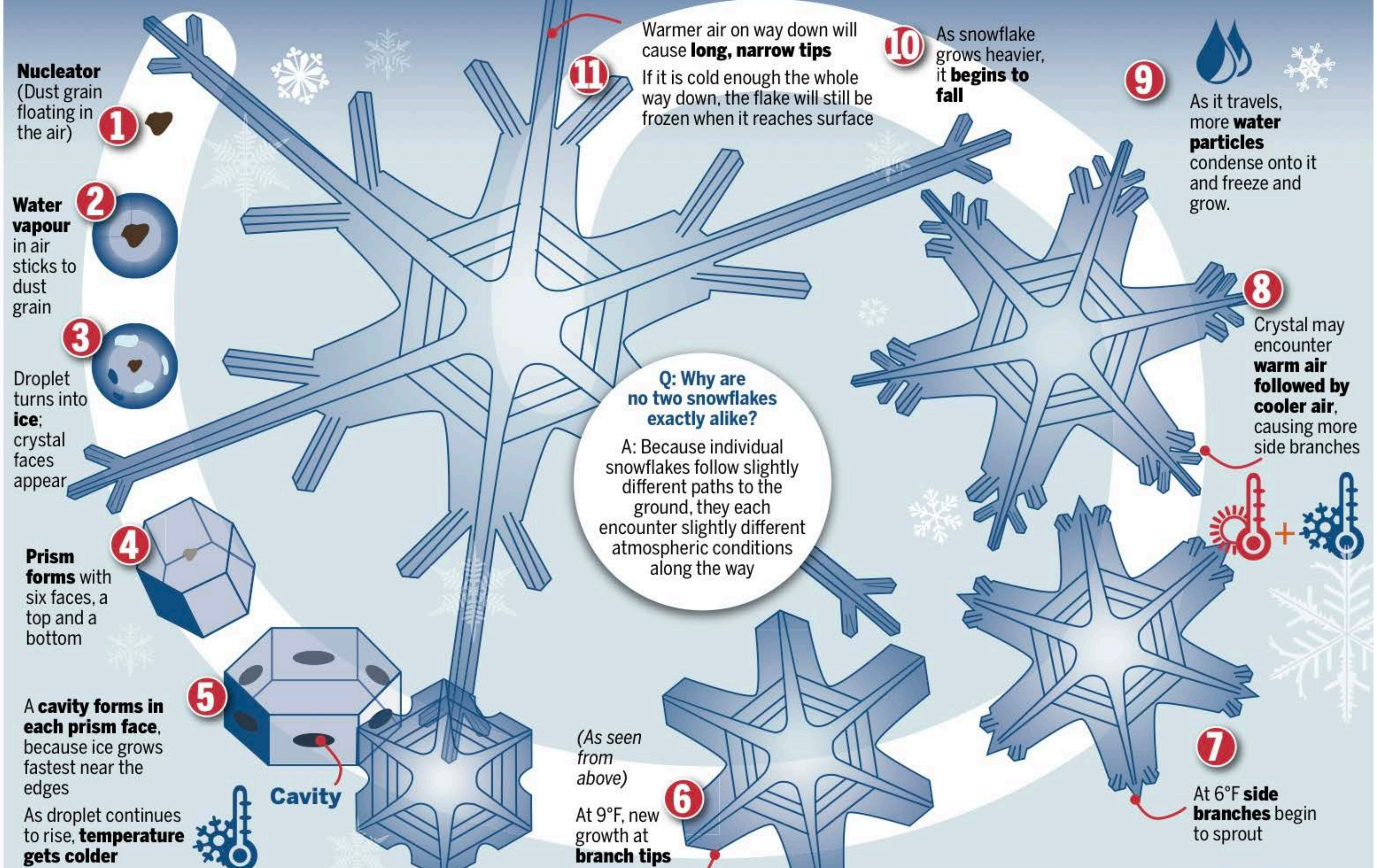

1 **Nucleator** (Dust grain floating in the air)

2 **Water vapour** in air sticks to dust grain

3 Droplet turns into **ice**; crystal faces appear

4 **Prism forms** with six faces, a top and a bottom

5 A **cavity forms in each prism face**, because ice grows fastest near the edges

Cavity

As droplet continues to rise, **temperature gets colder**

6 (*As seen from above*)

At 9°F, new growth at **branch tips narrows**

7 At 6°F **side branches** begin to sprout

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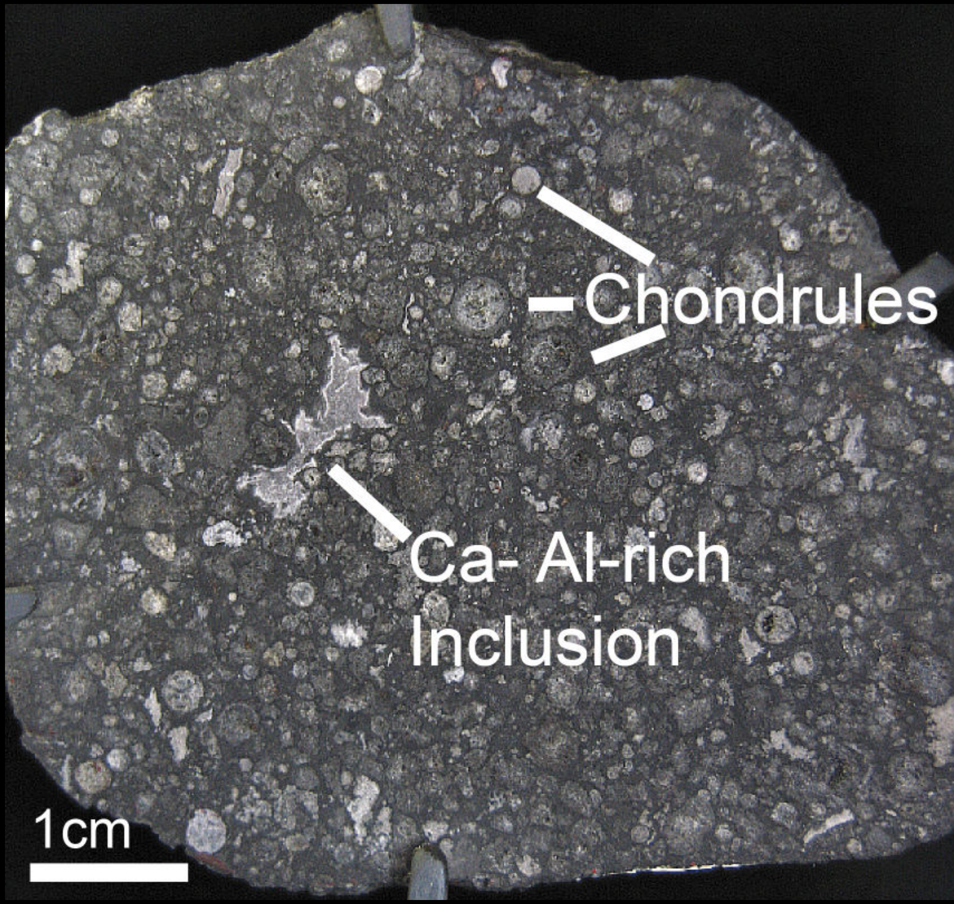
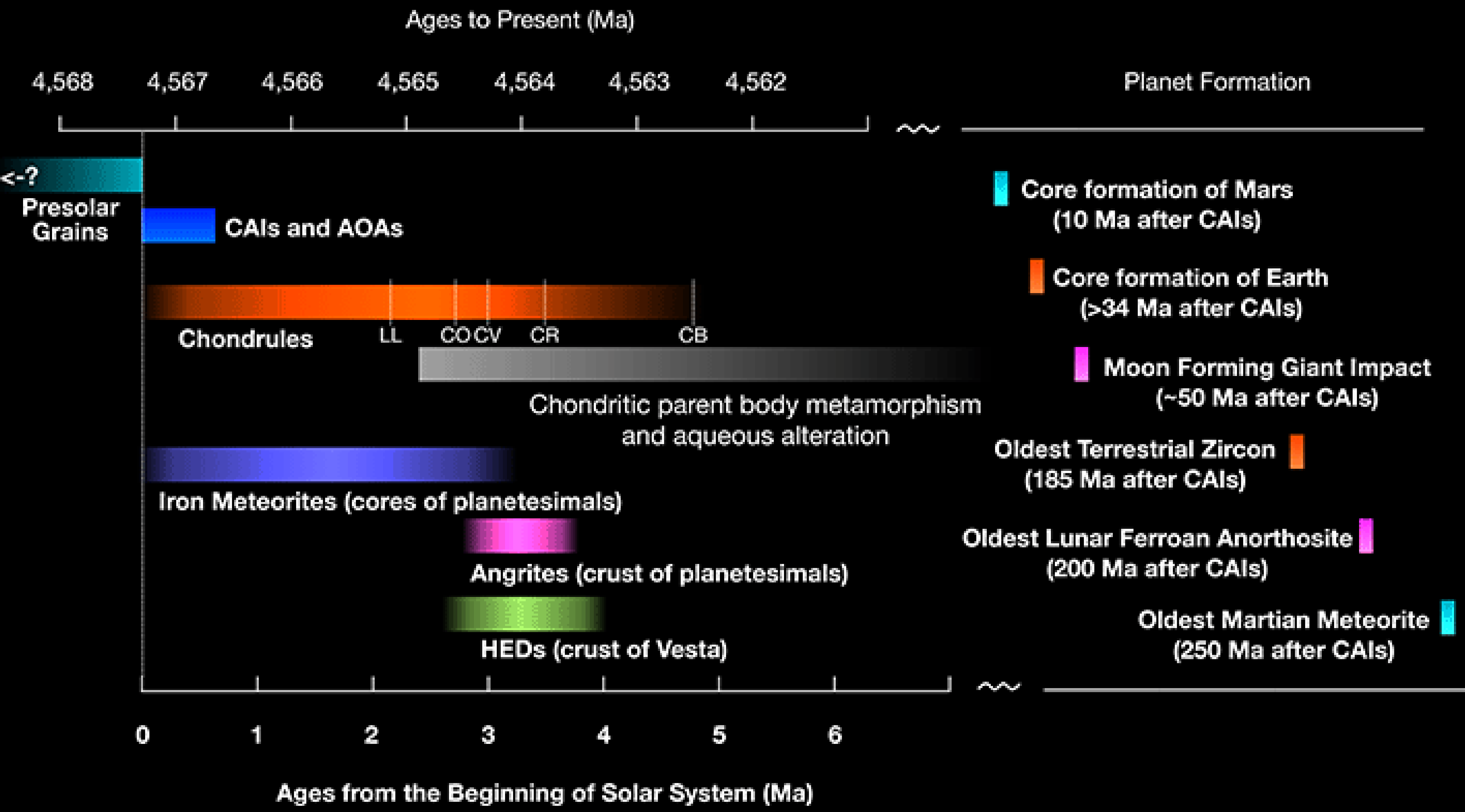
CONDENSATION

- A similar process occurs for refractory elements in protoplanetary discs as gas cools at very low pressures
  - First elements condense directly into mineral crystals at ~1,500 °C

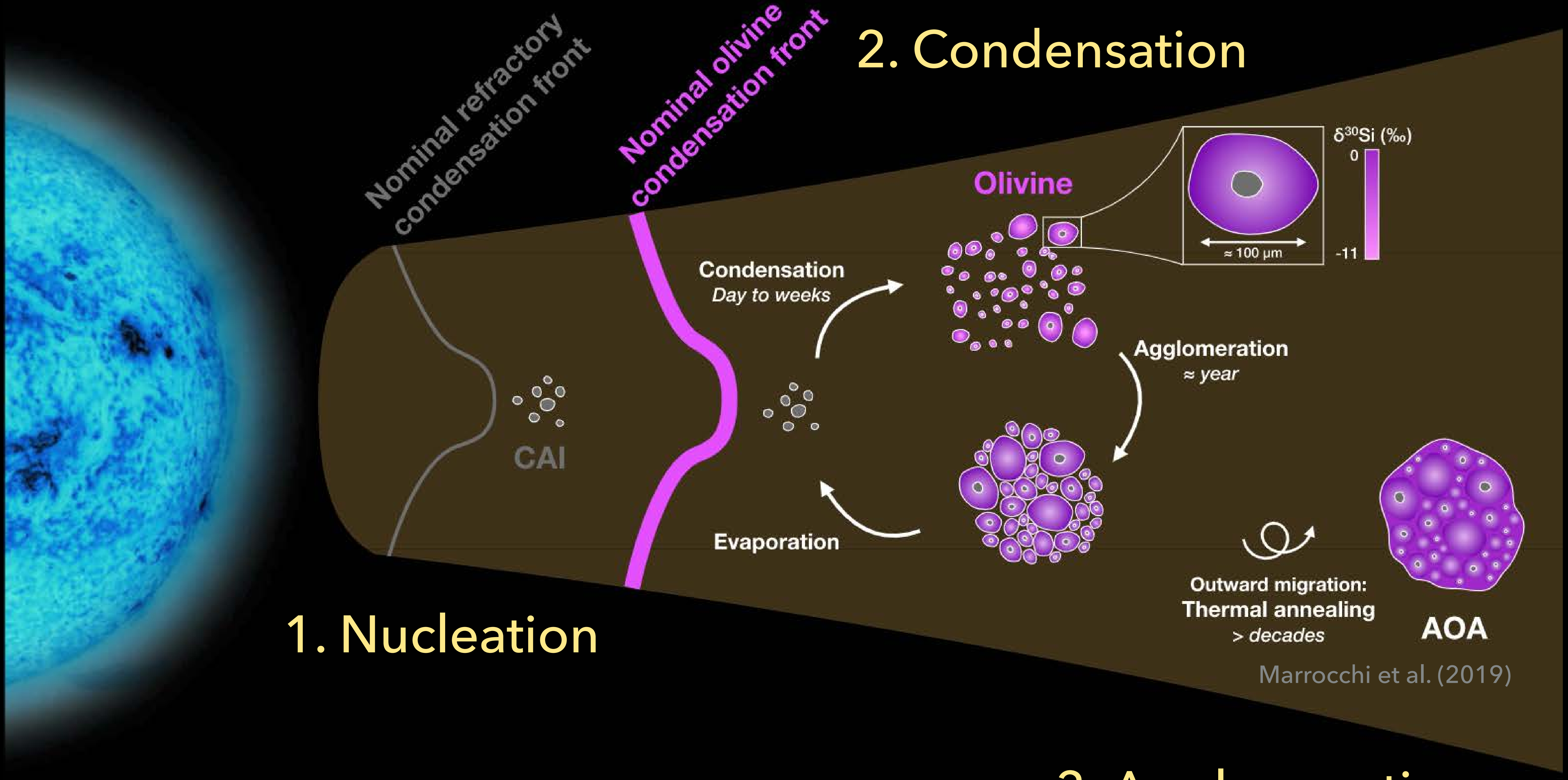


    - Oxide minerals rich in calcium, aluminium and titanium → Calcium-Aluminium Inclusions (CAIs)
  - At lower temperatures ($\lesssim 1$ Myr after CAIs), crystals of minerals containing magnesium, silicon and iron began to condense → chondrules

CONDENSATION



CONDENSATION



Marrocchi et al. (2019)

1. Nucleation

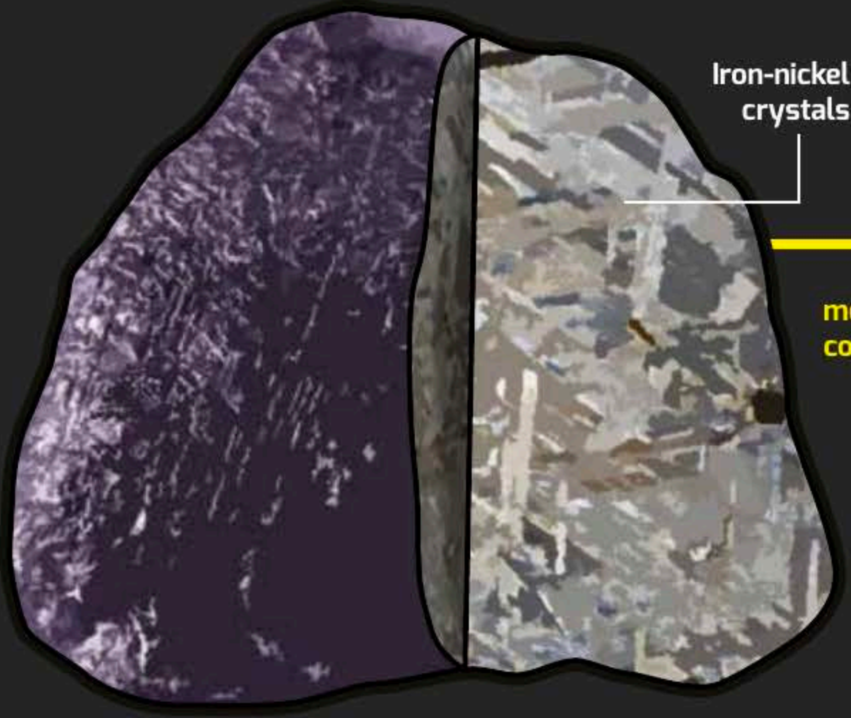
2. Condensation

3. Agglomeration

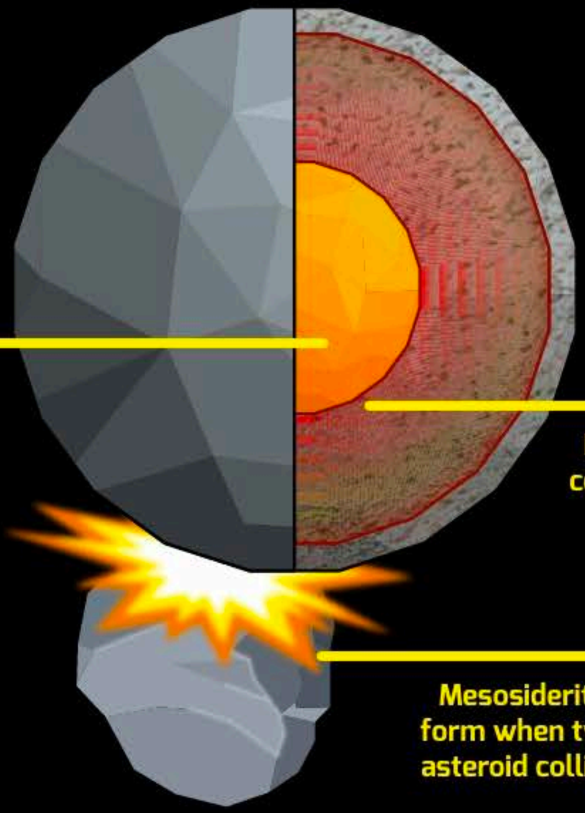
DIFFERENT TYPES OF METEORITE

LITTLE BOOK OF SPACE ROCKS

IRON METEORITES



Iron meteorites are made of about 90-95% iron with the rest made up of mostly nickel and some trace elements. It is thought that they come from the metallic cores of asteroids. Iron meteorites are rarer than stony meteorites but are easier to find because of their magnetism.



Iron meteorites come from the core

Pallasites come from near the core

Mesosiderites form when two asteroid collide

STONY-IRON METEORITES

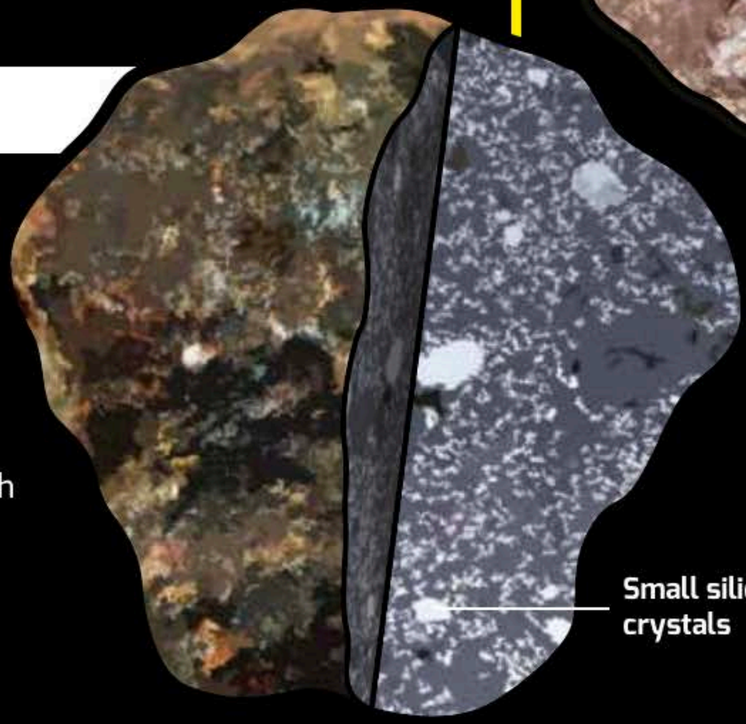
Stony-Iron meteorites are made of a mix of both metallic and rocky material. They probably formed when the metal cores and the rocky magmas inside asteroids mixed together, which makes them extremely rare. There are two types of stony-iron meteorites: pallasites and mesosiderites.

PALLASITE

Pallasites have solid bodies of nickel and iron but also contain large translucent crystals of olivine. Pallasites come from the area between the metallic core of an asteroid and the surrounding rocky magma.

MESOSIDERITE

Mesosiderites differ from pallasites in that their crystals are smaller and made of silicate minerals. It is thought that mesosiderites form when magma mixes with the core during a collision between two asteroids.



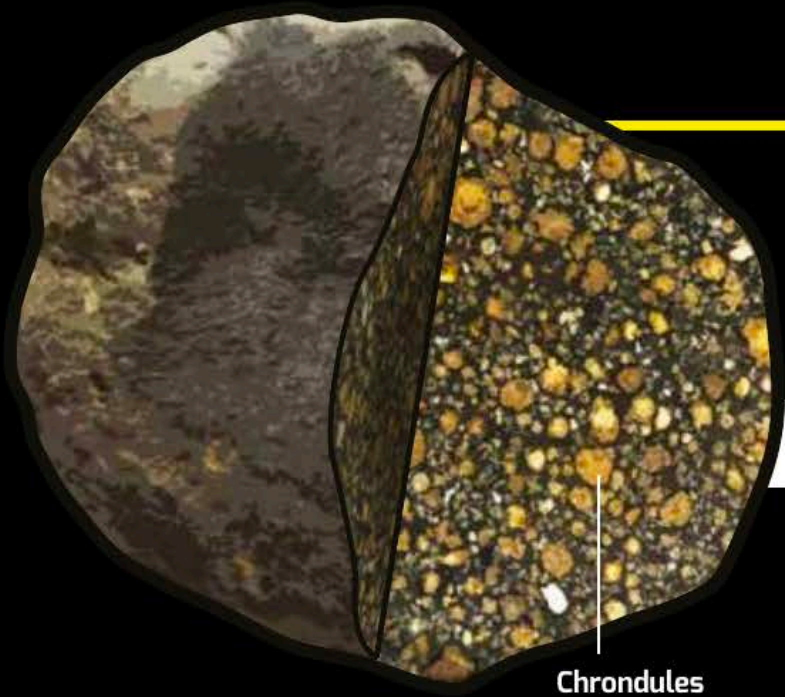
Credit: Ben Gilliland, STFC Borrow the Moon

DIFFERENT TYPES OF METEORITE

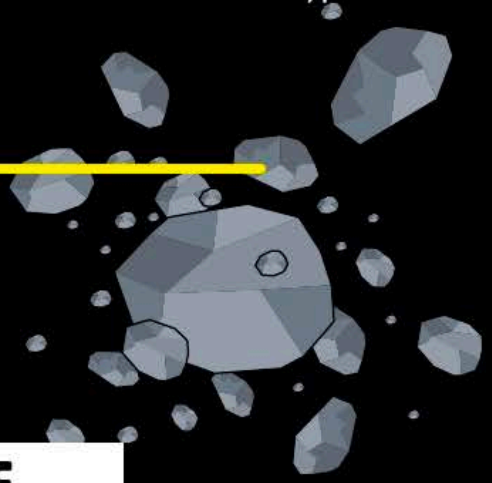
LITTLE BOOK OF SPACE ROCKS

STONY METEORITES

Stony meteorites are the most common type of meteorite. They are made of rock, but can also contain small amounts of iron. There are two types of stony meteorites: chondrites and achondrites.

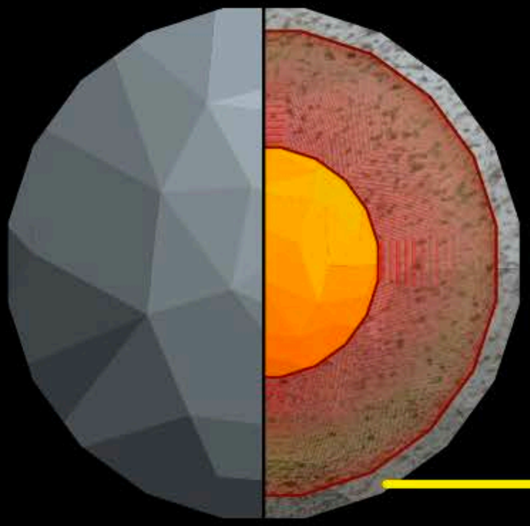


Chondrites come from undifferentiated asteroids



CHONDRITE

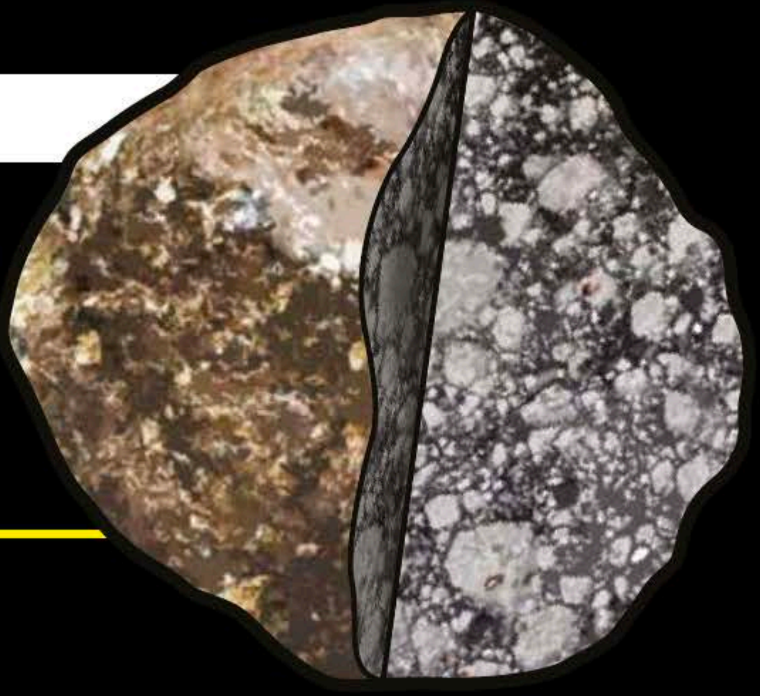
Chondrites contain rock that has changed little since the formation of the Solar System. They are made up of small mineral blobs called chondrules that formed in space billions of years ago and became clumped together.



ACHONDRITE

Achondrites are much younger than chrondrites. They contain minerals which have been melted, changed and altered since they were formed.

Achondrites come from the crust of asteroids or planets



NON-ASTEROID BELT

Not all meteorites come from the Asteroid Belt. Sometimes, very large collisions between an asteroid and a planet or moon can allow material from the surface to be ejected into space and sent on a trajectory that will overlap with the Earth's. This is, thus far, the only way we are able to get our hands on material from Mars.

Credit: Ben Gilliland, STFC Borrow the Moon

CHONDRITES

Carbonaceous Chondrite

- Carbonaceous chondrites show little chemical differentiation and fractionation
  - Primitive: provide clues to the initial chemical composition of the solar nebula
  - Contain volatile organic chemicals and water → no significant heating (>200 °C)

CHONDRITES

CI-Chondrite

- CI-chondrites (the I is for Ivuna): most primitive sub-class
    - Contain H_2O (17–22%; bound in silicates), Fe (25%; in form of iron oxides), C (3–5%), amino acids, and PAHs
    - Have not been heated above 50 °C (formed and remained beyond ~ 4 au)
    - Relative elemental abundances are similar to the Sun’s photosphere (notable exceptions are Li, used in nucleosynthesis, and volatile elements like H and O)

CONDENSATION

Near one-to-one correlation between elemental abundances found in Sun and CI chondrites

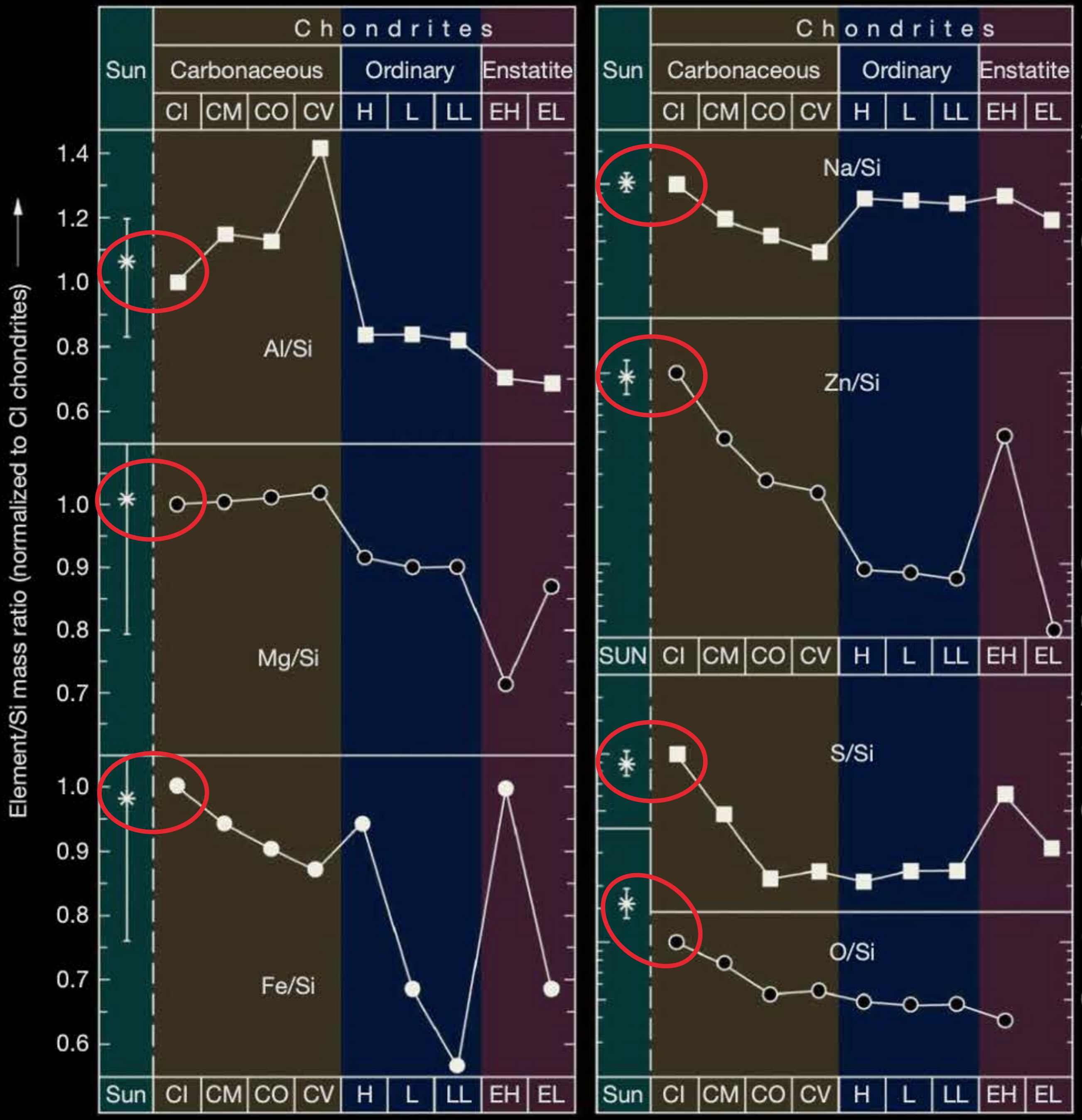
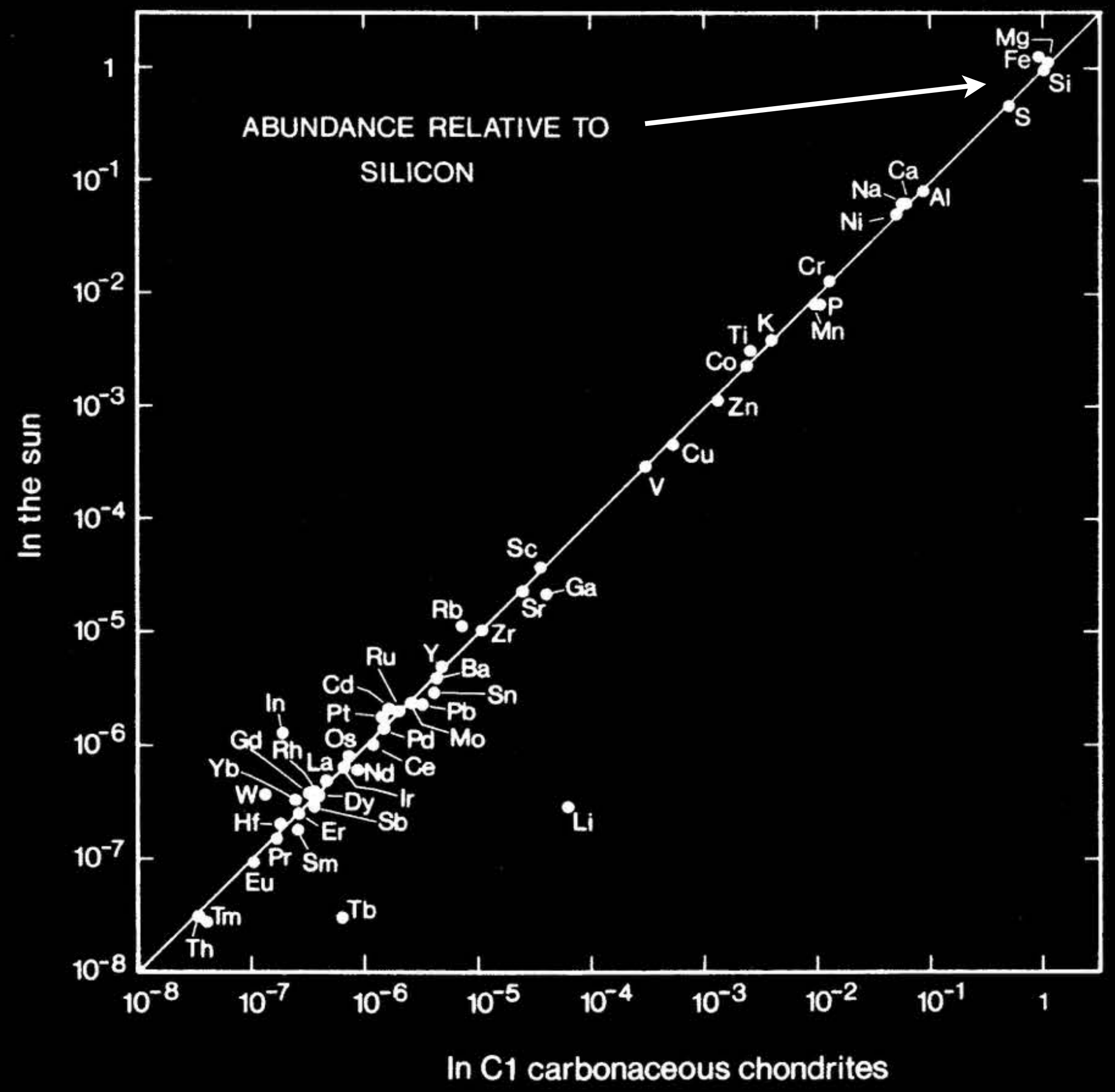
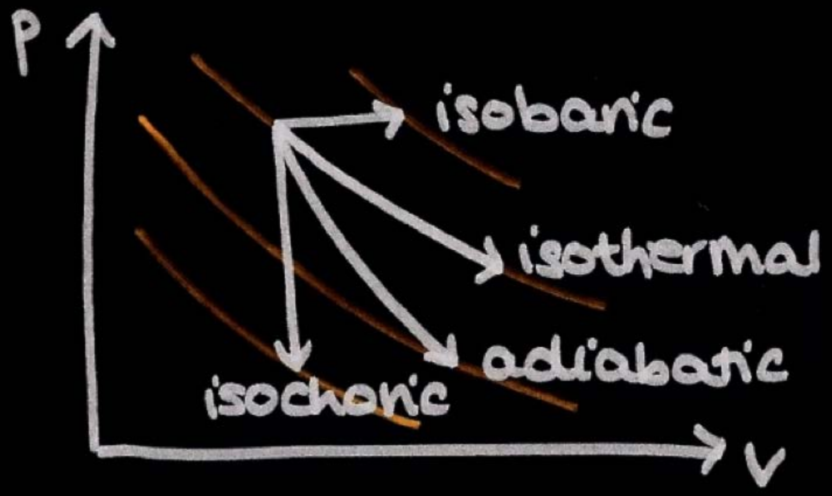


Figure 3 Element/Si ratios of characteristic elements in various groups of chondritic (undifferentiated) meteorites normalized to respective ratios in CI chondrites. Meteorite groups are arranged in order of decreasing oxygen content. The best match between solar photosphere measurements and meteoritic abundances is with CI chondrites (see text for details).



CONDENSATION

- In a thermodynamical system, processes will continue spontaneously until the relevant thermodynamical potential is minimised. In equilibrium, e.g.:
  - Helmoltz free energy is minimised for isothermal-isochoric systems: $F = U - TS$
  - Gibbs free energy (also called free enthalpy) is minimised for isothermal-isobaric systems: $G = F + PV = (U - TS) + PV = H - TS$ (as opposed to enthalpy $H = U + PV$)
- Chemical reactions typically occur in isothermal-isobaric conditions at thermodynamical equilibrium.



CONDENSATION

- Using the first law of thermodynamics ($dU = \delta Q - PdV$):

$$\begin{aligned} G &= H - TS &\longrightarrow\quad & dG = dH - TdS - SdT \\ H &= U + PV &\longrightarrow\quad & dH = dU + PdV + VdP = \delta Q + VdP \end{aligned}$$

- For reversible processes (where entropy is $dS = \delta Q_{\mathrm{rev}}/T$):

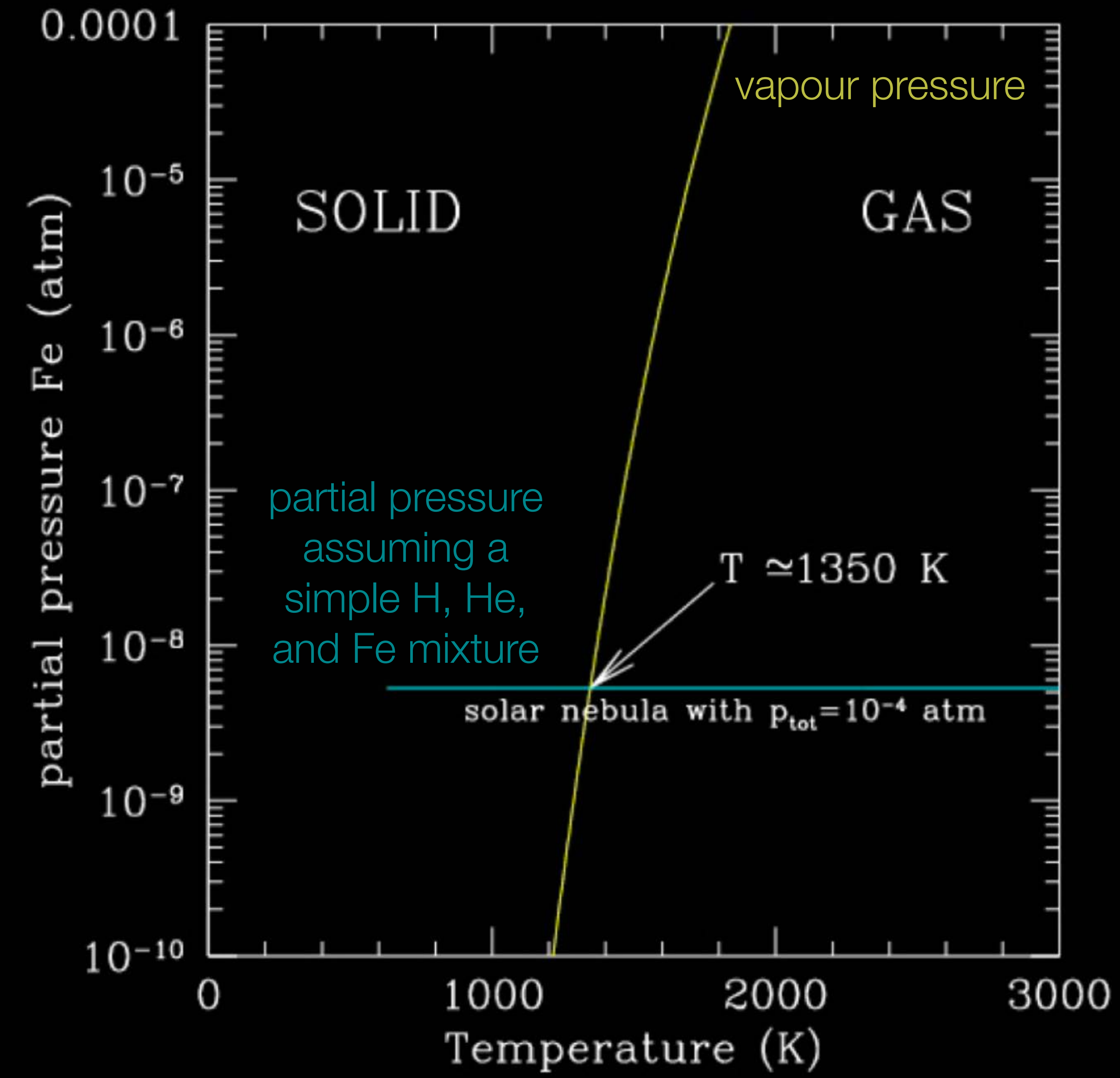
$$dG = \cancel{\delta Q} + VdP - \cancel{T}\left(\frac{\cancel{\delta Q}}{\cancel{T}}\right) - SdT = VdP - SdT$$

- In equilibrium, we can assume $dG = 0$ and the potential is defined to within a constant.

- Useful to define standard conditions to be used as a reference point, usually:

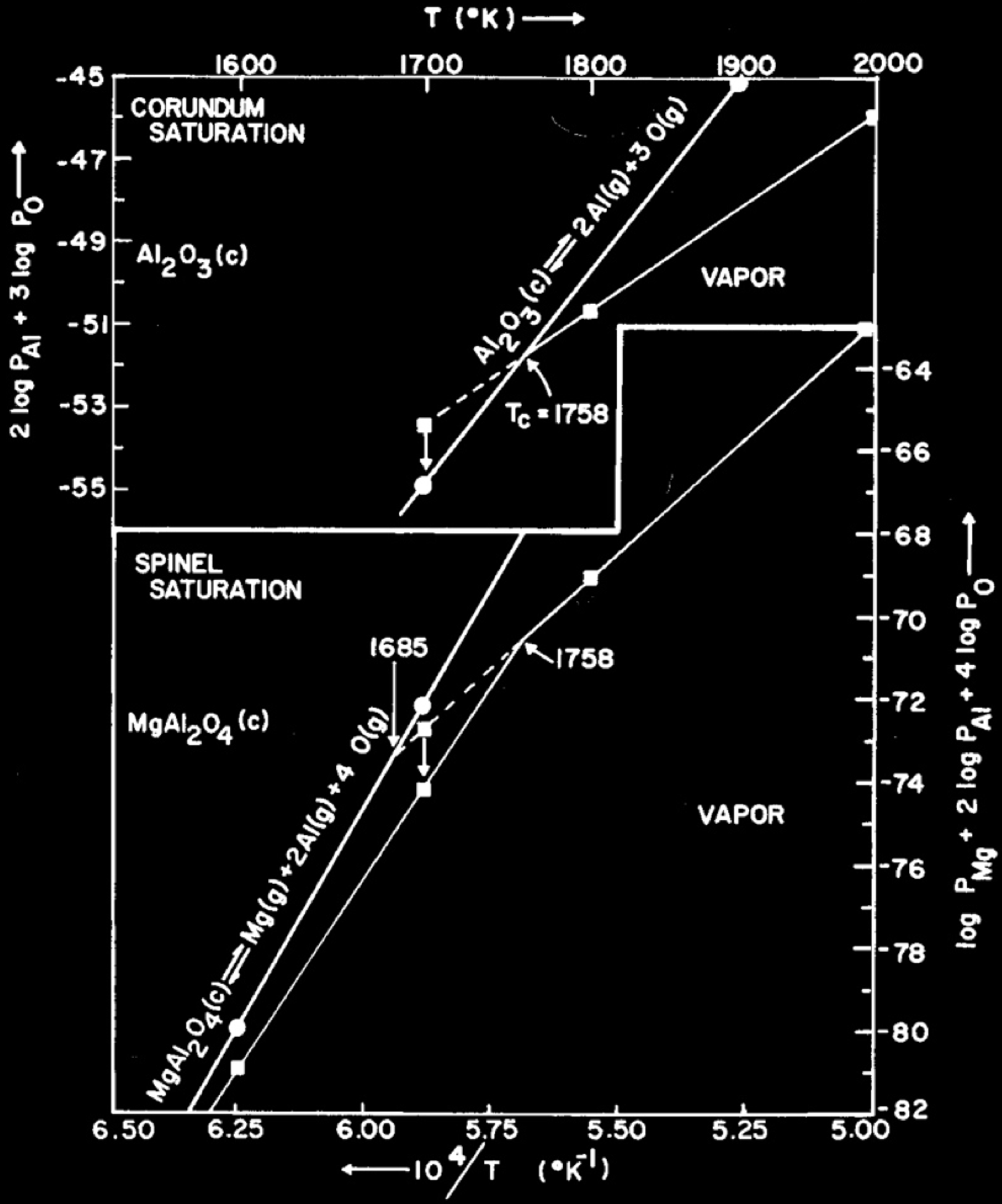
$$T_0 = 298\ \mathrm{K} \qquad\qquad P_0 = 1\ \mathrm{atm}$$

CONDENSATION: IRON EXAMPLE

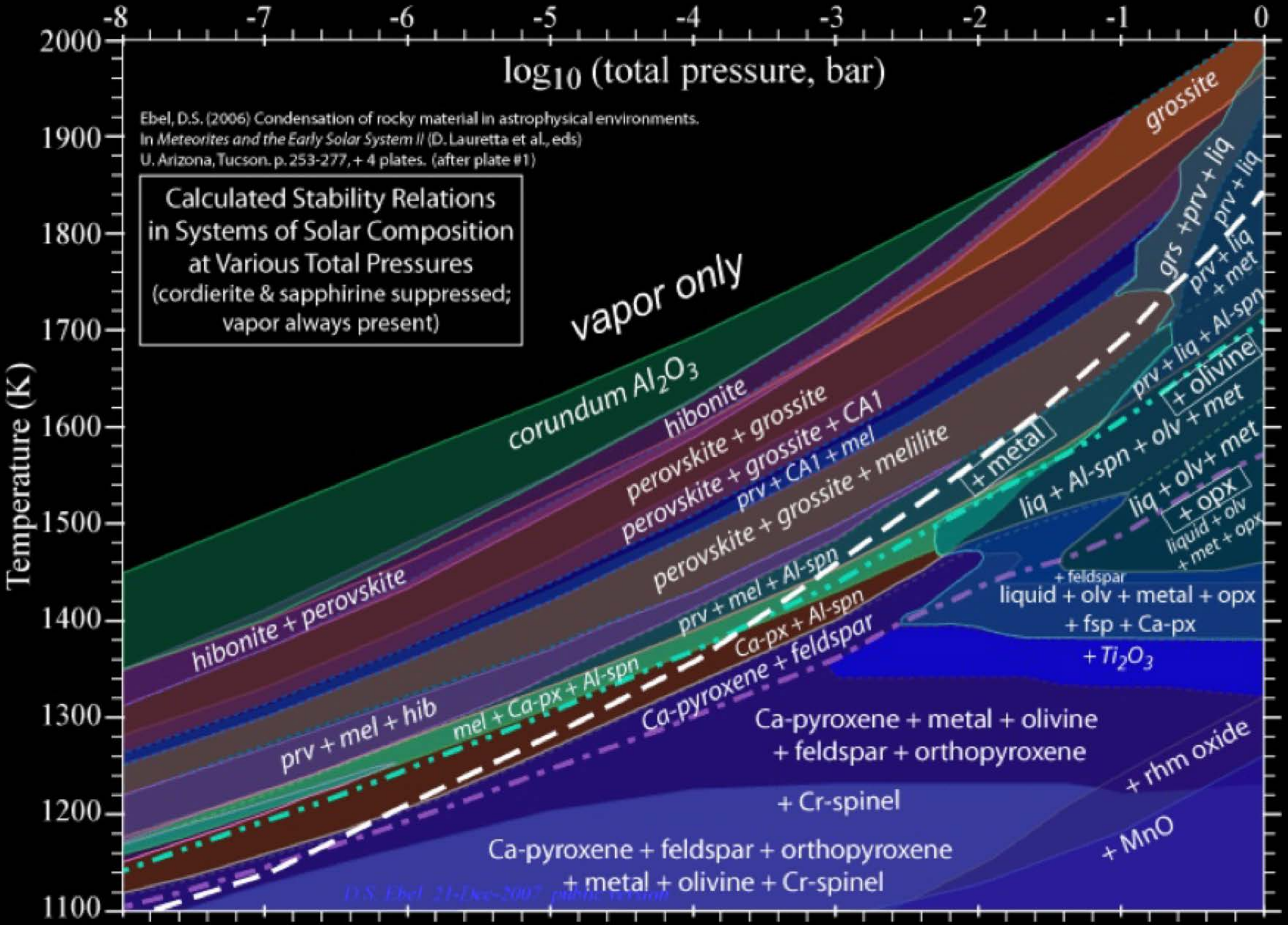


CONDENSATION: FULL SEQUENCE

- In more detailed models, the partial pressure is not a horizontal line (relative abundances depend on T and P).
- Normally, spinel would condense at $T = 1685\ \mathrm{K}$, but corundum condenses first and removes Al and O, causing the slope of the partial pressure to change.
- Condensation for spinel now happens at $T = 1500\ \mathrm{K}$.



CONDENSATION: FULL SEQUENCE



Ebel, D.S. (2006) Condensation of rocky material in astrophysical environments. In *Meteorites and the Early Solar System II* (D. Lauretta et al., eds) U. Arizona, Tucson. p. 253-277, + 4 plates. (after plate #1)

Calculated Stability Relations in Systems of Solar Composition at Various Total Pressures (cordierite & sapphirine suppressed; vapor always present)

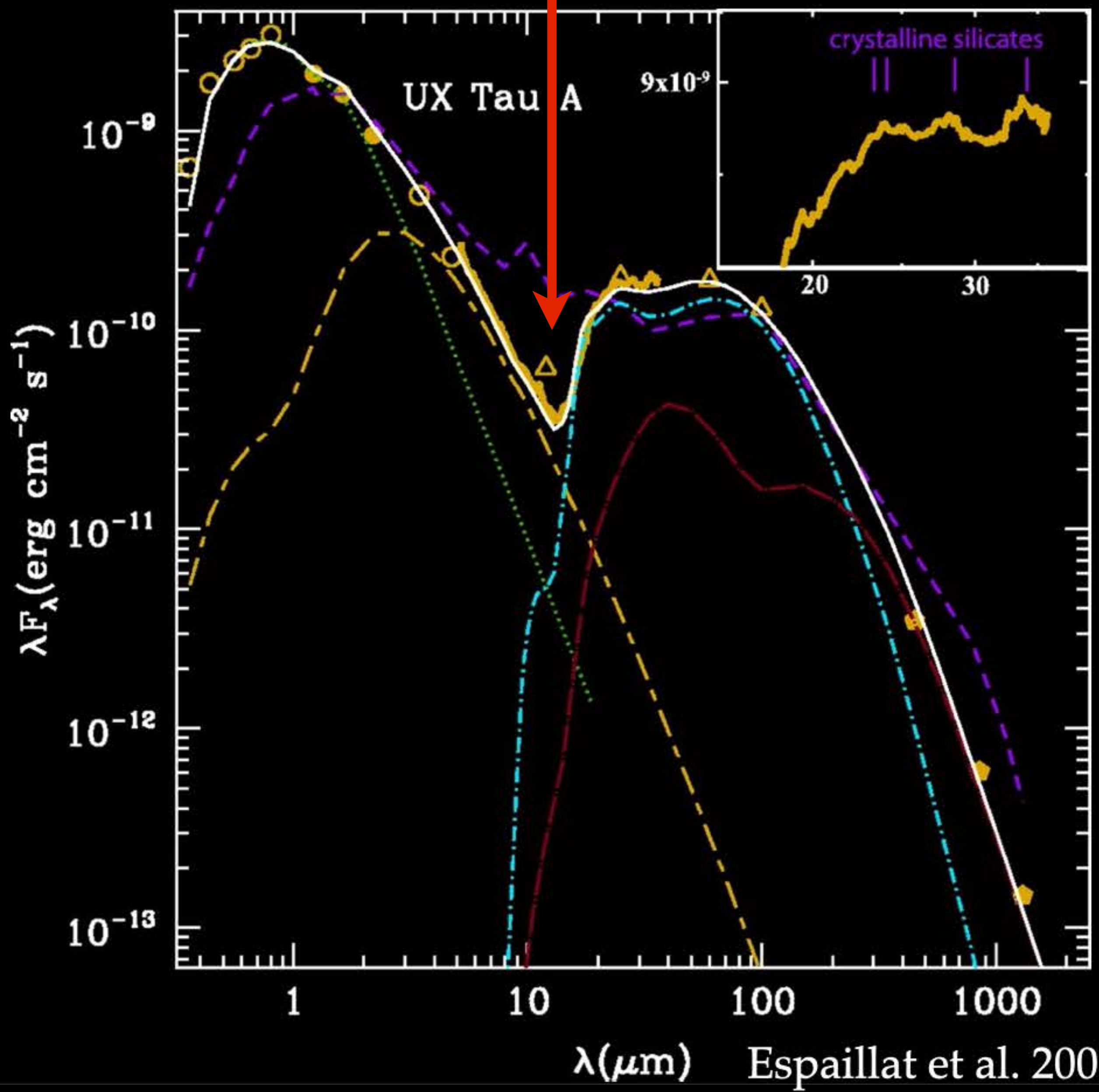
FROM UNIVERSE

TO PLANETS

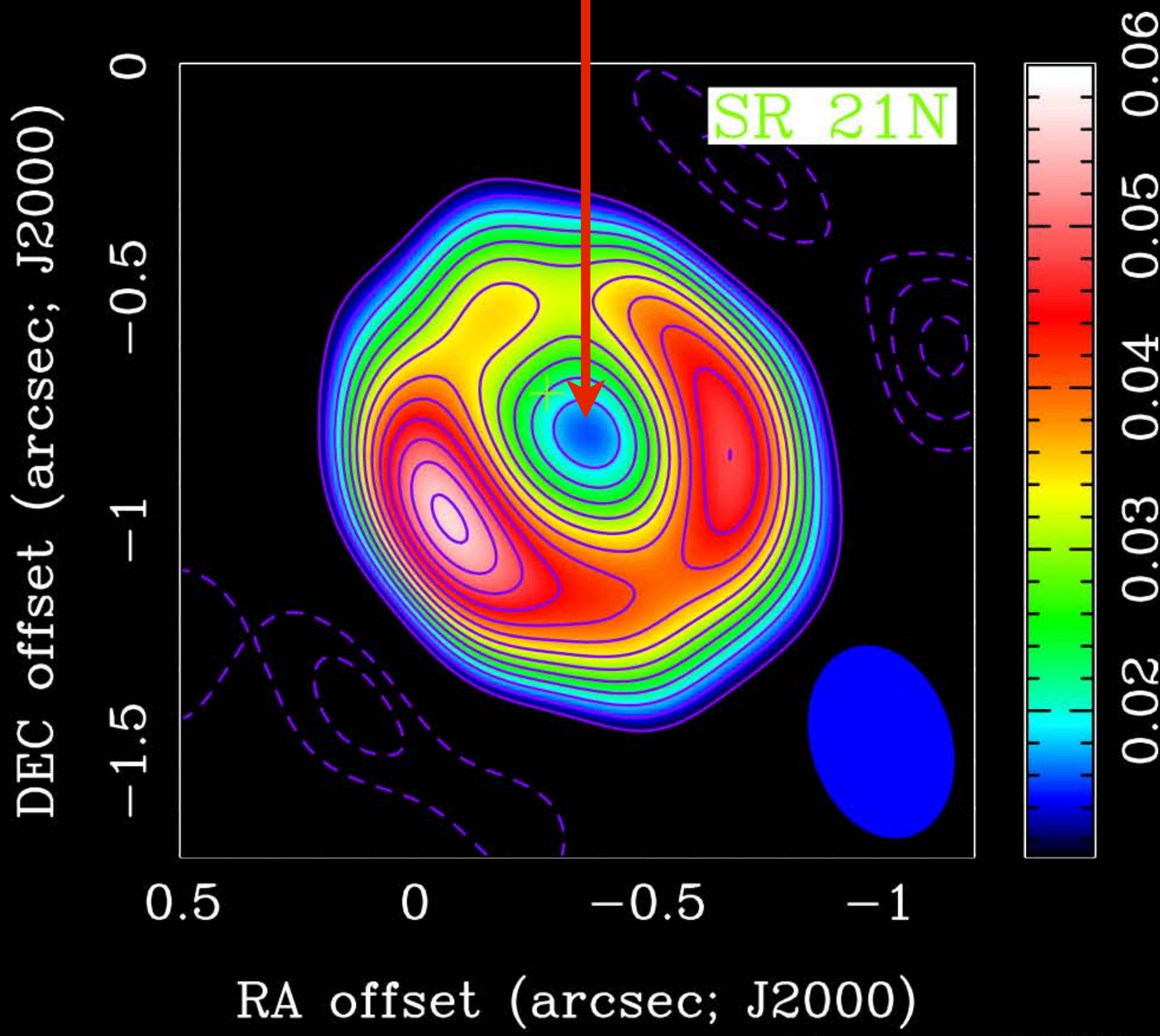
LECTURE 3.2: GROWTH/FRAGMENTATION

EVIDENCE OF GRAIN GROWTH

Warm dust in inner regions is missing



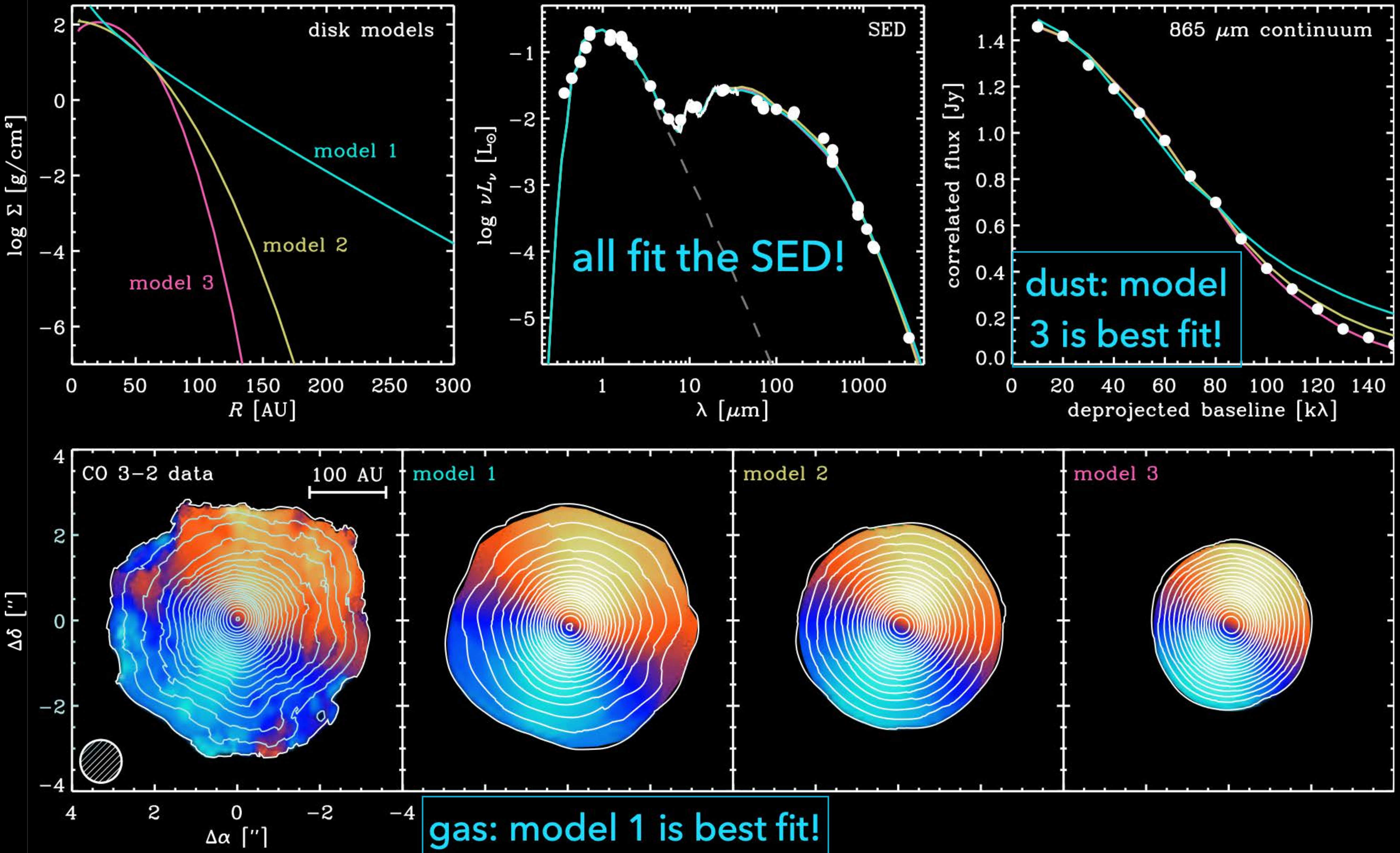
Espaillat et al. 2007



Brown et al. 2009

EVIDENCE OF GRAIN GROWTH

3 different models



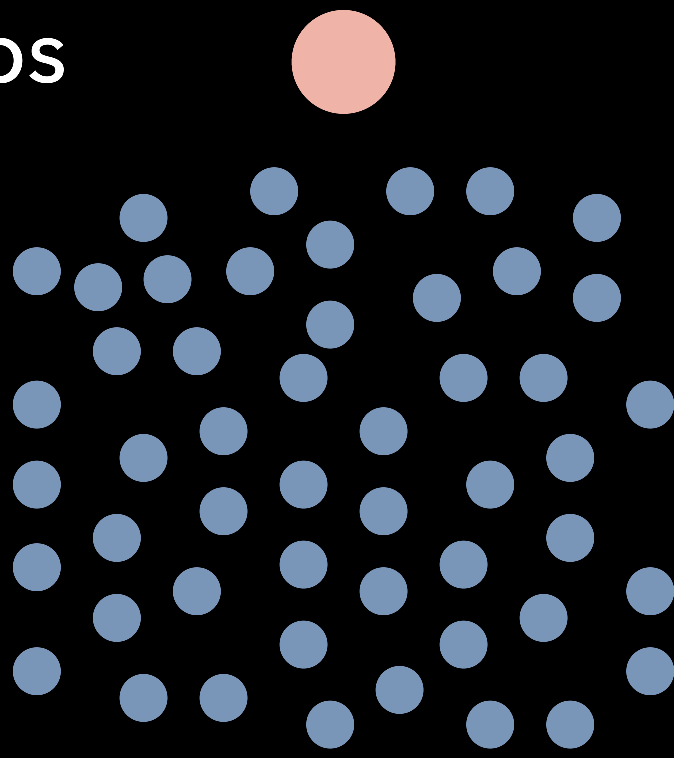
all fit the SED!

dust: model 3 is best fit!

gas: model 1 is best fit!

COAGULATION

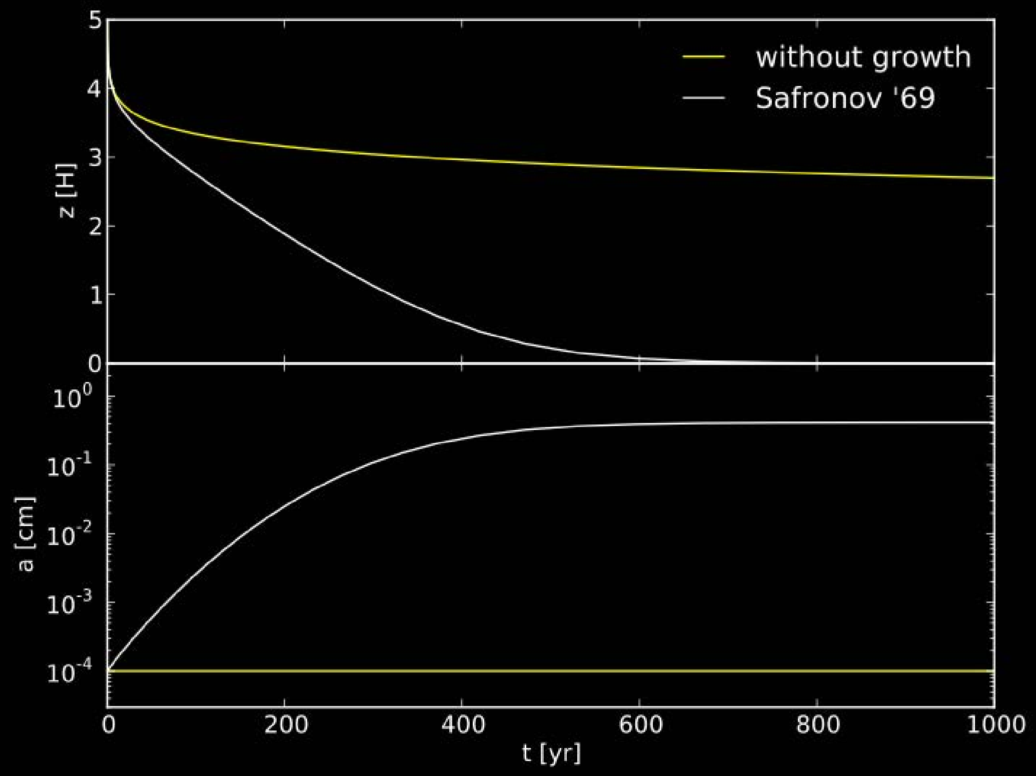
- Vertical settling timescale is much faster than the radial drift timescale. Simple model: the dust sweeps up grains as it settles at terminal velocity.



$$dm = \underbrace{\pi a^2 |v_z| dt}_{\text{volume}} \times \underbrace{\rho_g \varepsilon}_{\text{dust density}}$$

$$\frac{da}{dt} = \frac{\varepsilon \Omega_K^2}{4 v_{th}} z a \qquad v_d^z = -z \Omega_K \mathrm{St}(a, z)$$

- Solving this numerically:



- Differences in the condensation sequence can fractionate the disc.

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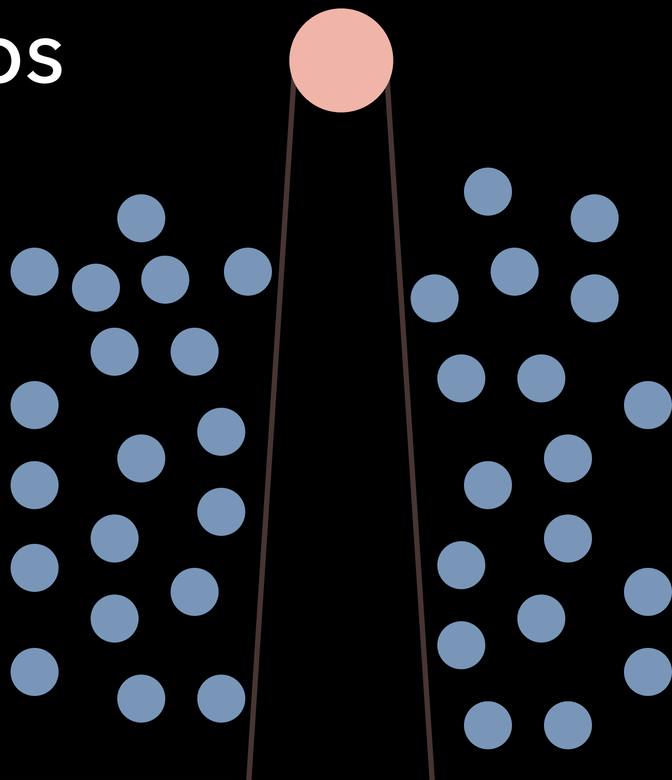

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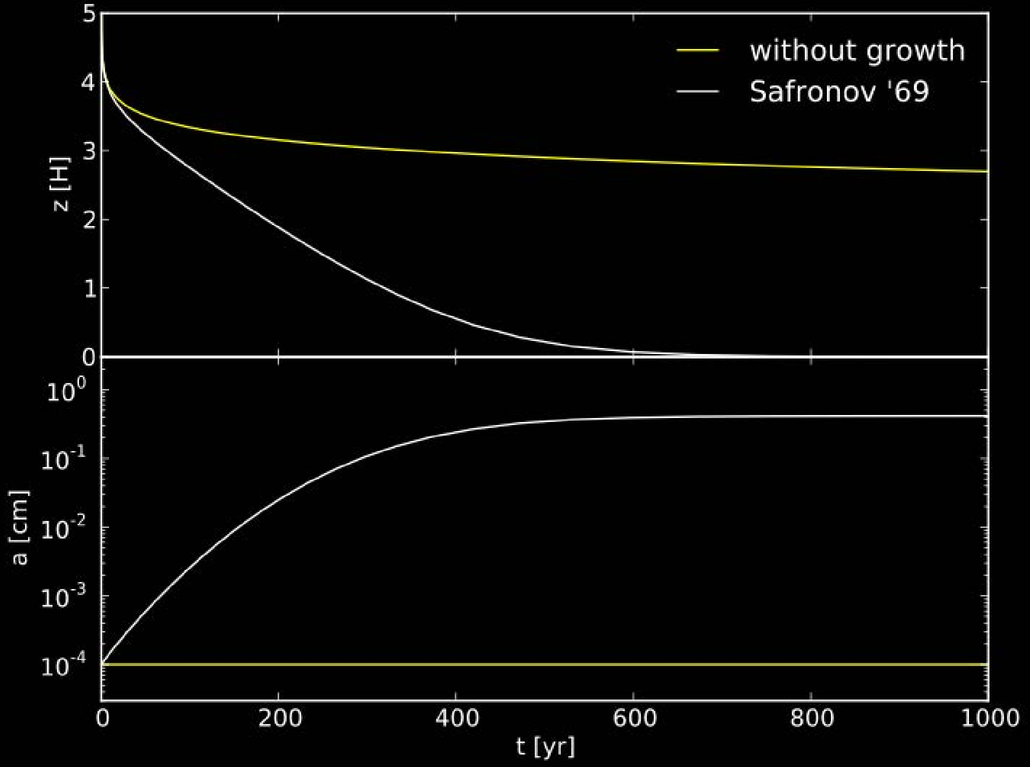
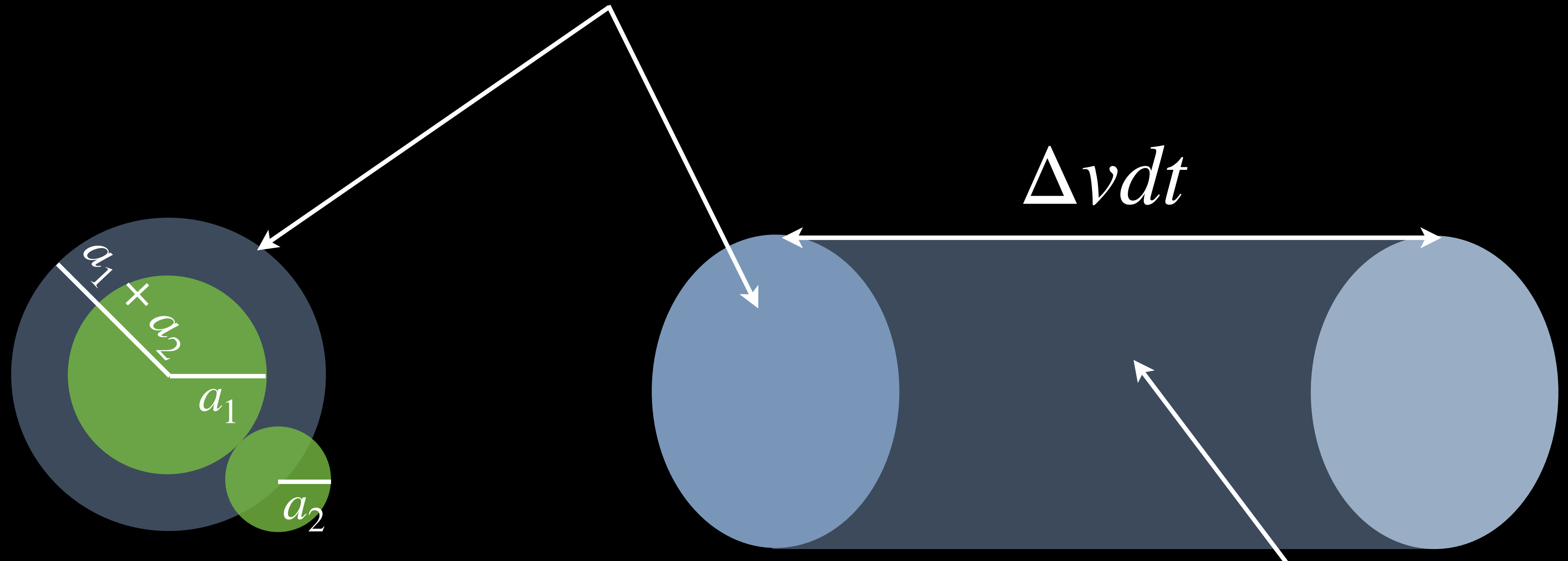


- Differences in the condensation sequence can fractionate the disc.

COAGULATION

cross section

$$\sigma = \pi(a_1 + a_2)^2$$



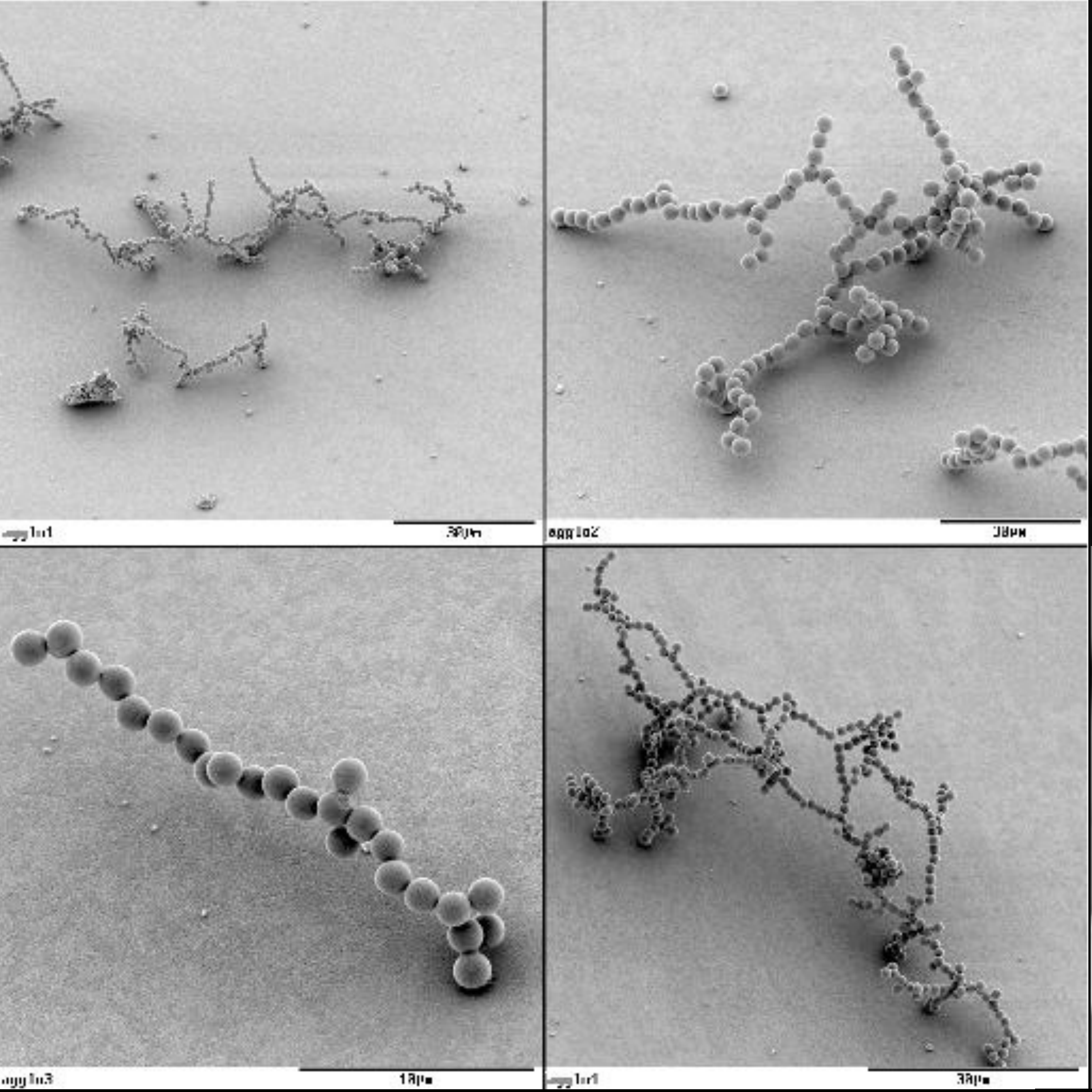
} volume
that can be
swept up
by particle 1

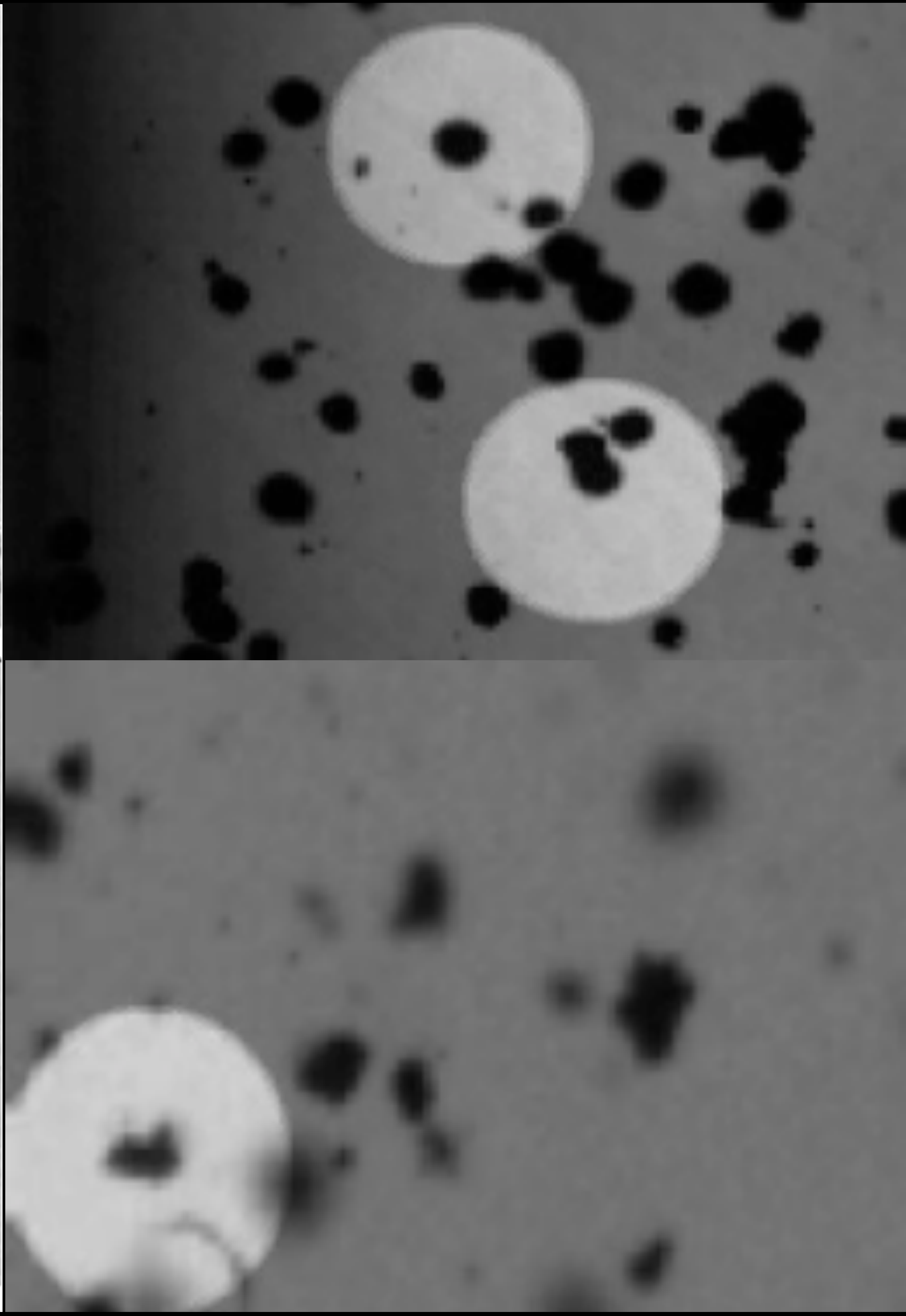
number density
n_2

- For one particle of m_1: $\dfrac{\#\ \text{collisions}}{\text{time}} = \sigma \Delta v n_2$
- But we have n_1 of them: $\dfrac{dn_3}{dt} = \sigma \Delta v n_1 n_2$
- The fraction S that lead to sticking: $\dfrac{dn_3}{dt} = \underbrace{S\sigma\Delta v}_{K\,=\,\text{coagulation kernel}}\ n_1 n_2$

Describes the rate at
which particles of size
1 coagulate with
particles of size 2.

COAGULATION + FRAGMENTATION

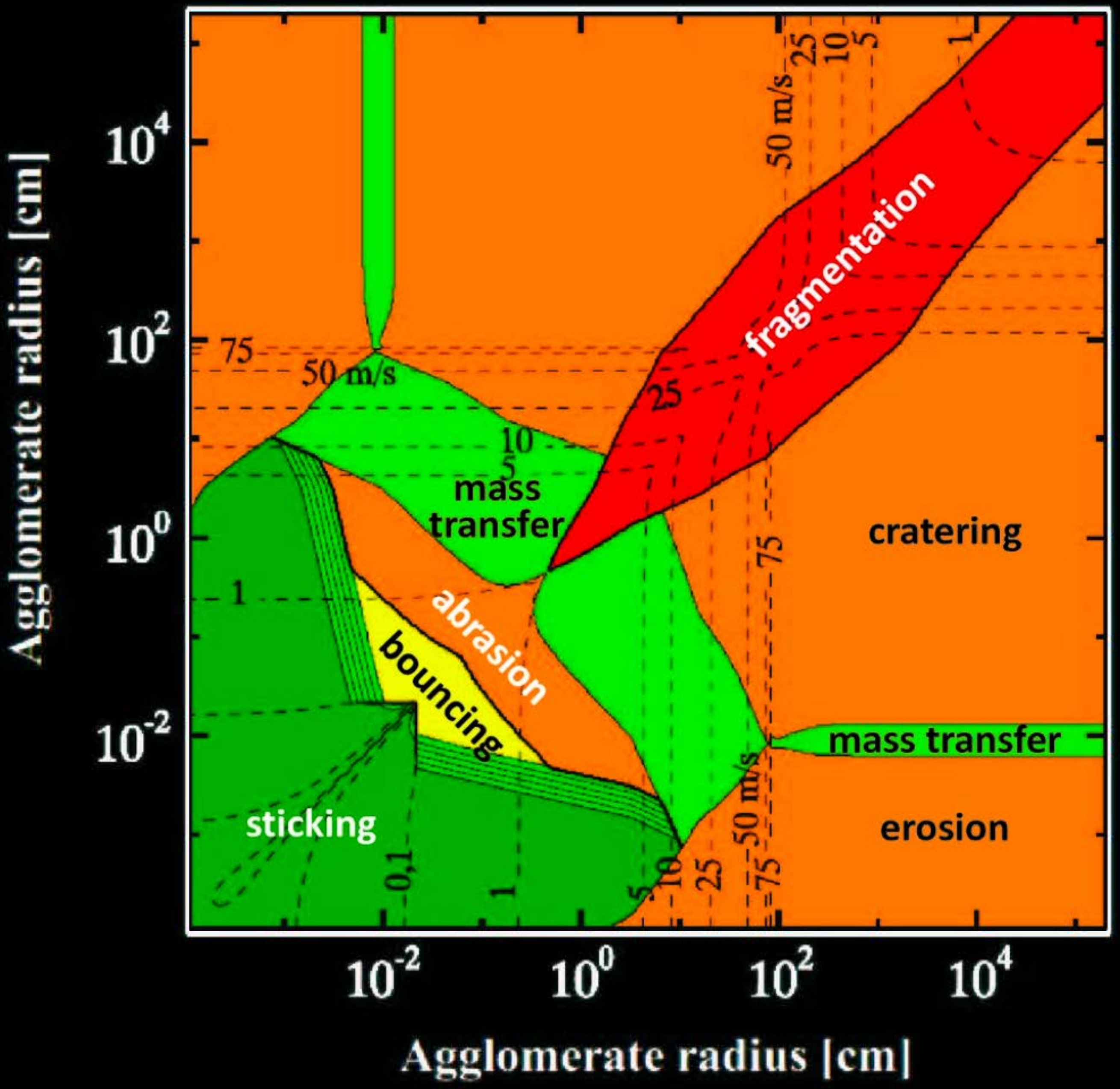


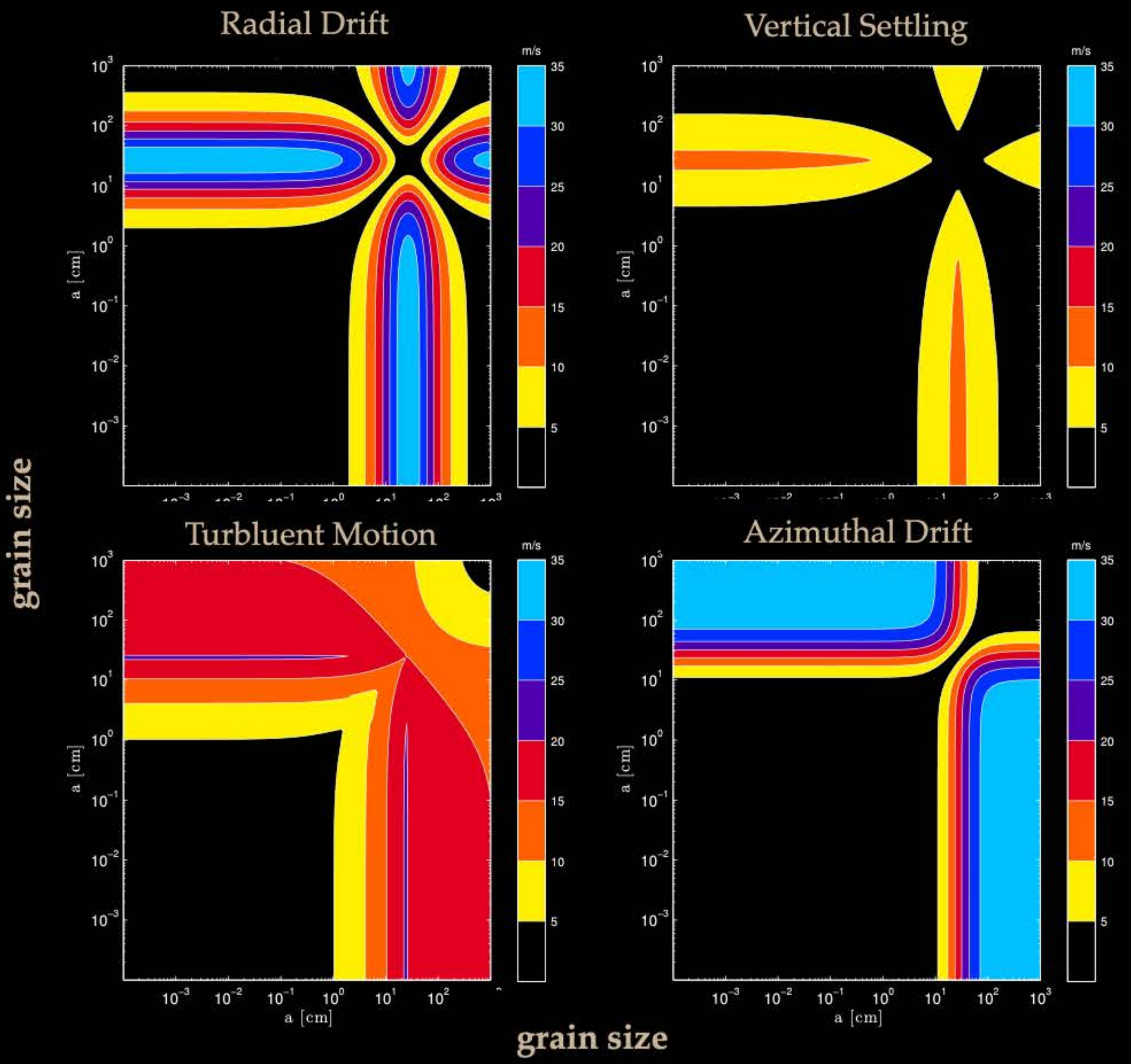


COAGULATION + FRAGMENTATION

COAGULATION + FRAGMENTATION

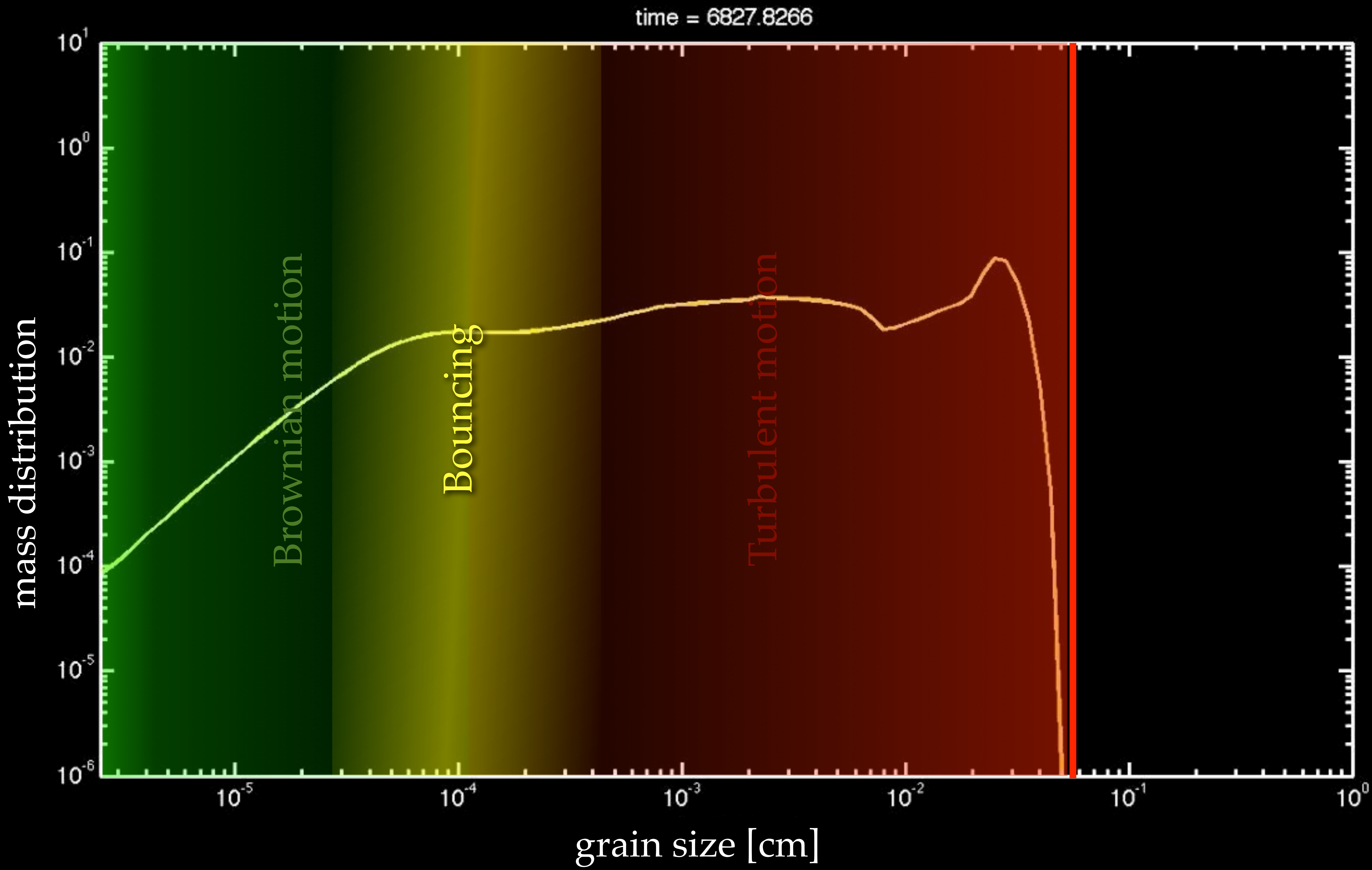
COAGULATION + FRAGMENTATION



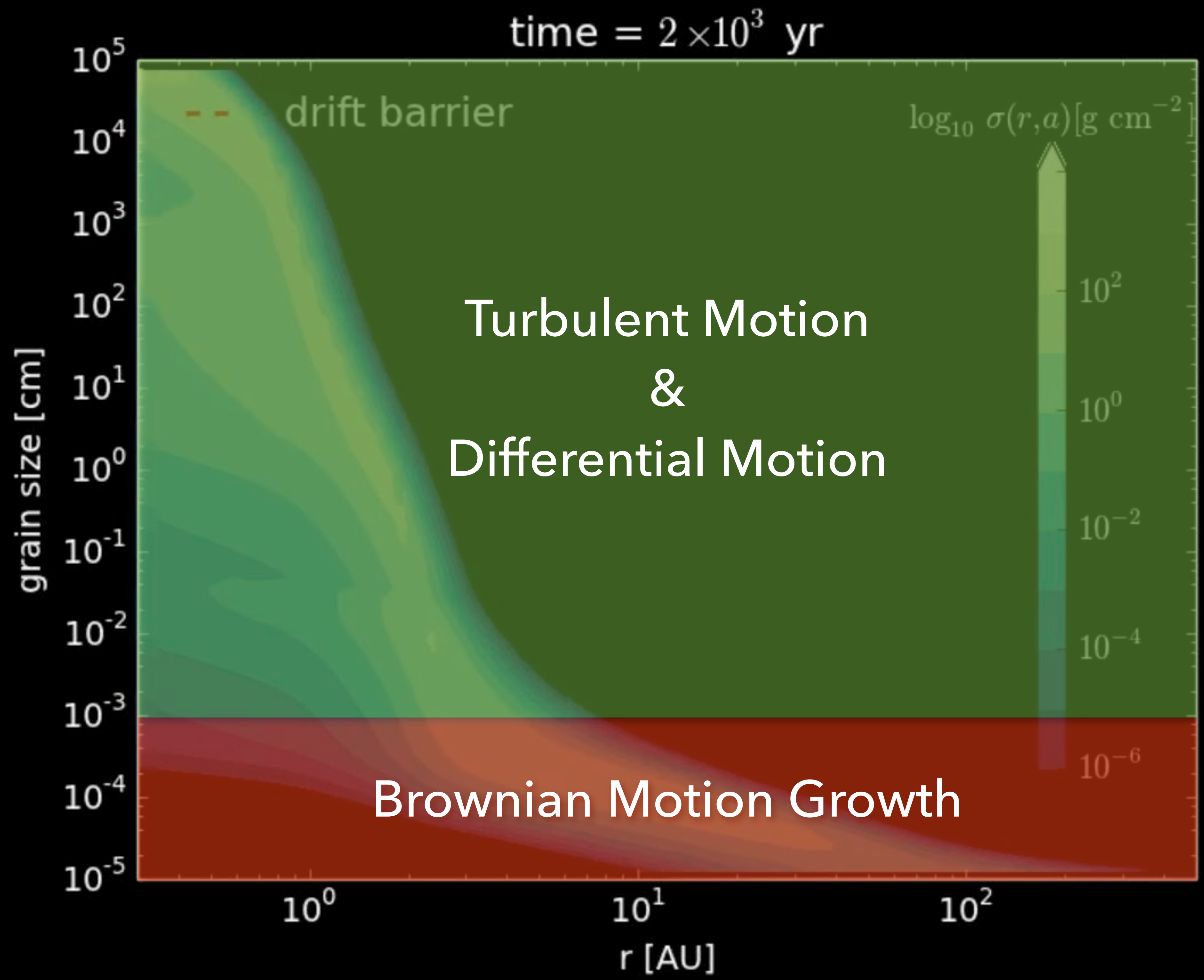


- Small particles are sticky, velocities given by Brownian motion.
- Turbulence and differential motion dominates for larger particles.
- Impact velocities increase with particle size → problem!

COAGULATION + FRAGMENTATION



GROWTH BARRIERS



Turbulent Motion

&

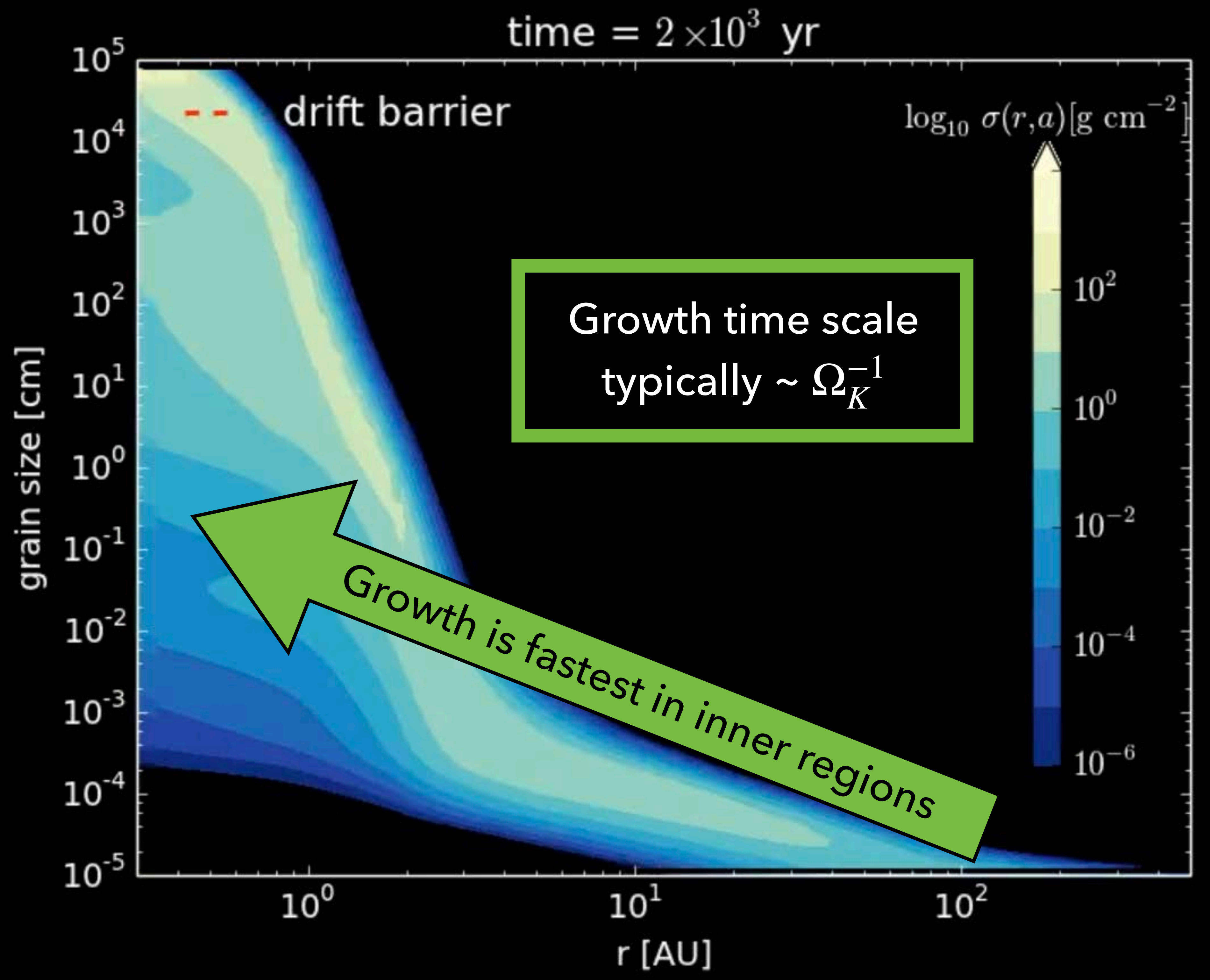
Differential Motion

Brownian Motion Growth

Birnstiel et al. 2010, 2012

GROWTH BARRIERS

Only grain growth



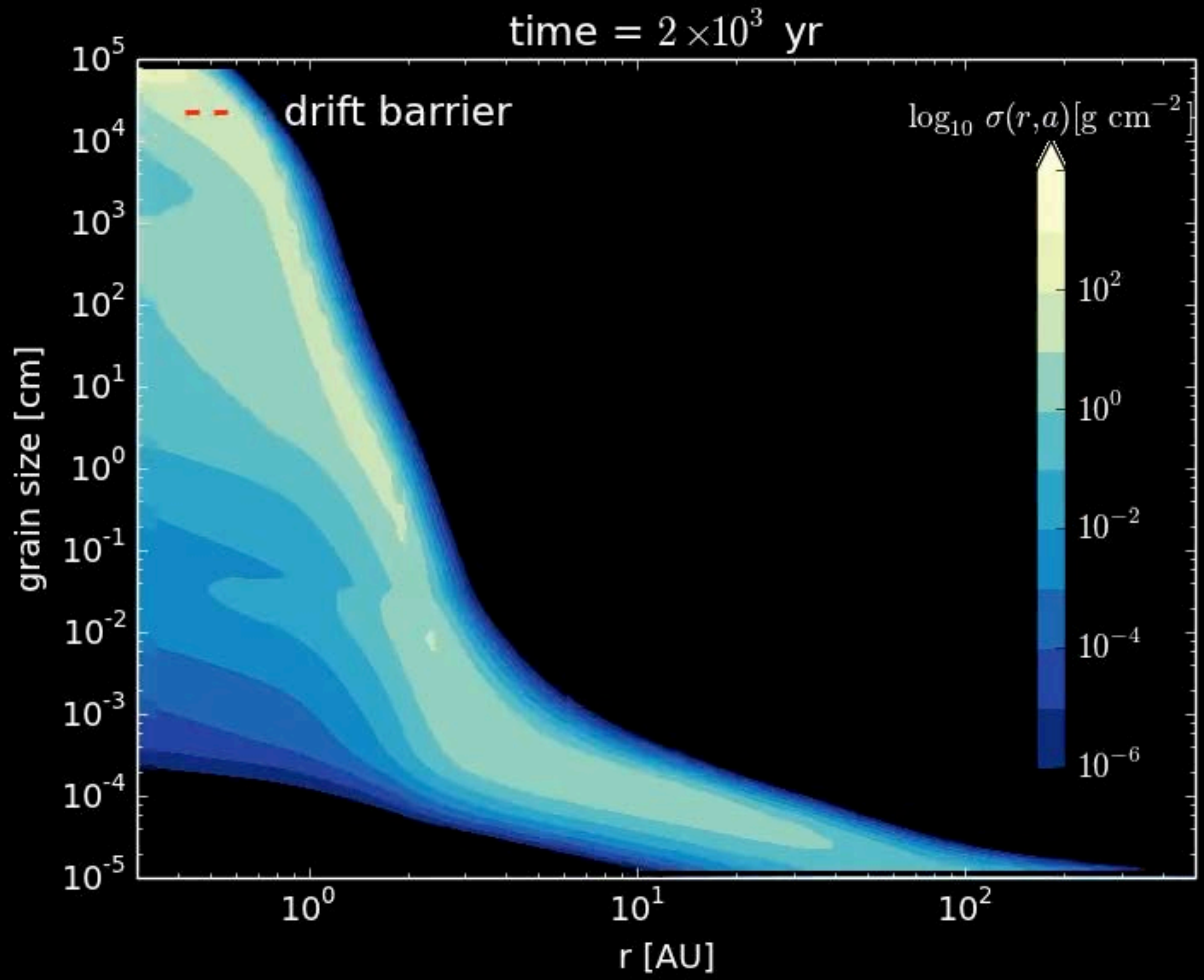
Growth time scale typically ~ Ω_K^{-1}

Growth is fastest in inner regions

Birnstiel et al. 2010, 2012

GROWTH BARRIERS

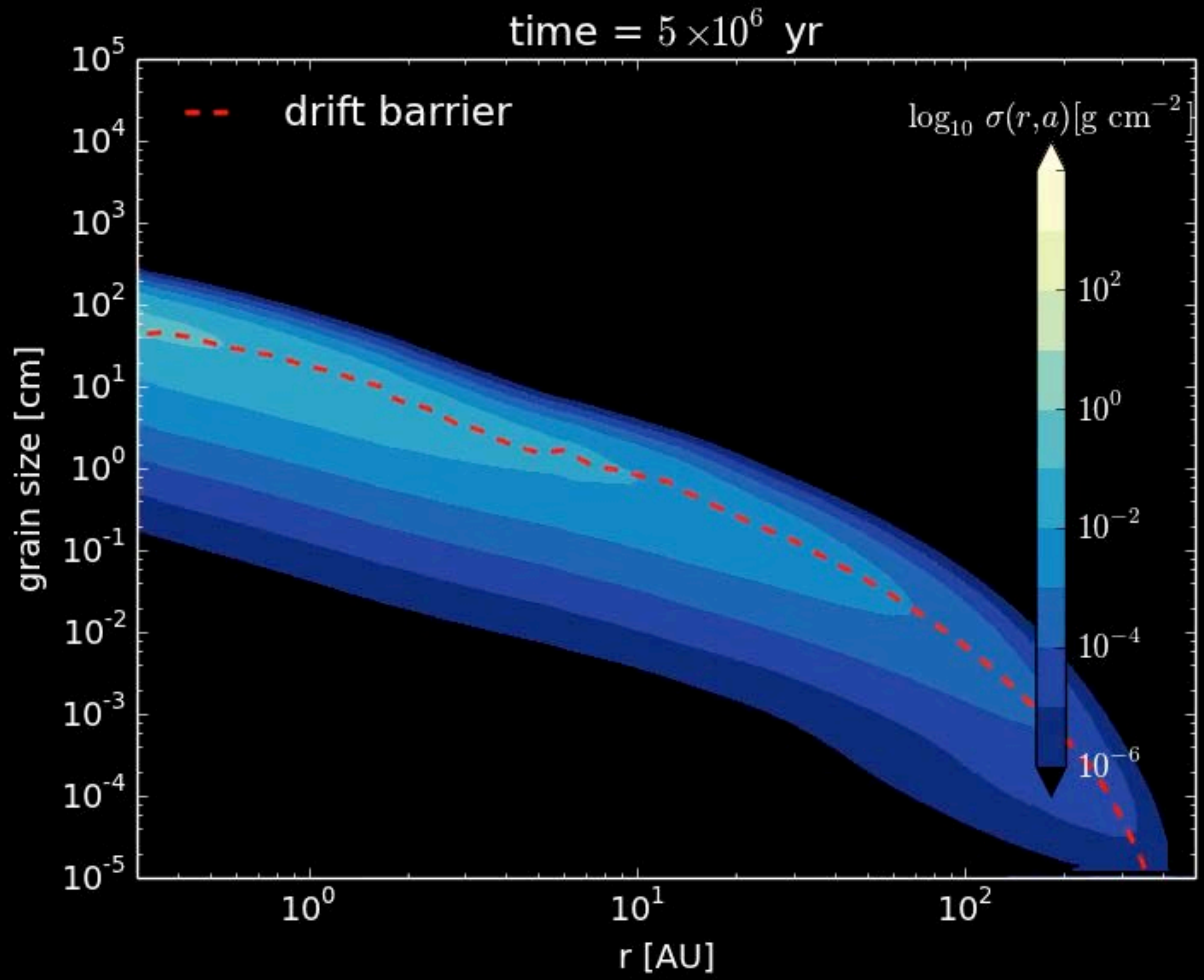
Only grain growth



Birnstiel et al. 2010, 2012

GROWTH BARRIERS

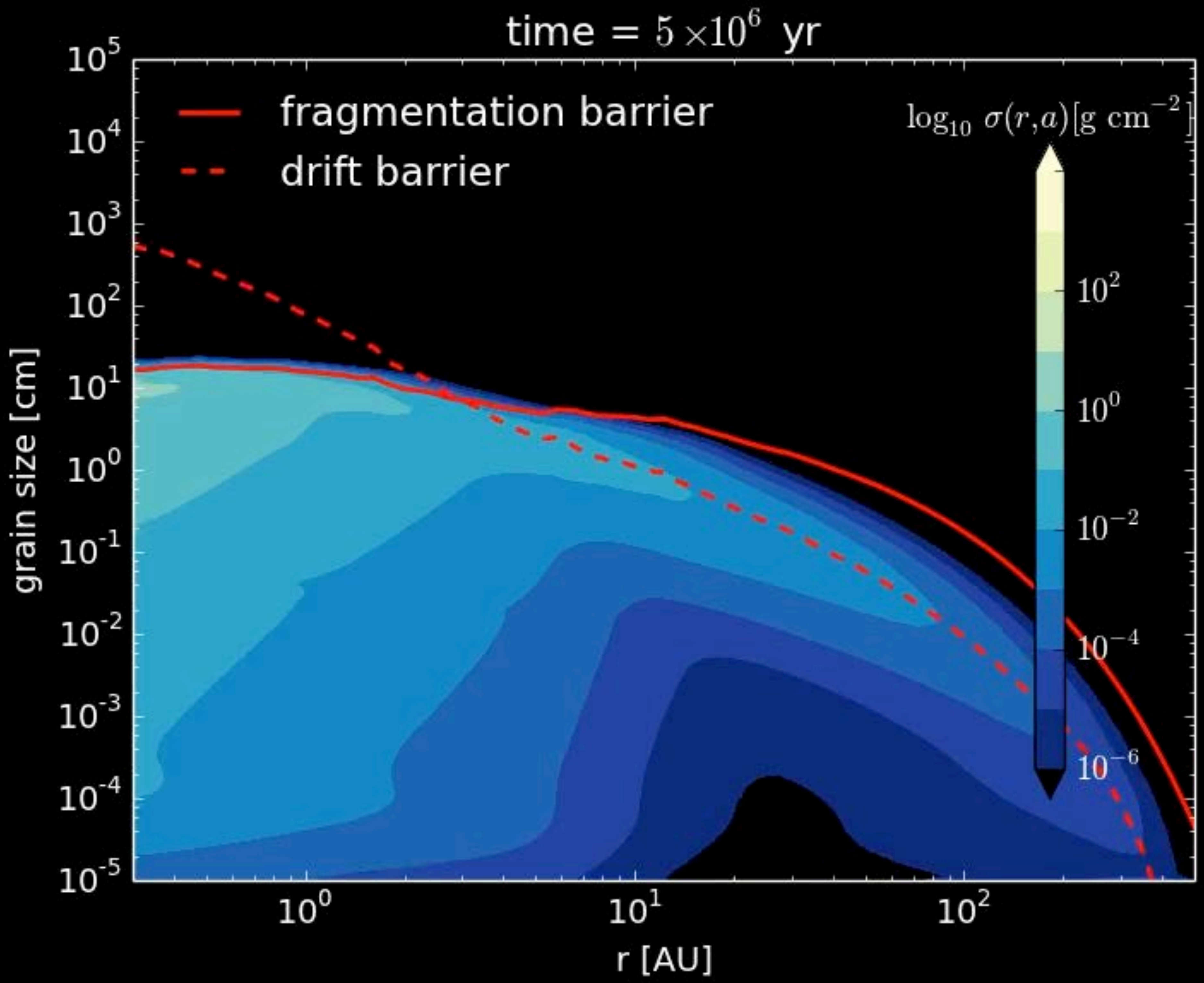
Grain growth and drift



Birnstiel et al. 2010, 2012

GROWTH BARRIERS

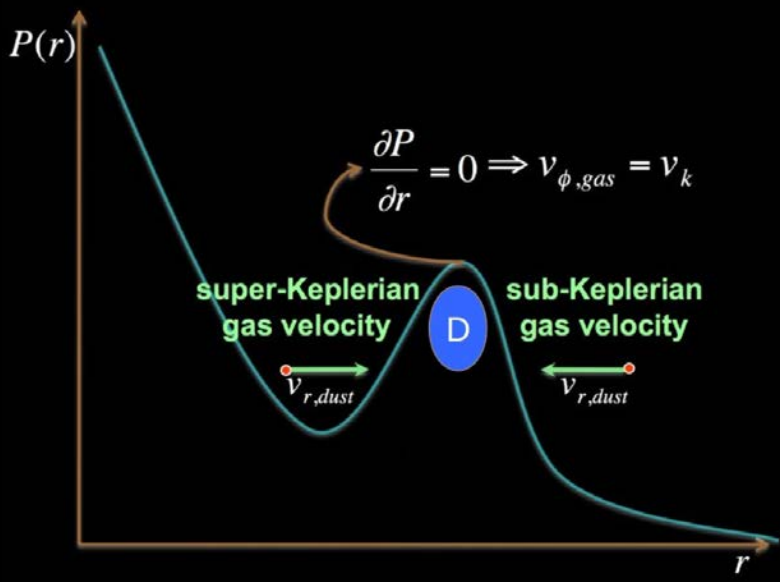
Grain growth, drift, and fragmentation

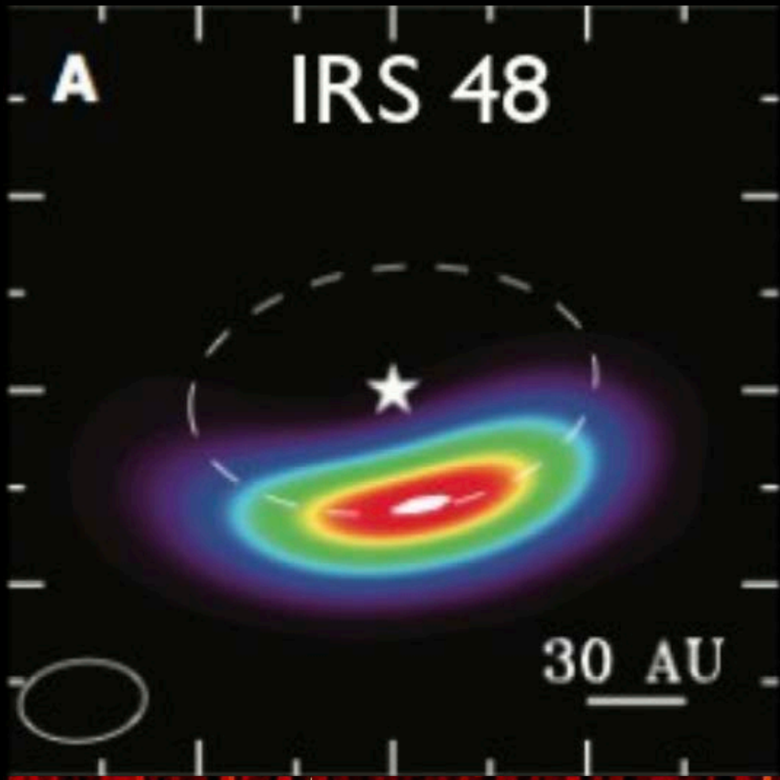


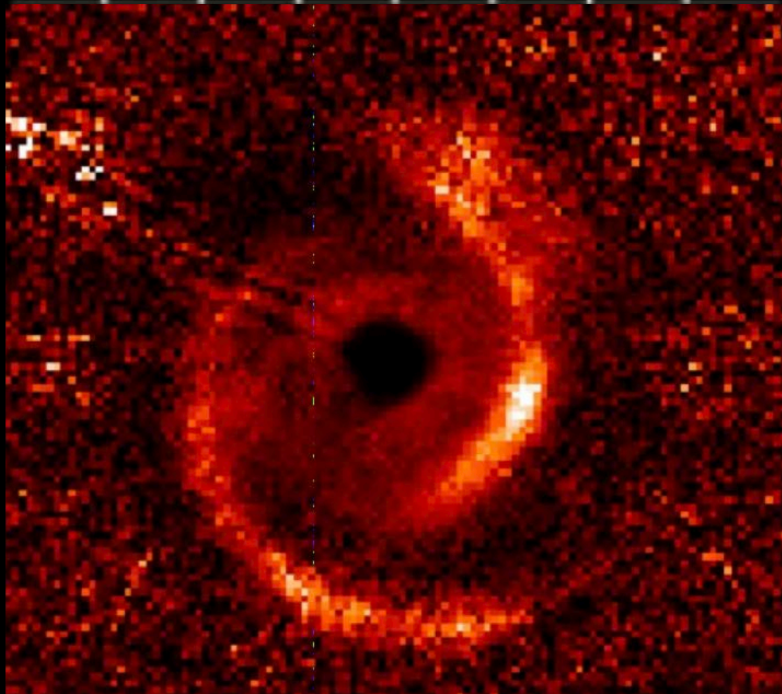
Birnstiel et al. 2010, 2012

OVERCOMING GROWTH BARRIERS

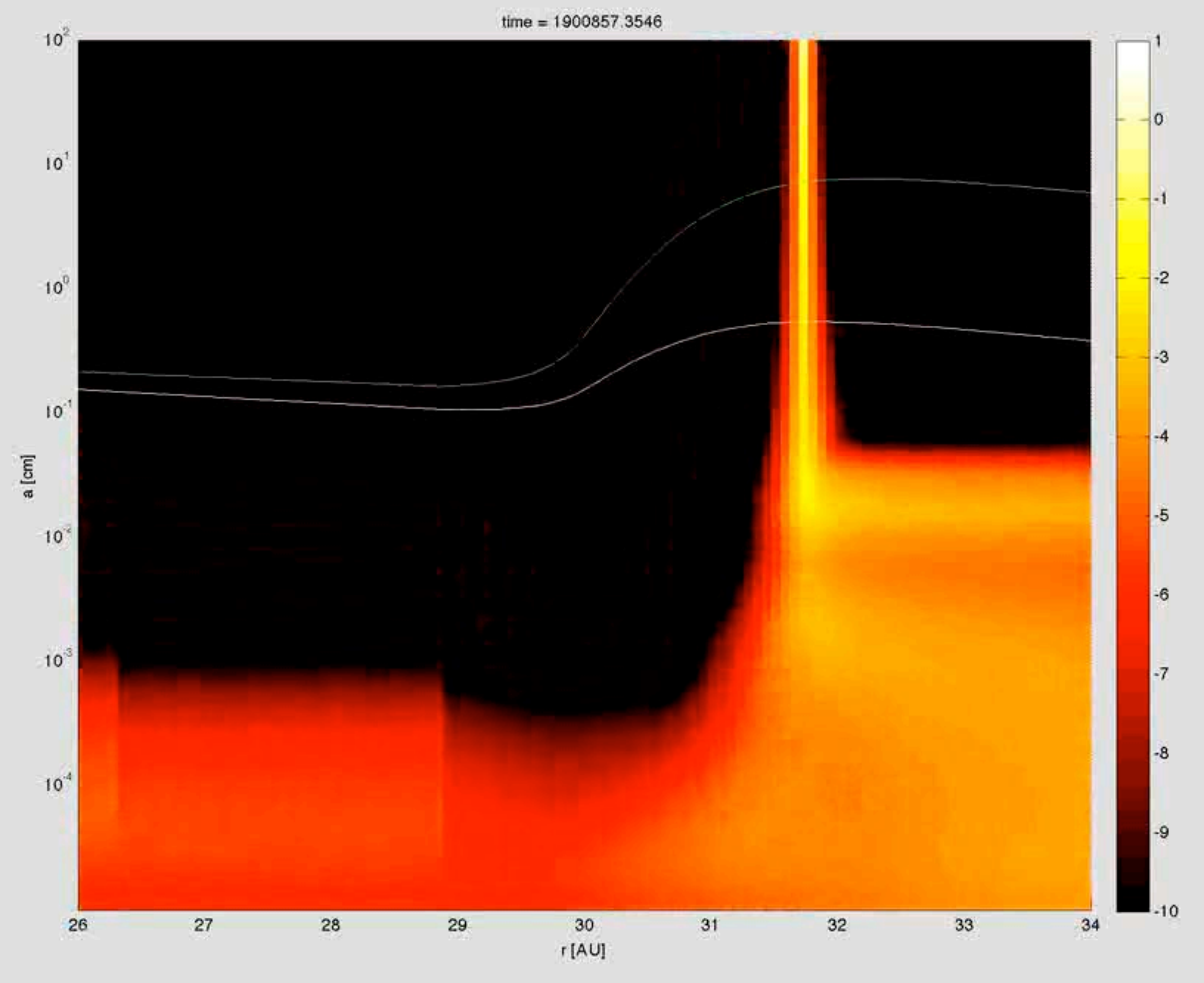
- Dust traps: usually associated with pressure maxima (zero gradient) → no radial or azimuthal drift.
    - Snow lines, turbulence, vortices, planet gaps, gravity, self-induced pile-ups.
- Trap larger grains, small grains follow gas (accretion and viscous spreading). Relative velocities only due to turbulence. Thus for small α, growth can continue.
- A few "lucky" particles in the tails of the velocity distribution may be able to grow to reach planetesimal sizes.







DUST TRAPS: PRESSURE BUMPS



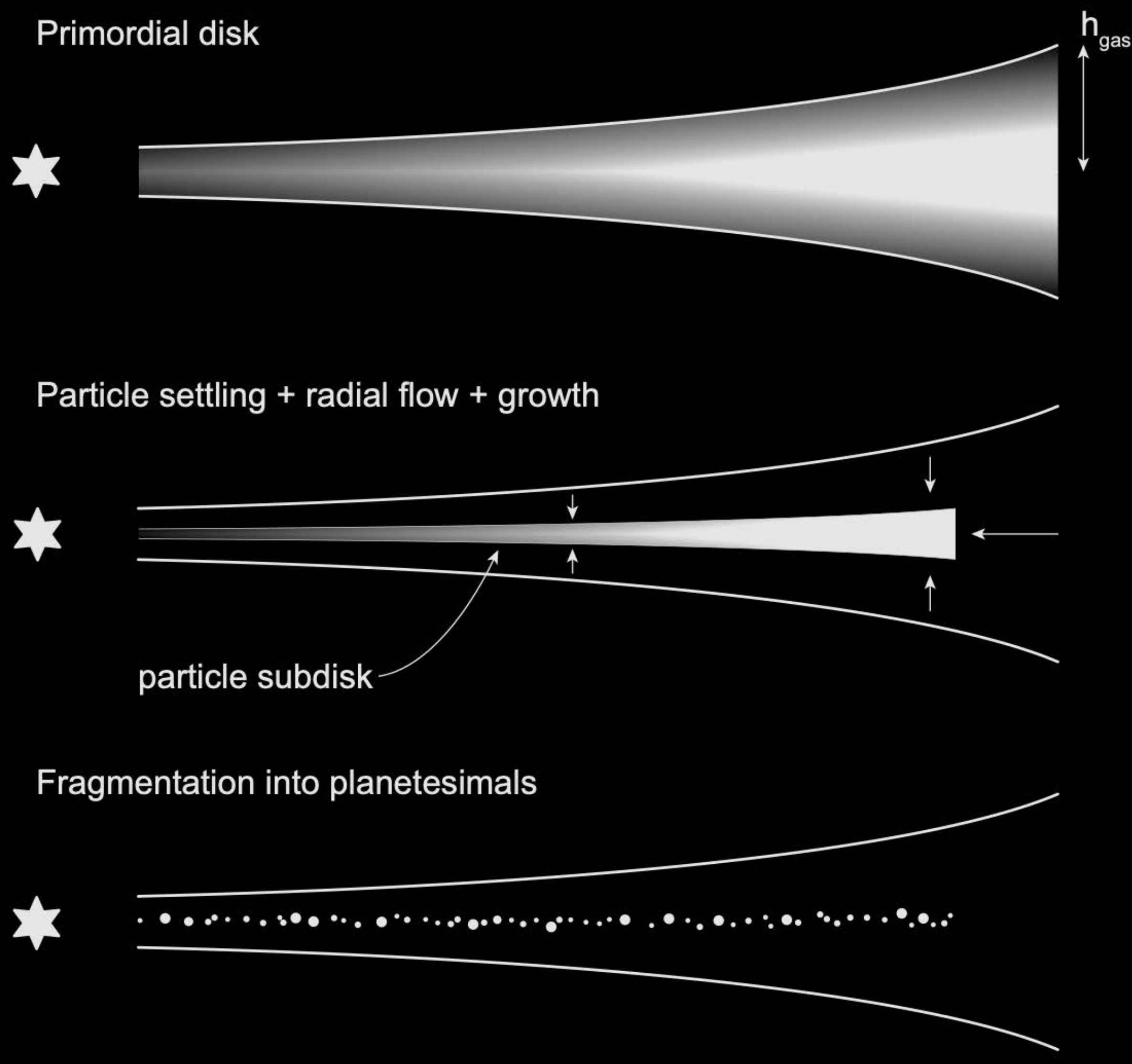
GRAVITATIONAL INSTABILITY (GI)

- **Goldreich-Ward instability**: settling of small grains increases the dust-to-gas ratio at the disc mid-plane, until the dust layer becomes gravitationally unstable and fragments.

- **Toomre criterion**: the disc is unstable for $Q \lesssim 1$, where

$$Q \equiv \frac{c_\mathrm{s}\Omega_\mathrm{K}}{\pi G \Sigma}$$

- If $\Sigma_\mathrm{gas} \sim 100\text{–}1000\ \mathrm{g\ cm^{-2}}$ and the dust-to-gas ratio is 0.01, then $Q < 1$ requires a disc temperature less than 1 K!



GRAVITATIONAL INSTABILITY (GI)

- For instability, gravity must dominant over rotational and thermal energies:

$$\frac{E_{\rm therm}}{E_{\rm grav}} \frac{E_{\rm rot}}{E_{\rm grav}} < 1$$

$$E_{\rm grav} \approx \frac{G(\pi R^2 \Sigma)}{R} = G\pi\Sigma R$$

$$E_{\rm rot} \approx \frac{1}{2}(\Omega_{\rm K} R)^2$$

$$E_{\rm therm} \approx \frac{3}{2}T \approx c_{\rm s}^2$$

- Plugging in expressions for the energies:

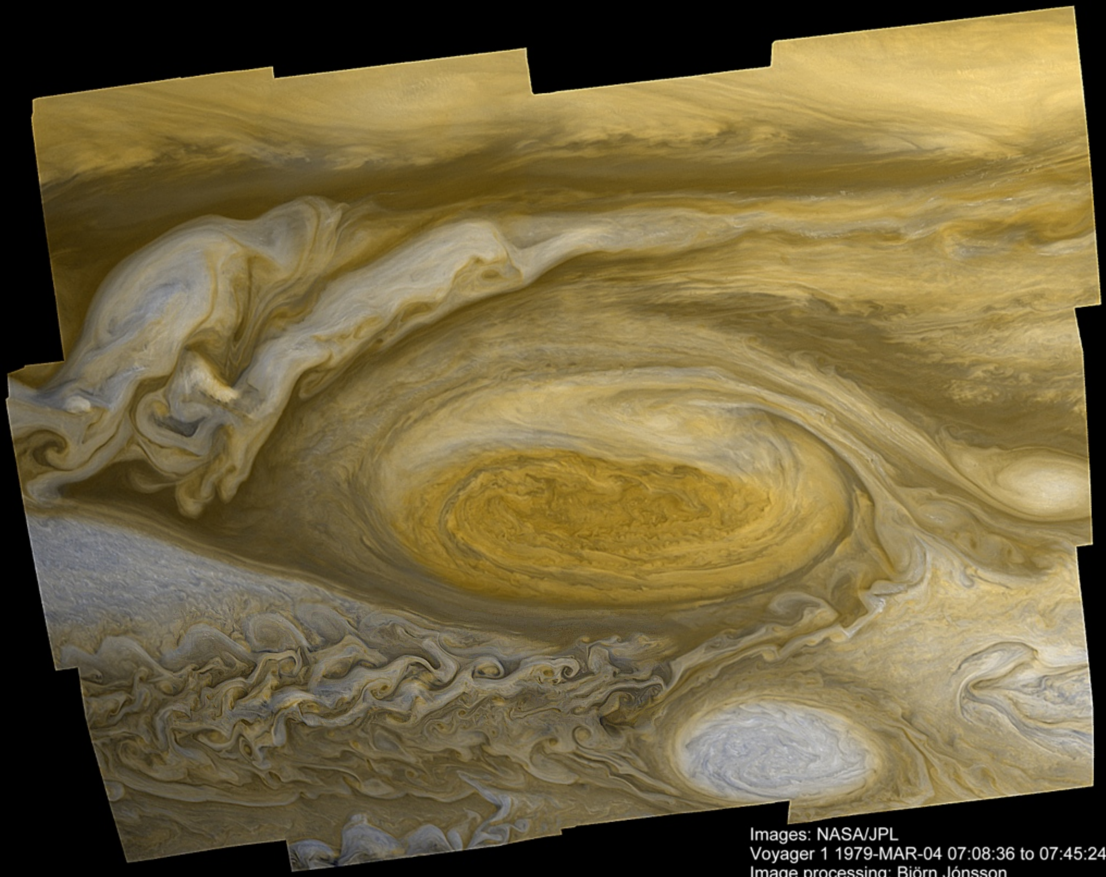
$$\frac{c_{\rm s}^2}{G\pi\Sigma R} \frac{\frac{1}{2}\Omega_{\rm K}^2 R^2}{G\pi\Sigma R} = \frac{1}{2}\left(\frac{c_{\rm s}\Omega_{\rm K}}{\pi G \Sigma}\right)^2 \longrightarrow Q \equiv \frac{c_{\rm s}\Omega_{\rm K}}{\pi G \Sigma} \lesssim 1$$

STREAMING INSTABILITY (SI)

- Dust experiences a headwind in discs, but if the dust layer of large grains (pebbles) is sufficiently compact and dense ($\sim 10^4 \times$ thinner and $\sim 100 \times$ denser than the gas!) then the dust accelerates the gas and reduces the headwind it feels. This has two consequences:
  - Radial drift is halted and dust drifting in from outside piles up.
  - The accelerated gas causes a pressure bump (dust trap).
- The process rapidly runs away until the clump becomes self-gravitating and collapses to form planetesimals.

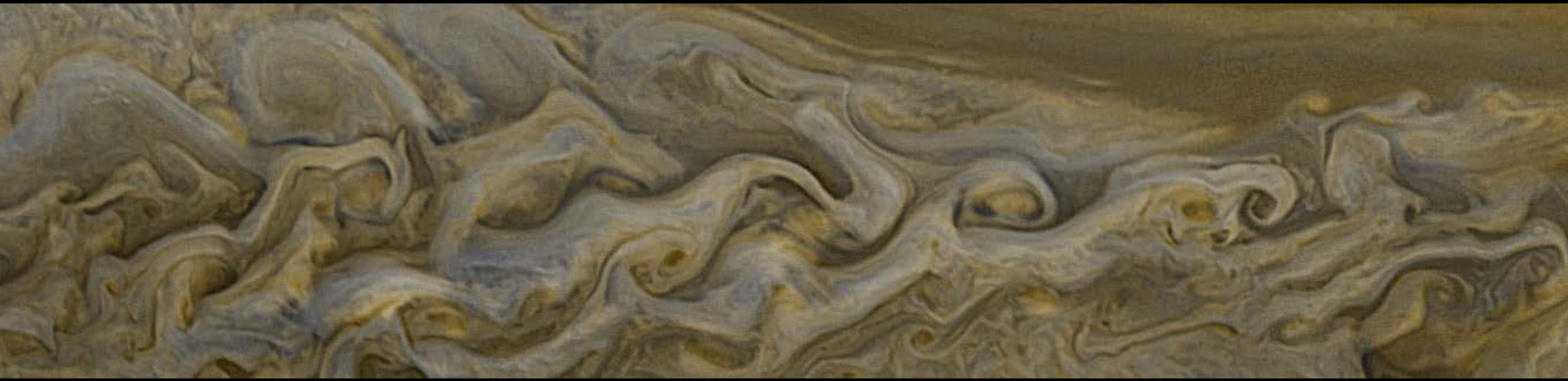
STREAMING INSTABILITY (SI)

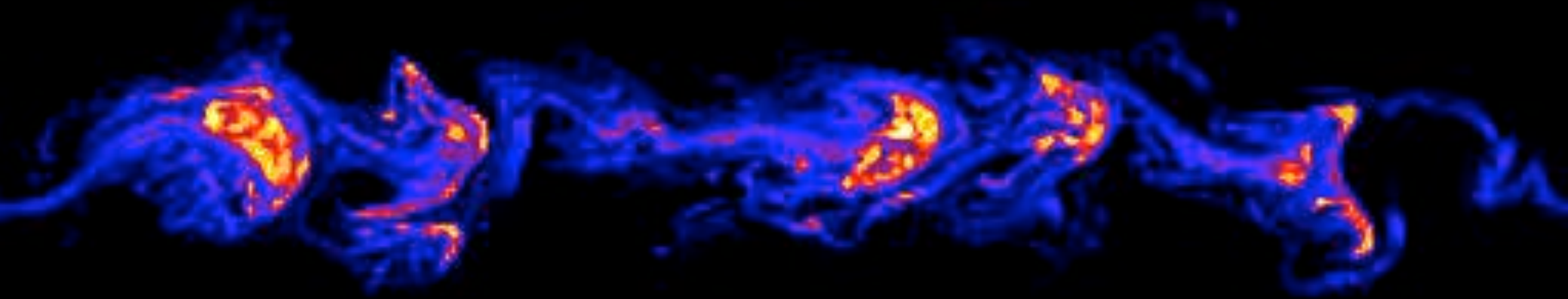
- While the compact dust layer is dynamically dominated by the dust, the layers above are still dominated by the gas → large vertical shear.
- Kelvin-Helmholtz Instability develop which increases the velocity dispersion of the dust layer.



Images: NASA/JPL
Voyager 1 1979-MAR-04 07:08:36 to 07:45:24
Image processing: Björn Jónsson

STREAMING INSTABILITY (SI)

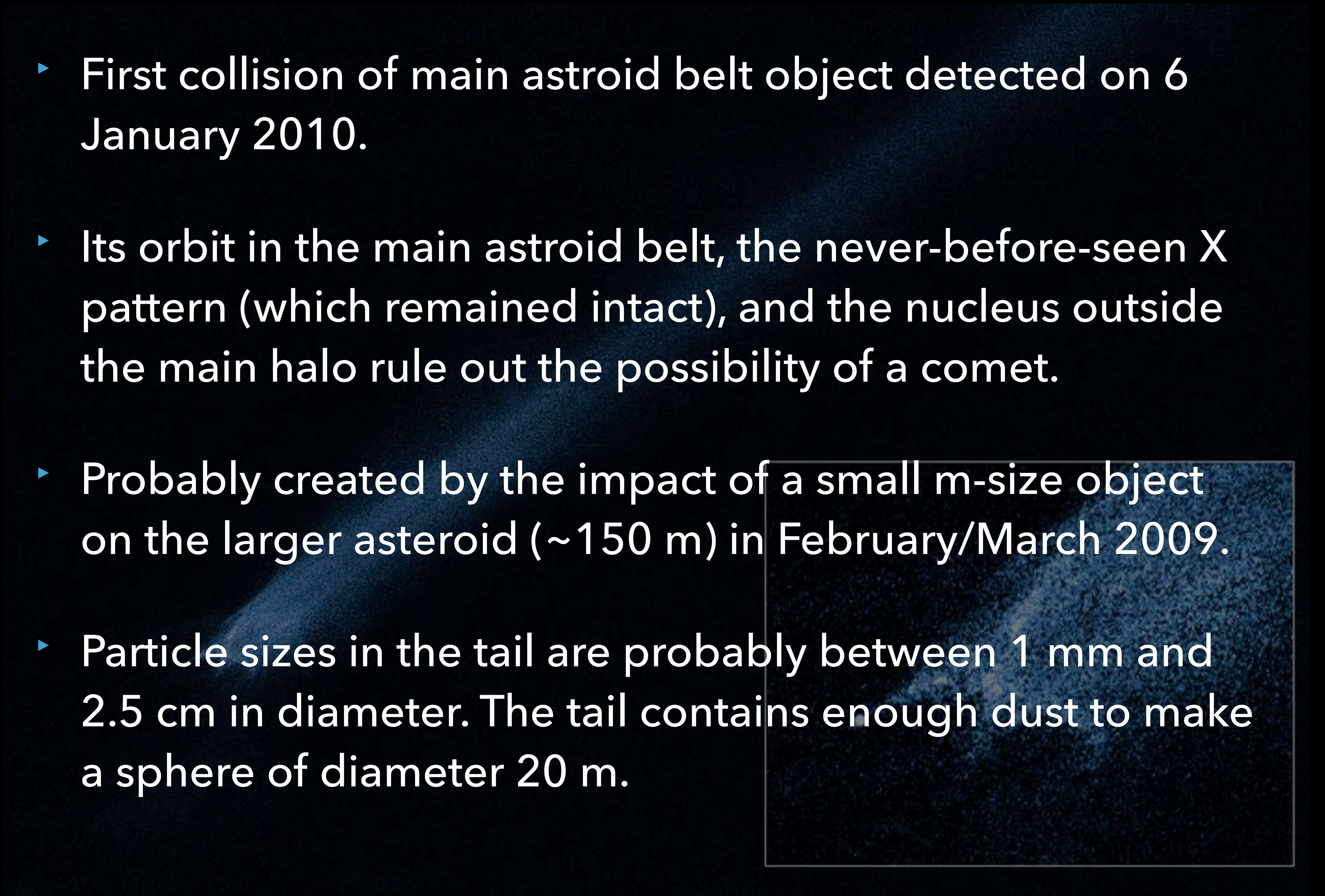




PLANETESIMAL FORMATION

354P/LINEAR

PLANETESIMAL FORMATION



- First collision of main astroid belt object detected on 6 January 2010.

- Its orbit in the main astroid belt, the never-before-seen X pattern (which remained intact), and the nucleus outside the main halo rule out the possibility of a comet.

- Probably created by the impact of a small m-size object on the larger asteroid (~150 m) in February/March 2009.

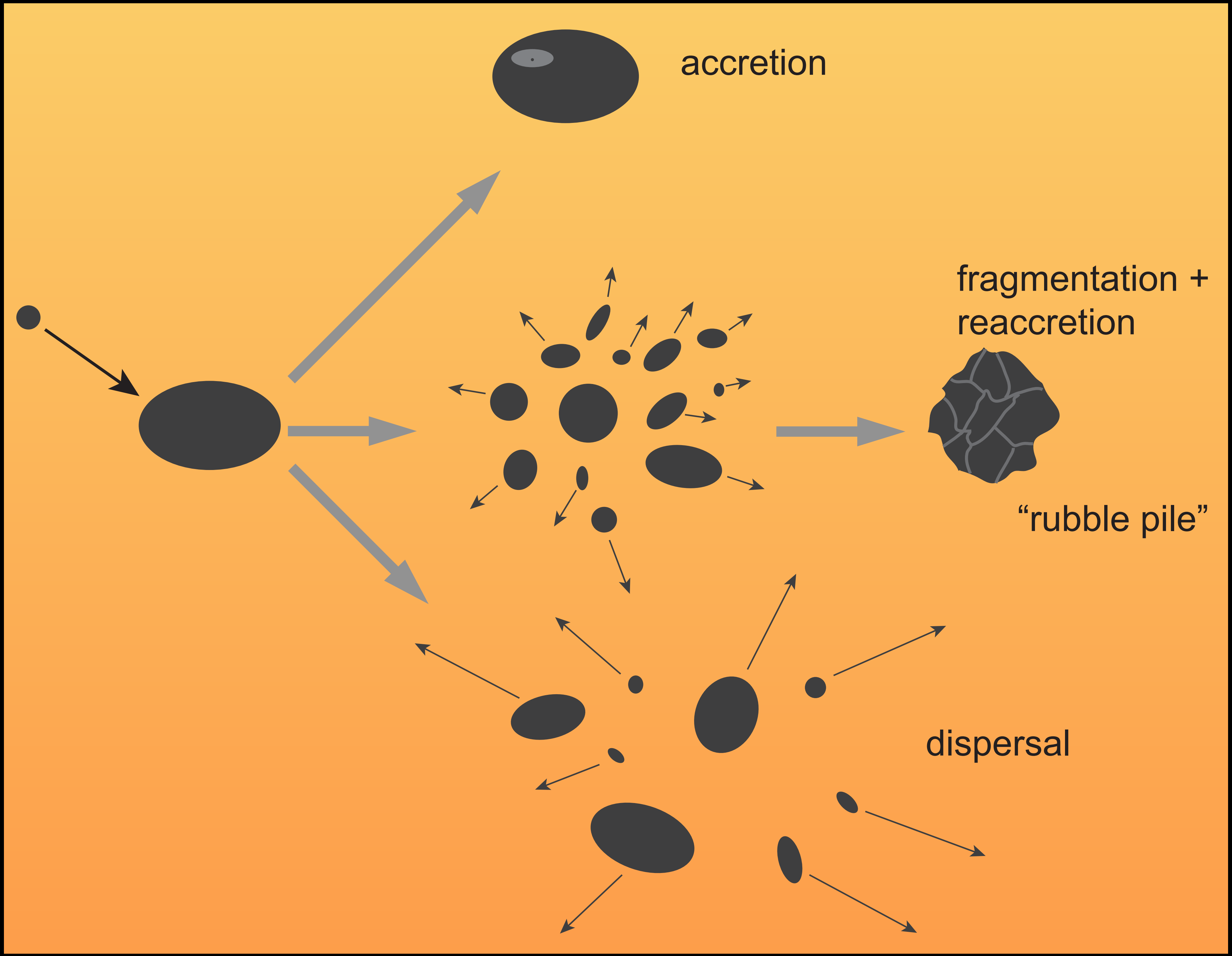
- Particle sizes in the tail are probably between 1 mm and 2.5 cm in diameter. The tail contains enough dust to make a sphere of diameter 20 m.

FROM UNIVERSE

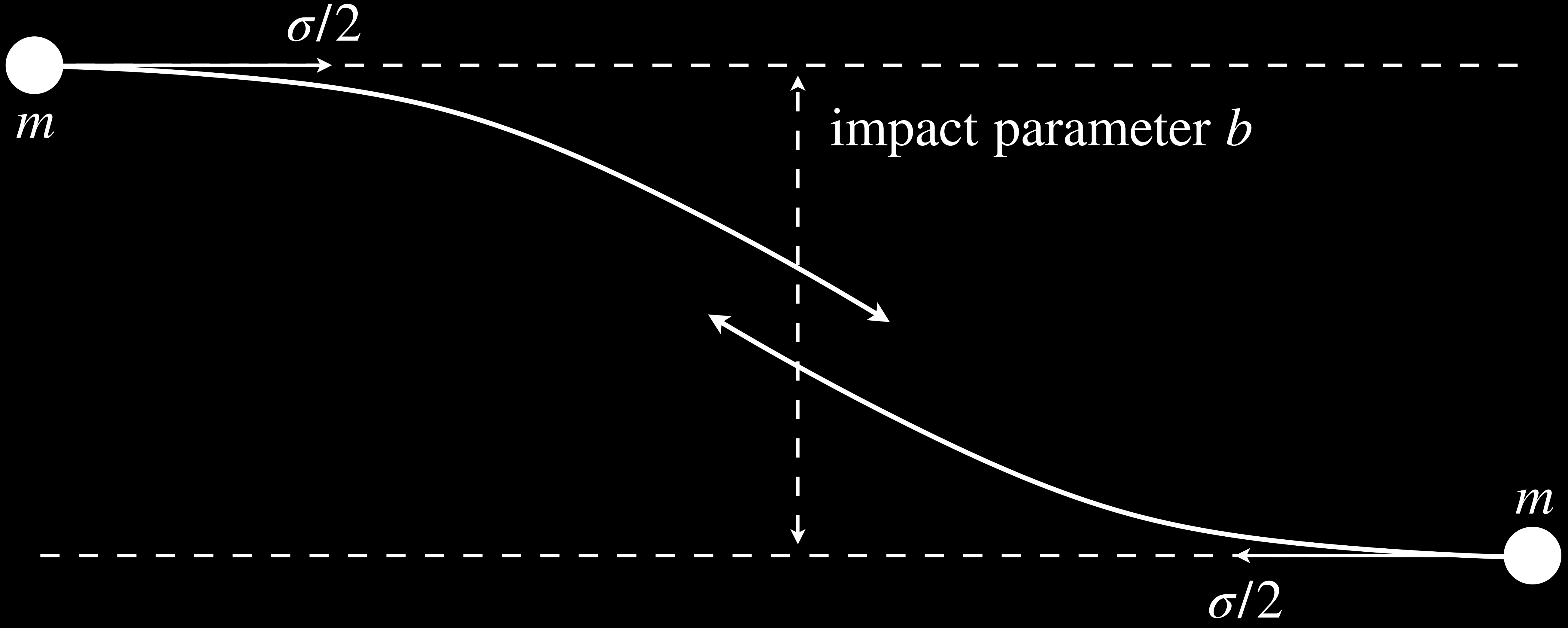
TO PLANETS

LECTURE 3.3: PLANETESIMALS

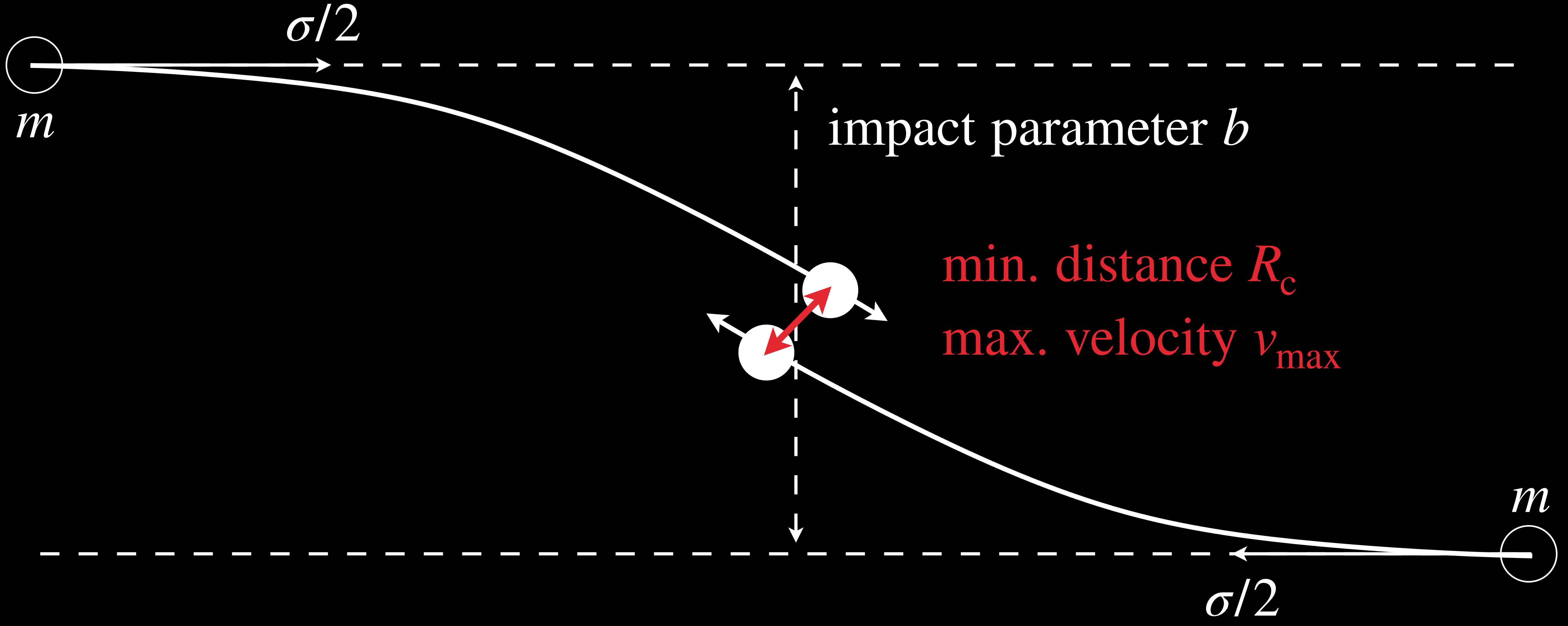
PLANETESIMAL COLLISIONS



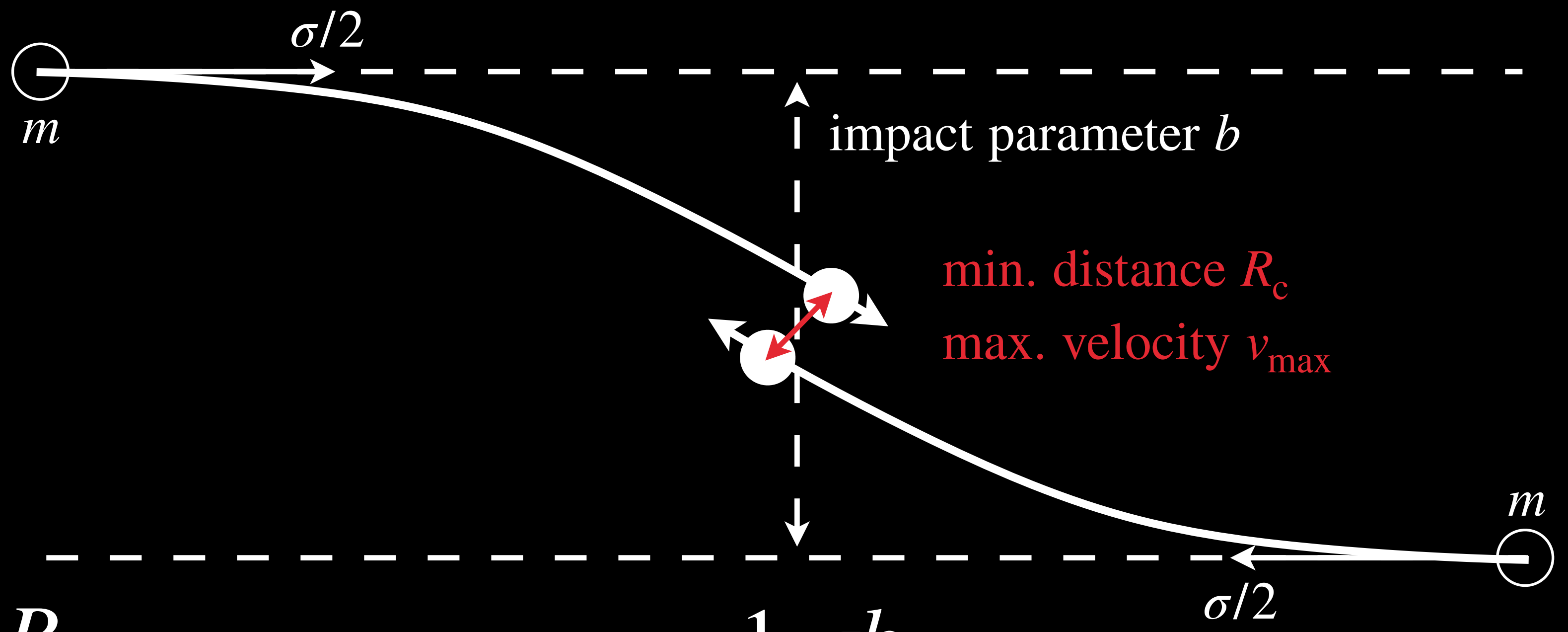
GRAVITATIONAL FOCUSING



GRAVITATIONAL FOCUSING



GRAVITATIONAL FOCUSING



- Angular momentum conservations gives:

$$J = 2 \cdot m\frac{\sigma}{2} \cdot \frac{b}{2} = 2 \cdot m v_\mathrm{max} \frac{R_\mathrm{c}}{2} \quad \longrightarrow \quad v_\mathrm{max} = \frac{1}{2}\frac{\sigma b}{R_\mathrm{c}}$$

- Conservation of energy gives (upon inserting v_max):

$$E = 2 \cdot \frac{1}{2} m \left(\frac{\sigma}{2}\right)^2 = 2 \cdot \frac{1}{2} m v_\mathrm{max}^2 - \frac{Gm^2}{R_\mathrm{c}} \quad \longrightarrow \quad b^2 = R_\mathrm{c}^2 + \frac{4GmR_\mathrm{c}}{\sigma^2}$$

- Collisions only occur if $R_\mathrm{c} < R_\mathrm{s}$, where R_s is the sum of the sizes. Using the escape velocity ($v_\mathrm{esc}^2 = 4Gm/R_\mathrm{s}$):

maximum distance leading to a collision

$$b^2 = R_\mathrm{s}^2 \left(1 + \frac{v_\mathrm{esc}^2}{\sigma^2}\right)$$

collision cross-section (also valid for different m)

$$\Gamma = \underbrace{\pi R_\mathrm{s}^2}_{\Gamma_\mathrm{geo}} \left(1 + \frac{v_\mathrm{esc}^2}{\sigma^2}\right)$$

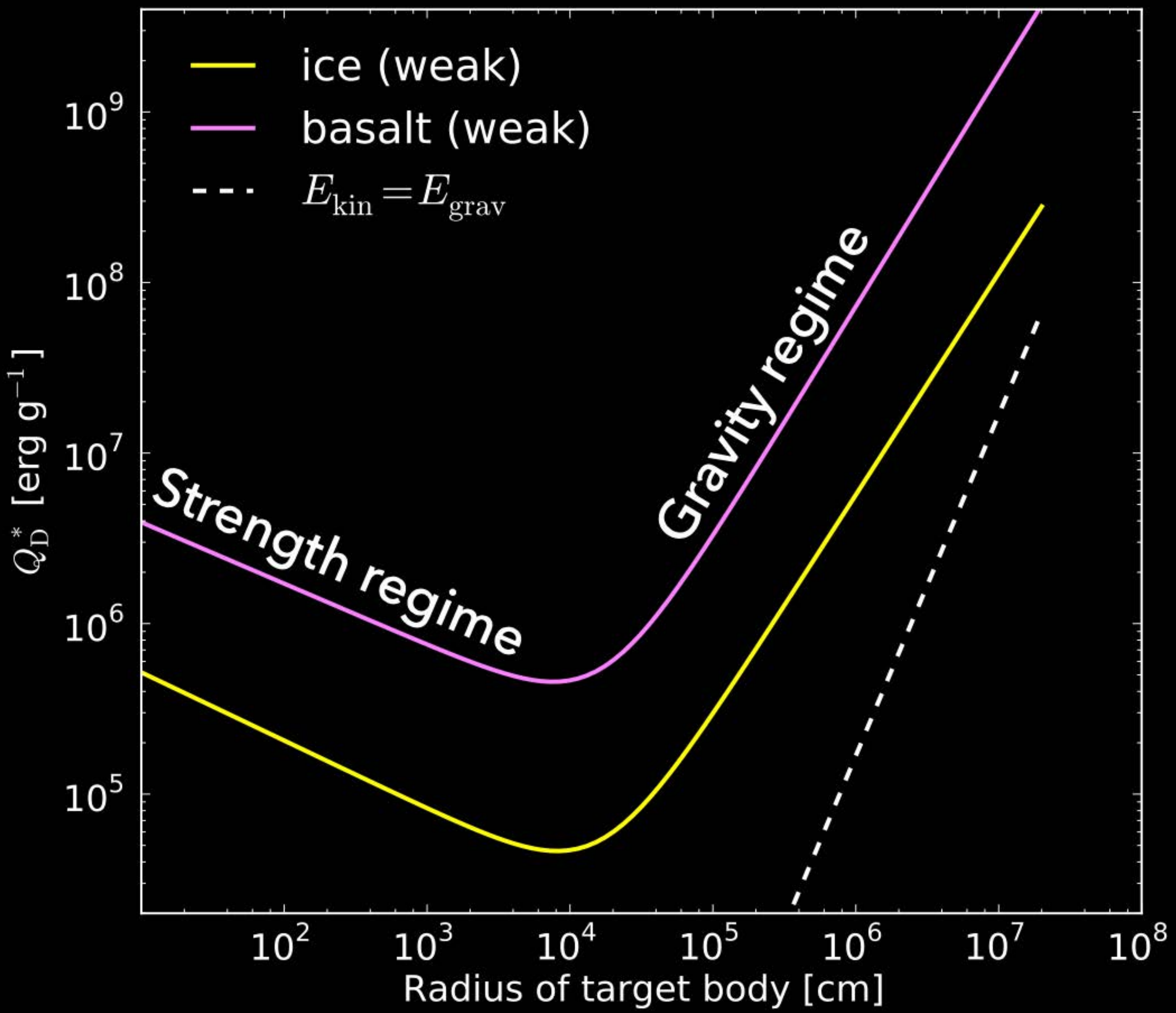
GRAVITATIONAL BINDING ENERGY

- Specific energy of the impact: $$Q \equiv \frac{mv^2}{2M} = \frac{\text{impactor energy}}{\text{target mass}}$$

- The gravitational binding energy for a sphere of uniform density:

$$E_{\mathrm{grav}} = \frac{3}{5}\frac{GM^2}{R}$$

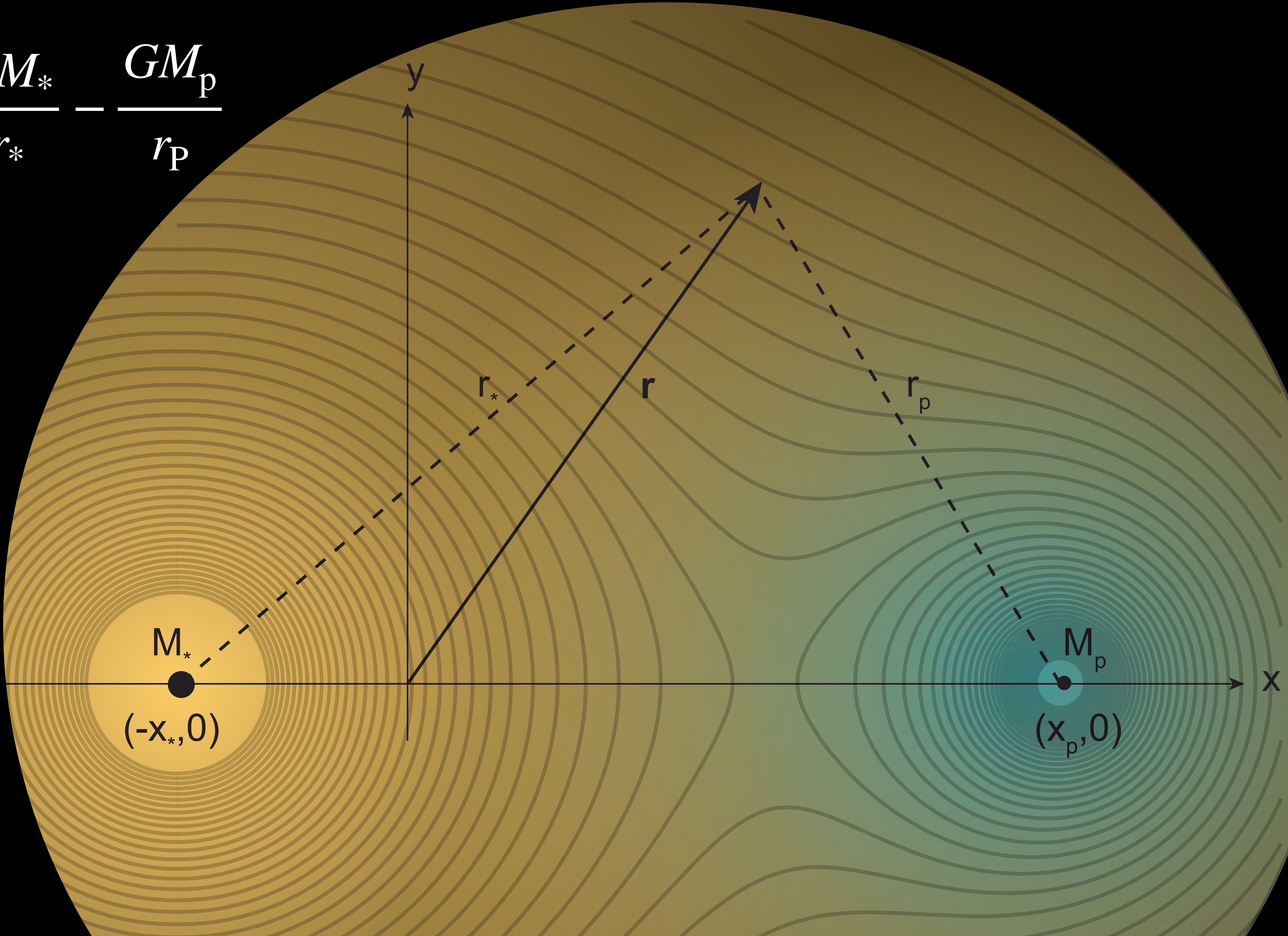
- Energy goes into heating phase changes, ejecta…



HILL RADIUS

$$\ddot{\mathbf{r}} = -\nabla\Phi \underbrace{-2(\boldsymbol{\Omega}_K \times \dot{\mathbf{r}})}_{\text{Coriolis Force}} \underbrace{-\boldsymbol{\Omega}_K \times (\boldsymbol{\Omega}_K \times \mathbf{r})}_{\text{Centrifugal Force}}$$

$$\Phi = -\frac{GM_*}{r_*} - \frac{GM_p}{r_P}$$

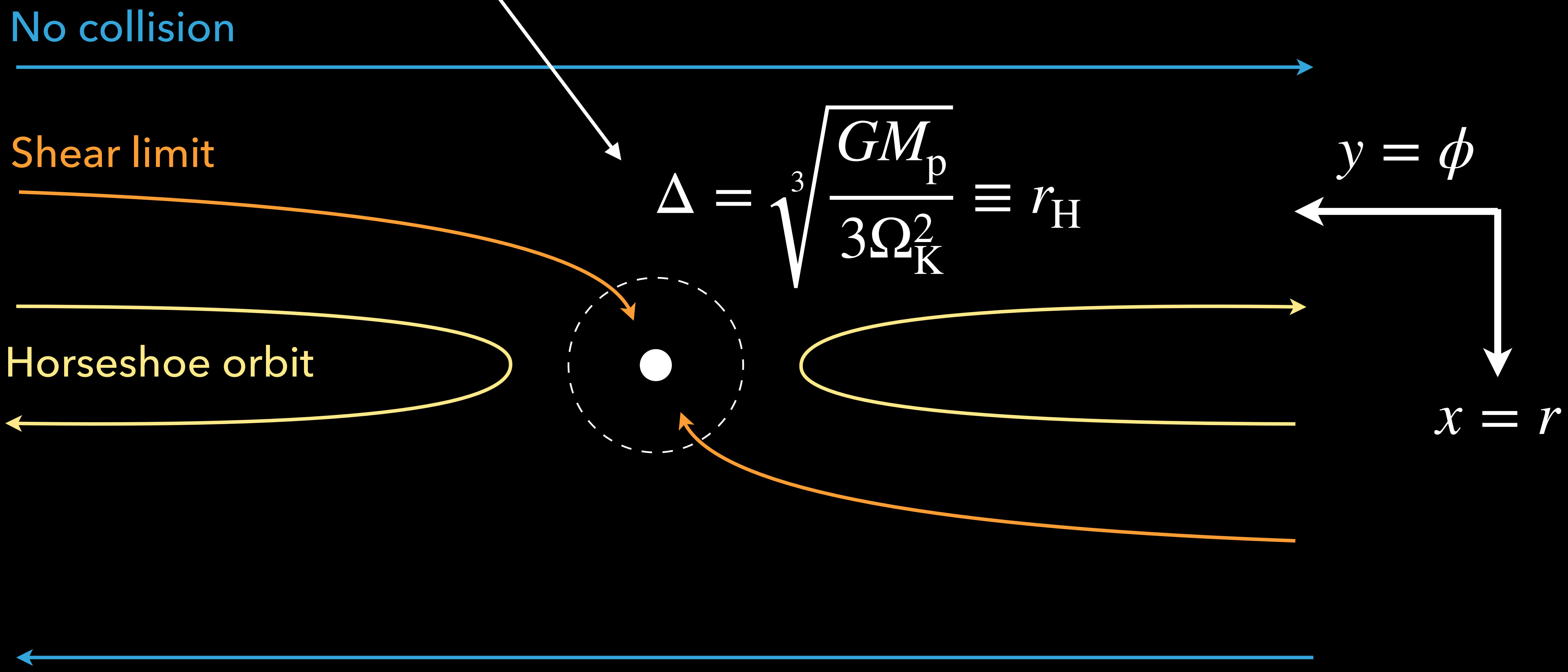


HILL RADIUS

- Assuming $M_* \gg M_\mathrm{P}$ and $\Delta = |\mathbf{r} - \mathbf{r}_\mathrm{P}|$, we can simplify:

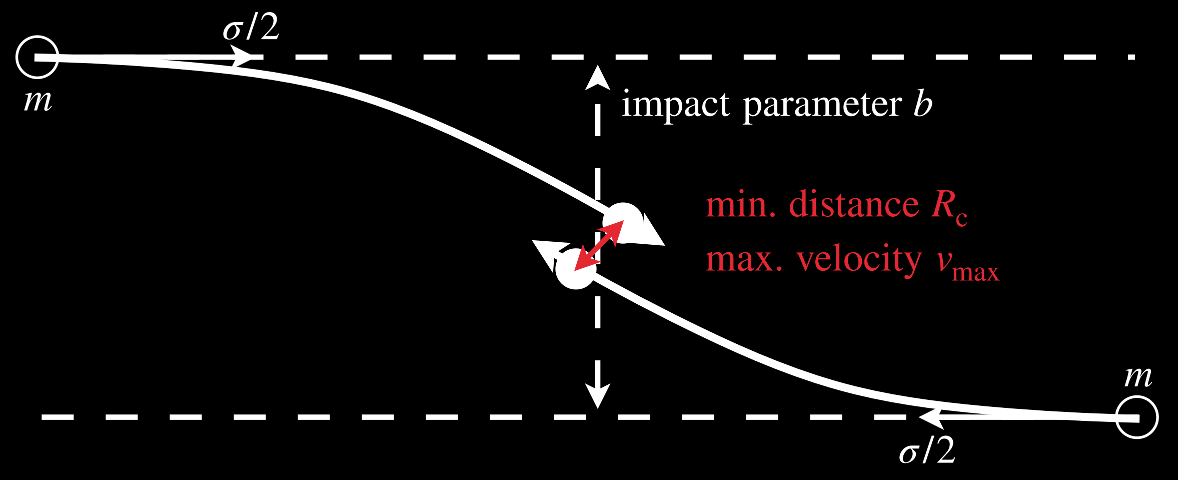
$$\ddot{x} - 2\Omega_\mathrm{K}\dot{y} = \underbrace{\left(3\Omega_\mathrm{K}^2 - \frac{GM_\mathrm{p}}{\Delta^3}\right)}_{=0} x \qquad \ddot{y} + 2\Omega_\mathrm{K}\dot{x} = -\frac{GM_\mathrm{p}}{\Delta^3} y$$

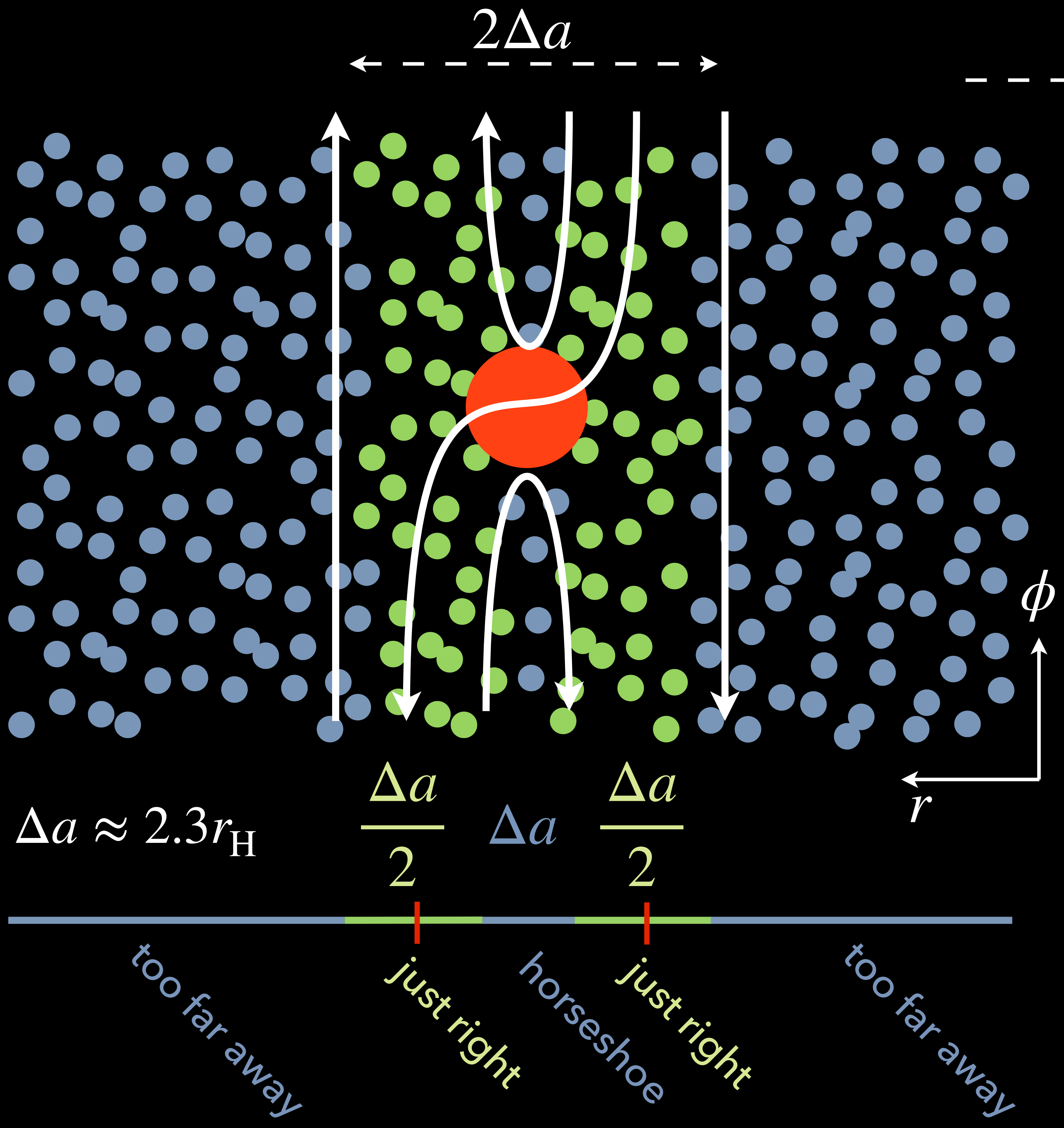
look for where the radial force vanishes (at $y = 0$)



$$\Delta = \sqrt[3]{\frac{GM_\mathrm{p}}{3\Omega_\mathrm{K}^2}} \equiv r_\mathrm{H}$$

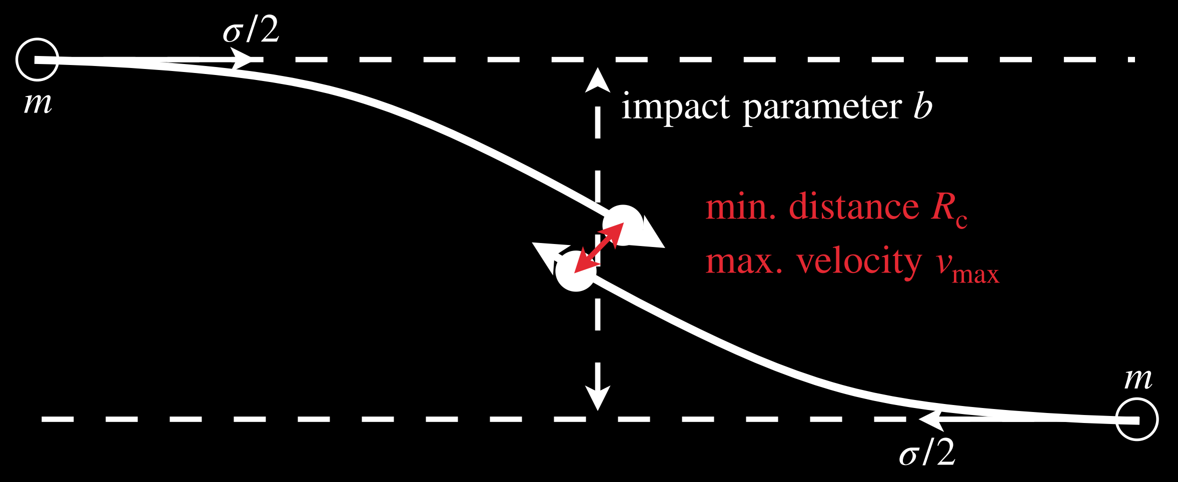
PLANETESIMAL GROWTH

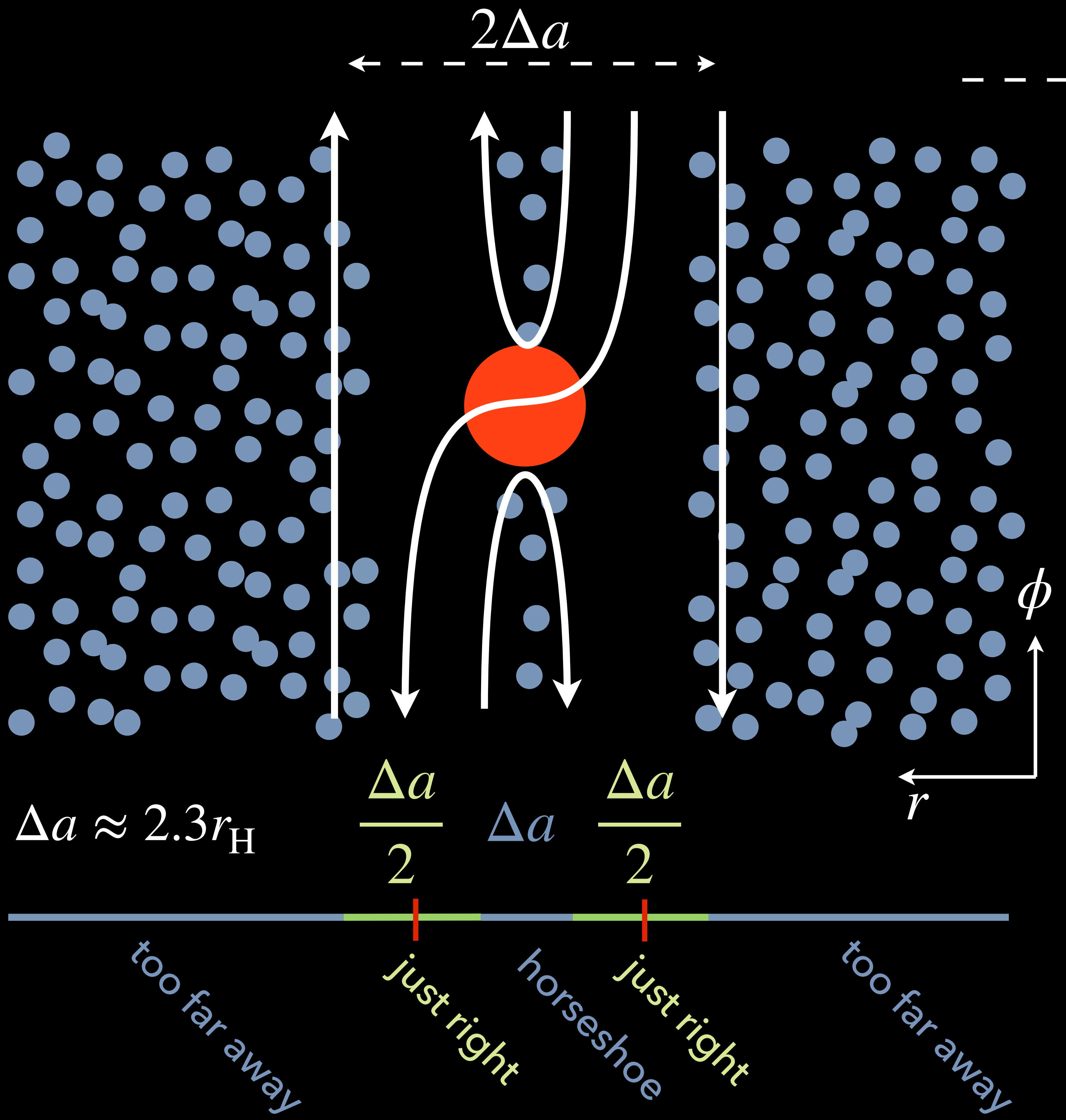




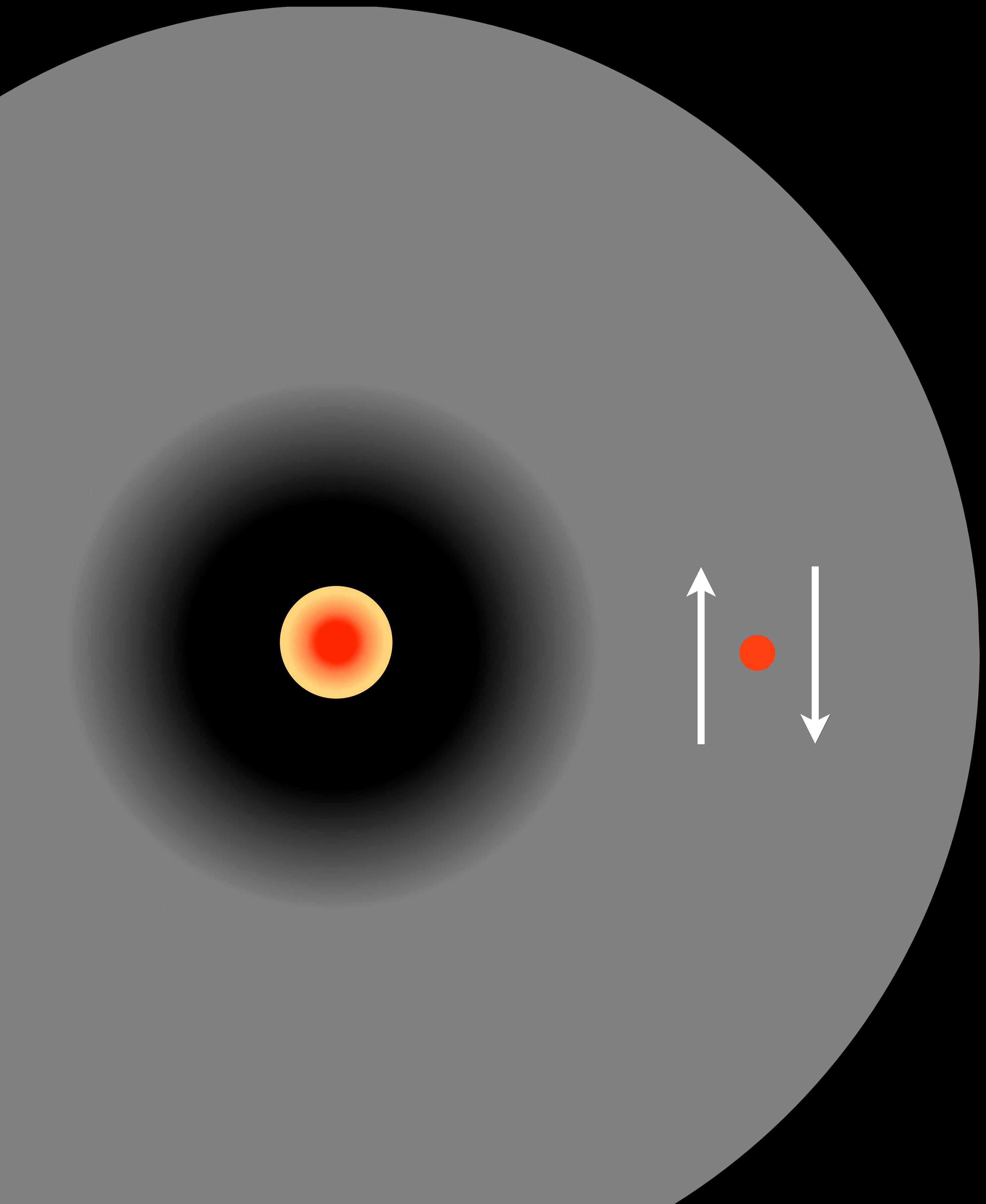
$$\Delta a \approx 2.3 r_\mathrm{H}$$

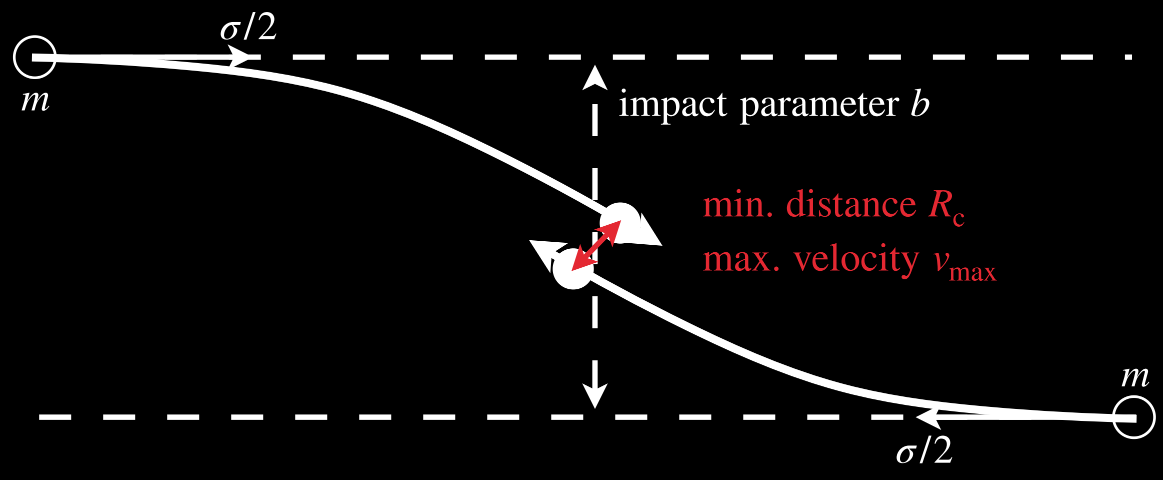
PLANETESIMAL GROWTH

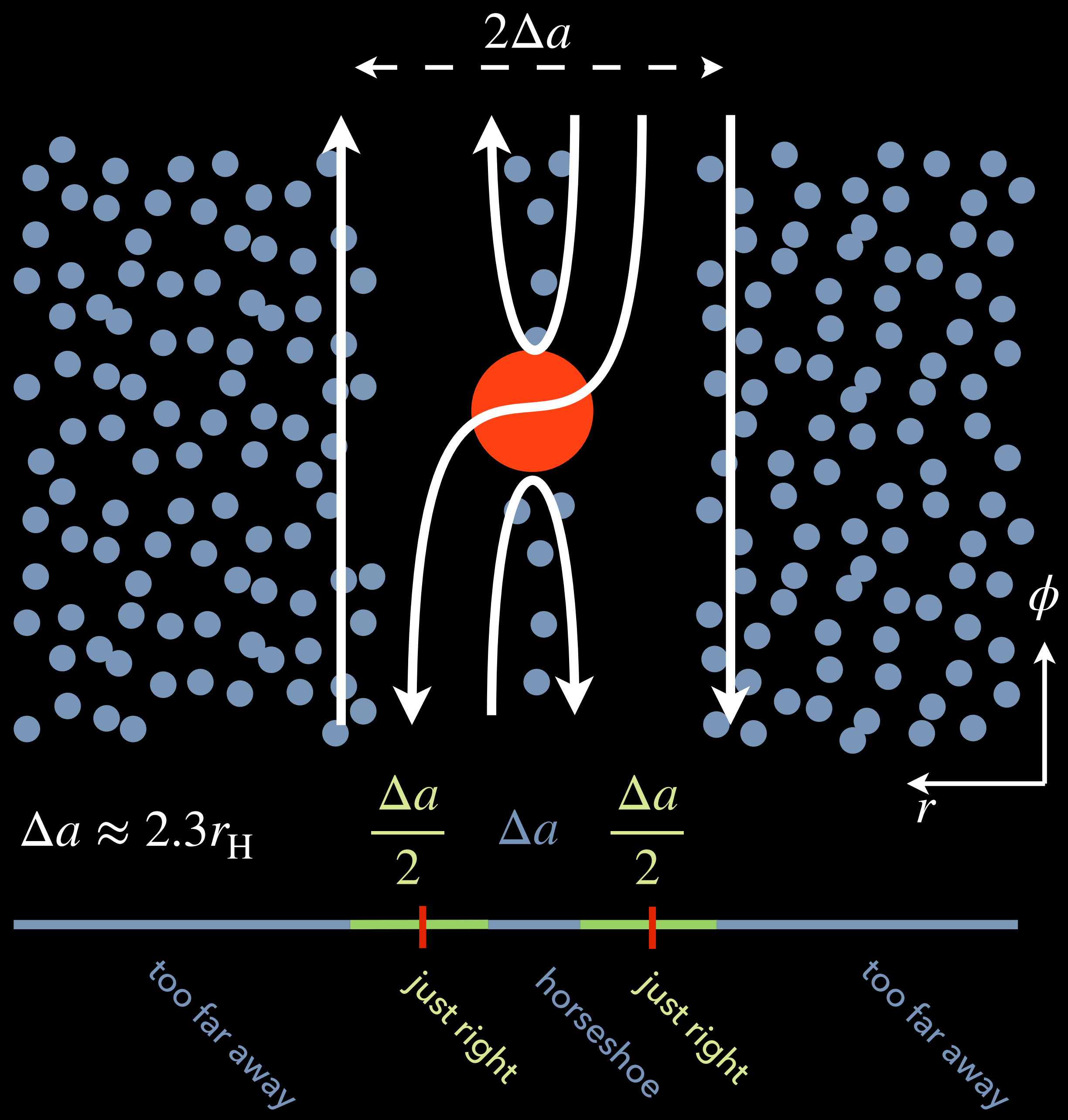




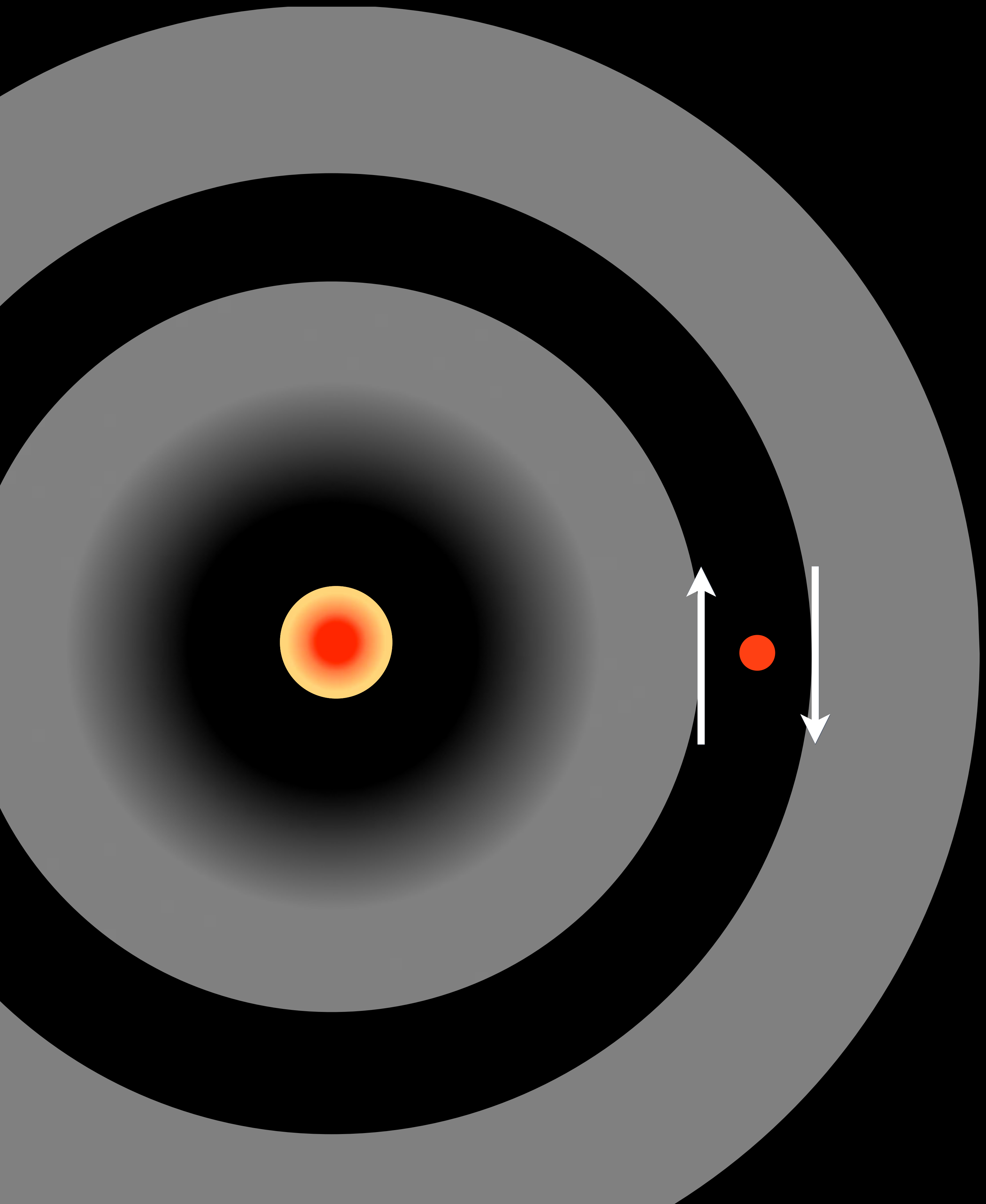
PLANETESIMAL GROWTH

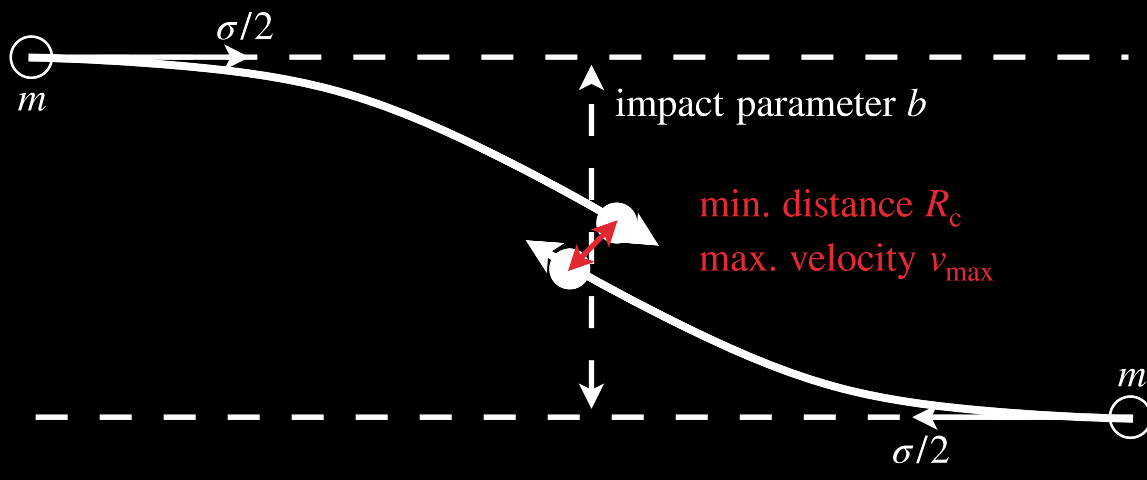


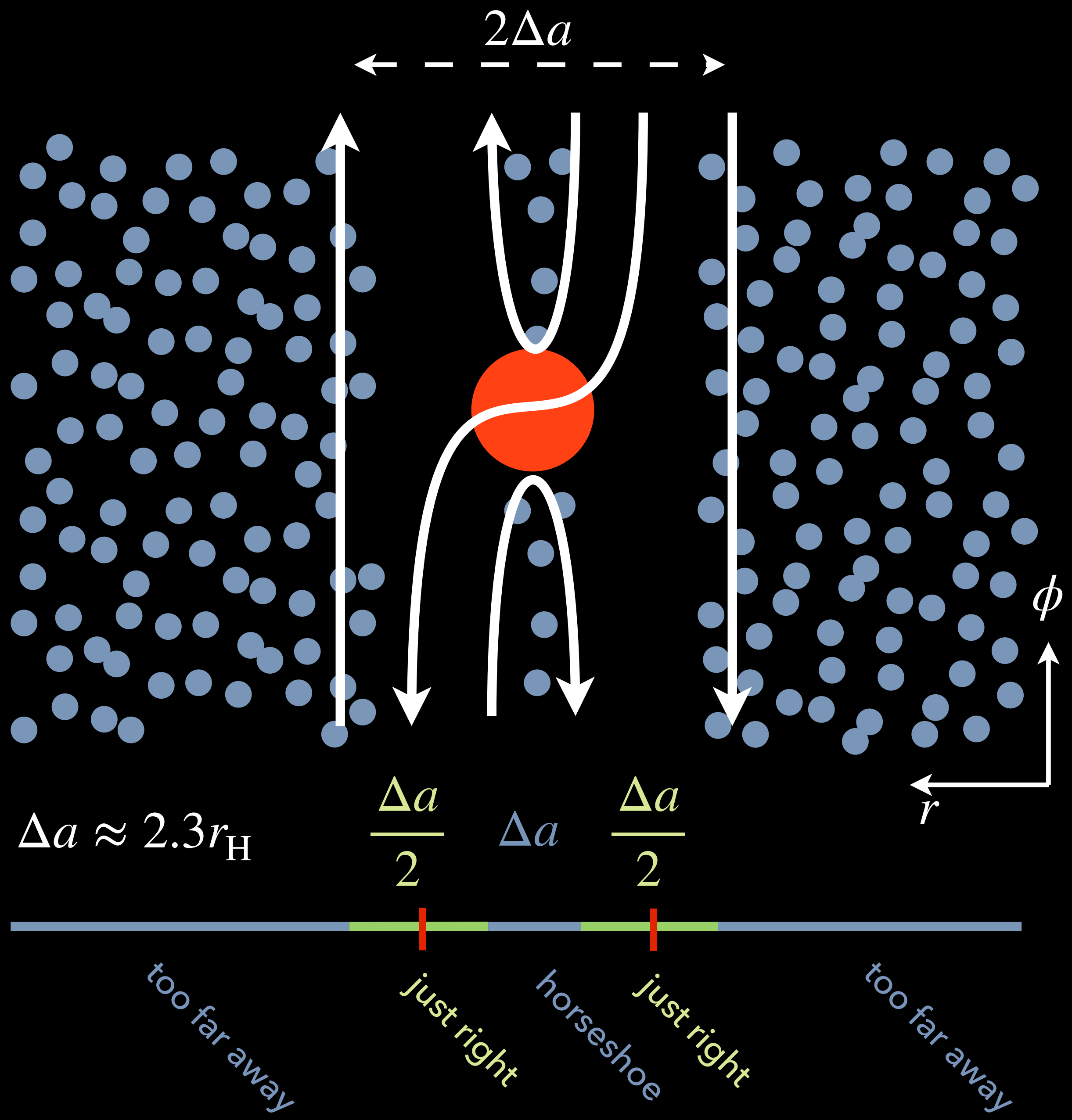




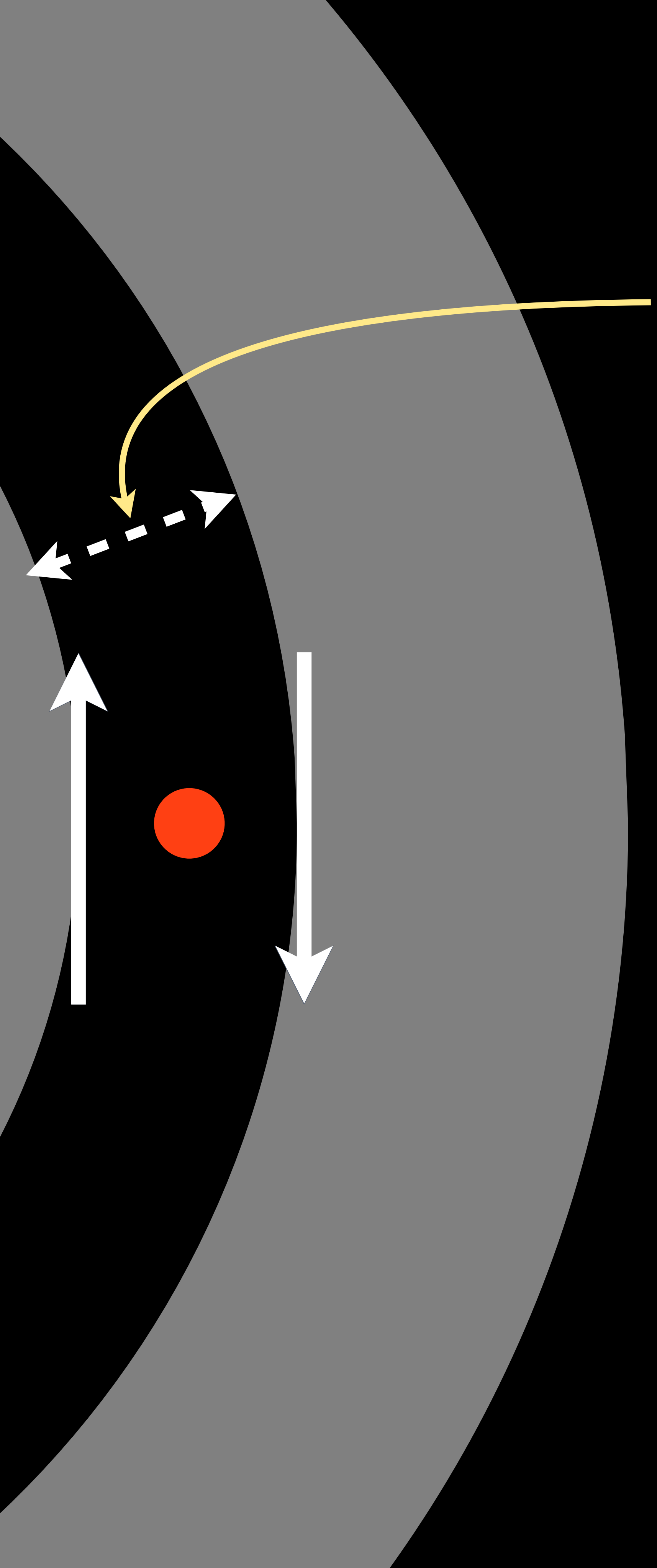
PLANETESIMAL GROWTH







PLANETESIMAL GROWTH



- Width scales with hill radius: $r_H = a_p \sqrt[3]{\frac{M_p}{3M_*}}$

$$\Delta a_{\max} \approx C r_H$$

- Mass in the feeding zone grows with planet mass:

$$M_{fz} \approx 2\pi a_p \cdot 2\Delta a_{\max} \cdot \Sigma_p \propto M^{1/3}$$

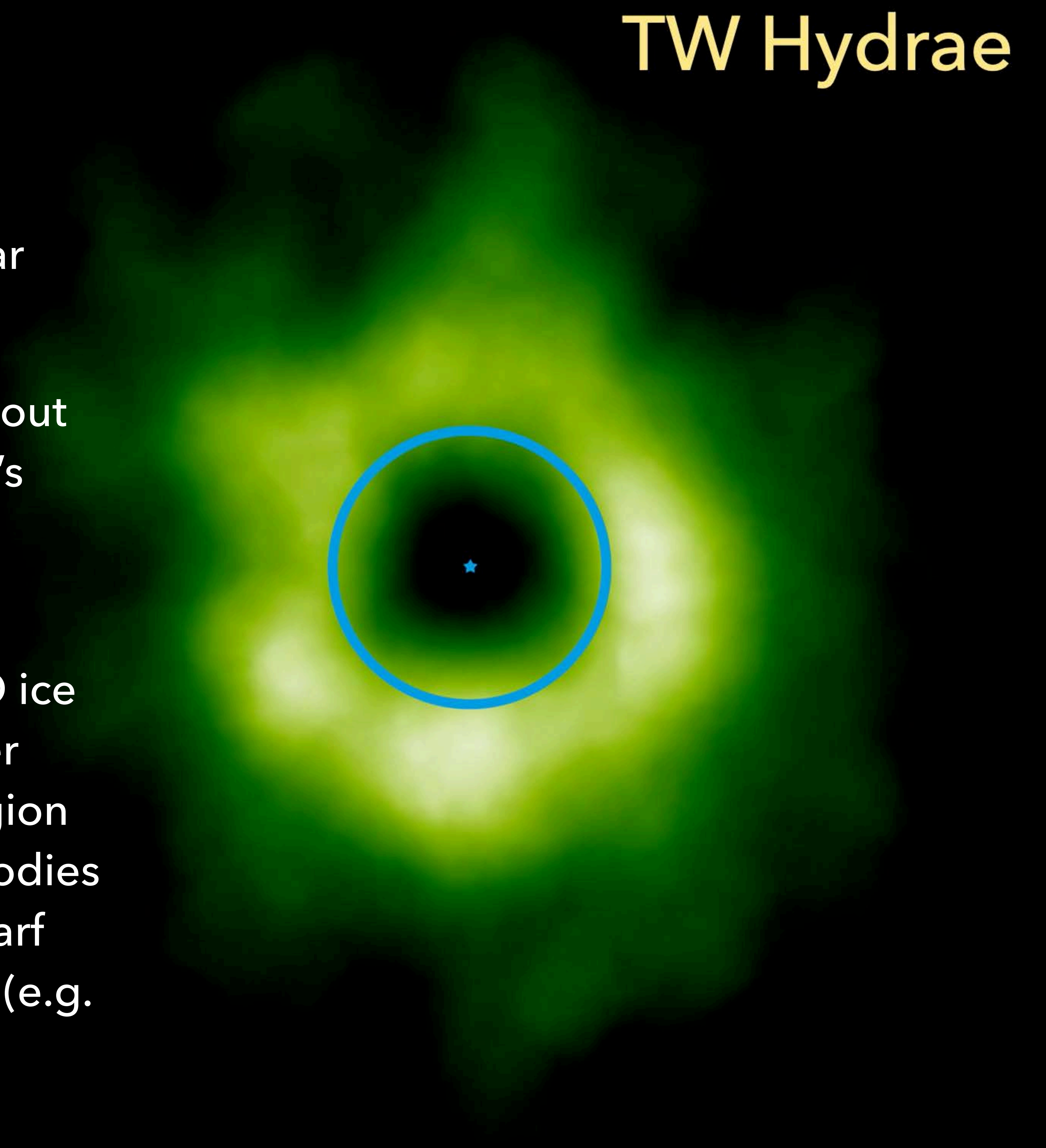
- Isolation mass (maximum mass a body can achieve through planetesimal accretion) grows with cylindrical radius:

$$M_{iso} = \frac{8}{\sqrt{3}} \pi^{3/2} C^{3/2} M_*^{-1/2} \Sigma^{3/2} a_p^3$$

$M_{iso} \approx 0.07\ M_\oplus$ in the terrestrial region

$M_{iso} \approx 9\ M_\oplus$ in the giant planet region

SNOW LINES

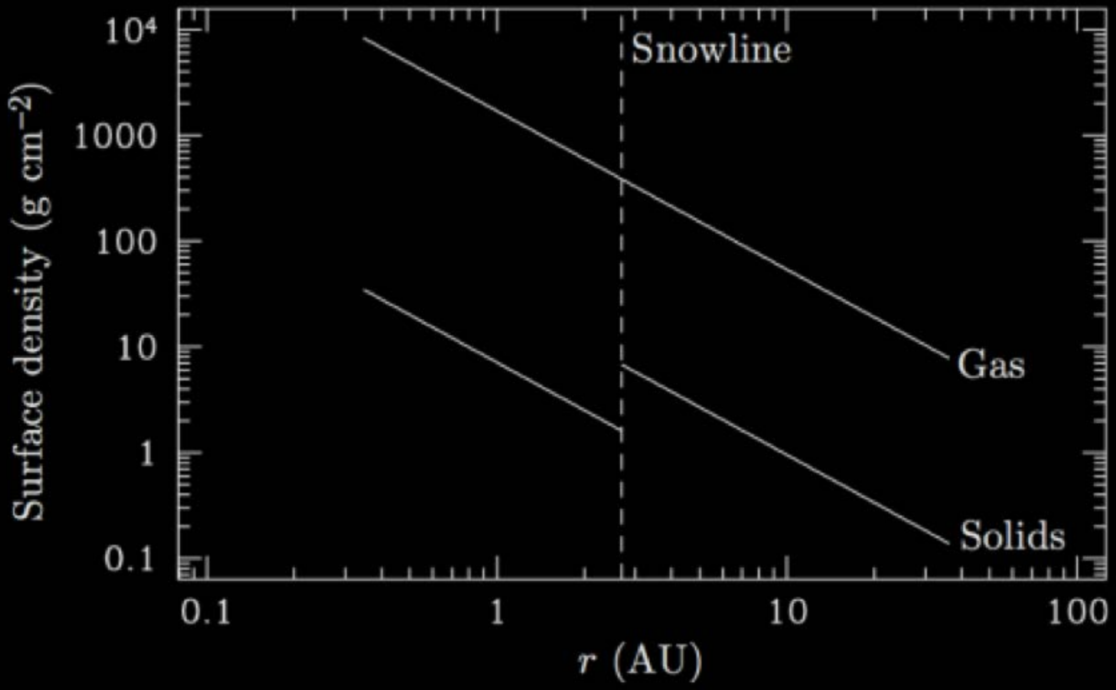


- ALMA image of CO snow around the star TW Hydrae.
- The blue circle is about the size of Neptune's orbit in our Solar System.
- The transition to CO ice could mark the inner boundary of the region where smaller icy bodies like comets and dwarf planets would form (e.g. Pluto and Eris).

SNOW LINES

- For water ice: $T_{\text{snow}} \sim 150\text{–}170\ \text{K}$, corresponding to $R \sim 1\text{–}3\ \text{au}$. The snow line for the Solar System was probably at $R = 2.7\ \text{au}$ (since the outer asteroids are icy and the inner asteroids are largely devoid of water).

- At the snow line, the density of solid particles increases suddenly. This increase in solid-particle surface density affects the time-scales and mass-scales of planets that form beyond the snow line.



- Gas giants form more easily beyond the snow line, since cores that form beyond the snow line are more massive and have a longer time to accrete gas from the disk before it dissipates.

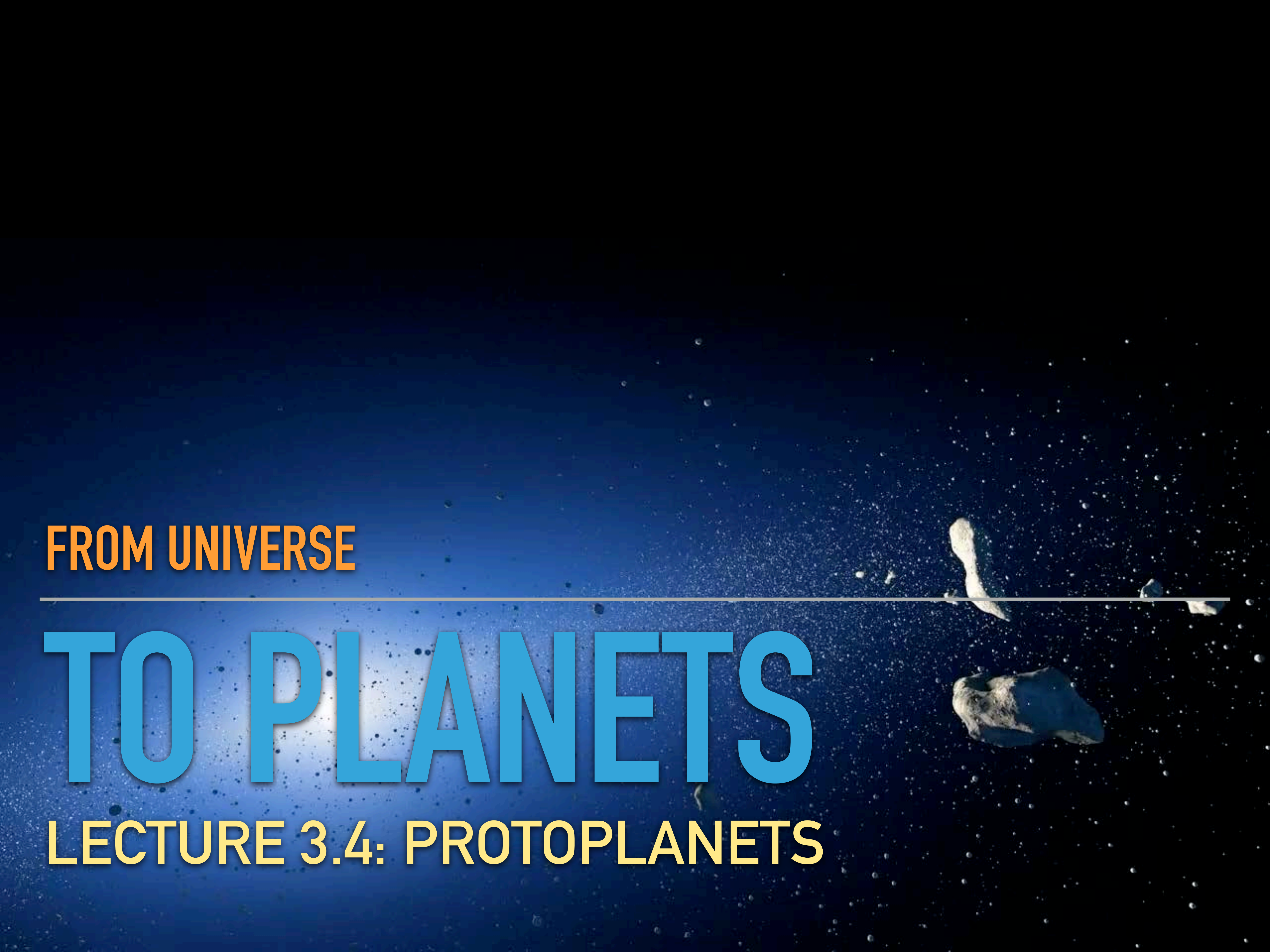
ISOLATION MASS

- The timescale for planet formation is roughly $\tau \propto 1/\Sigma$ so planetary cores which form beyond the snow-line are much larger than those that form within it.

- Isolation mass: maximum mass a body can achieve through planetesimal accretion ($M_{\mathrm{iso}} \propto \Sigma^{3/2} a_P^3$).

- Amplification of the solid surface density by a factor of ~3-4 at the snow line leads to an amplified isolation mass by a factor of ~5-8.

- The snow-line facilitates gas giant formation by helping cores to reach runaway gas accretion sooner. Timing is crucial because they must accrete the gas before the disc is dispersed.



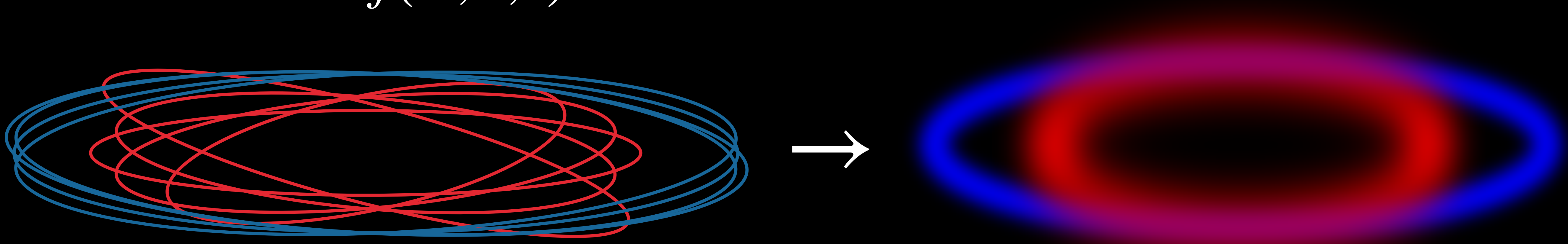
FROM UNIVERSE

TO PLANETS

LECTURE 3.4: PROTOPLANETS

RANDOM VELOCITIES

- Large number of planetesimals: statistical treatment
- Similar to gas molecules and kinetic gas theory
- “Hot” (many collisions, high velocity) vs. “cold” distributions
    - “Heating”: mutual gravitational scattering
    - “Cooling”: collisions, gas drag, ejection
- Treat eccentricity e, inclination i, and mass m using a distribution $f(m, e, i)$

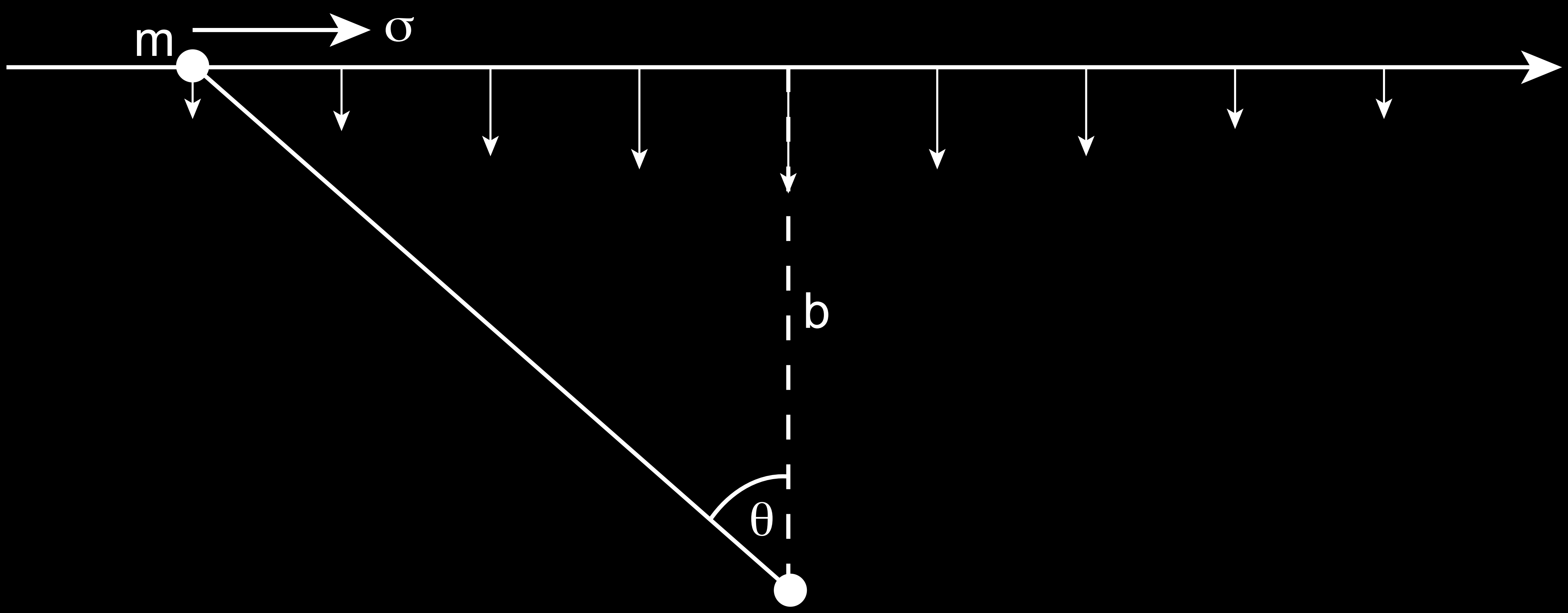


RANDOM VELOCITIES

- The random velocities set the growth regime: orderly, runaway, or oligarchic. The key ingredients are:
  - Viscous stirring (increase of random velocities) through collisions or gravitational scattering between planetesimals and protoplanets.
  - Dynamical friction: energy transfer from large to small bodies
    - Establishes energy equipartition between bodies of different sizes.
  - Damping due to dissipation in inelastic collisions and through gas drag.

VISCOUS STIRRING

- Distant fly-bys produce small angle deflections.
- Transforms some of the forward velocity (kinetic energy) to a random perpendicular velocity/energy.
- Integration over all encounters gives the increase of the random kinetic energy.



VISCOUS STIRRING

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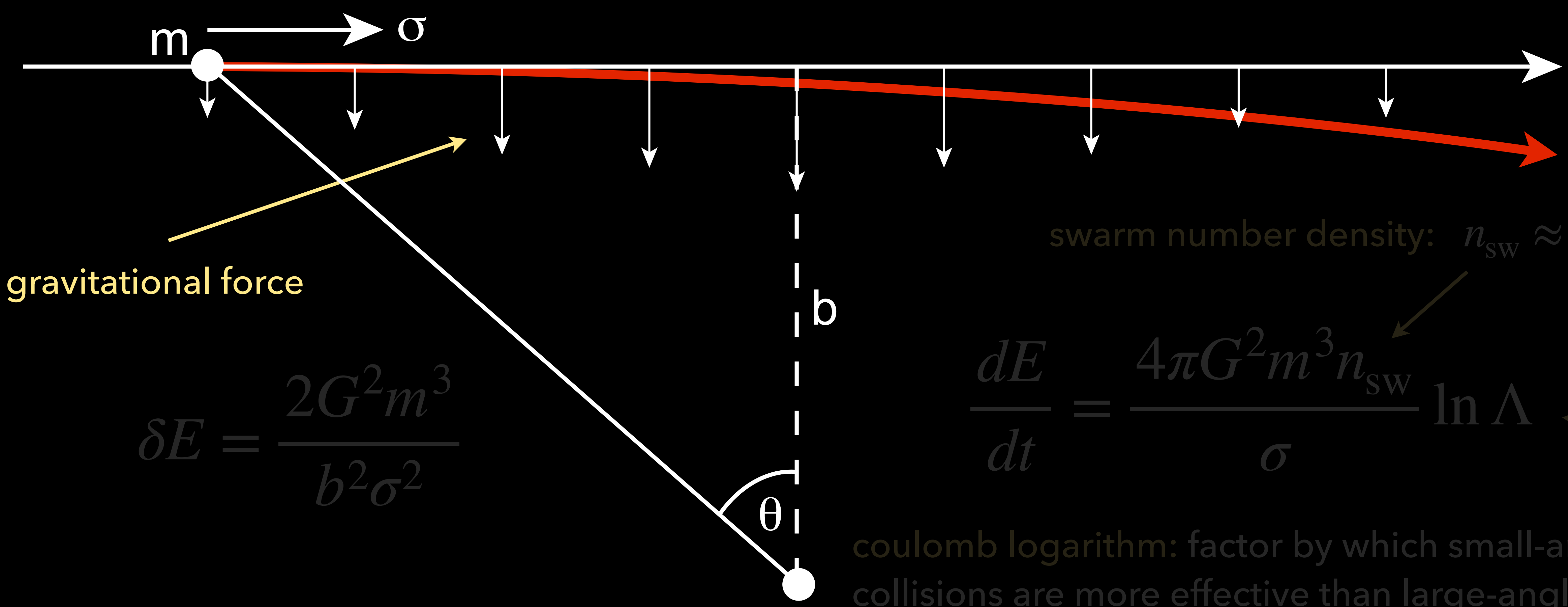


$$\delta E = \frac{2G^2m^3}{b^2\sigma^2}$$

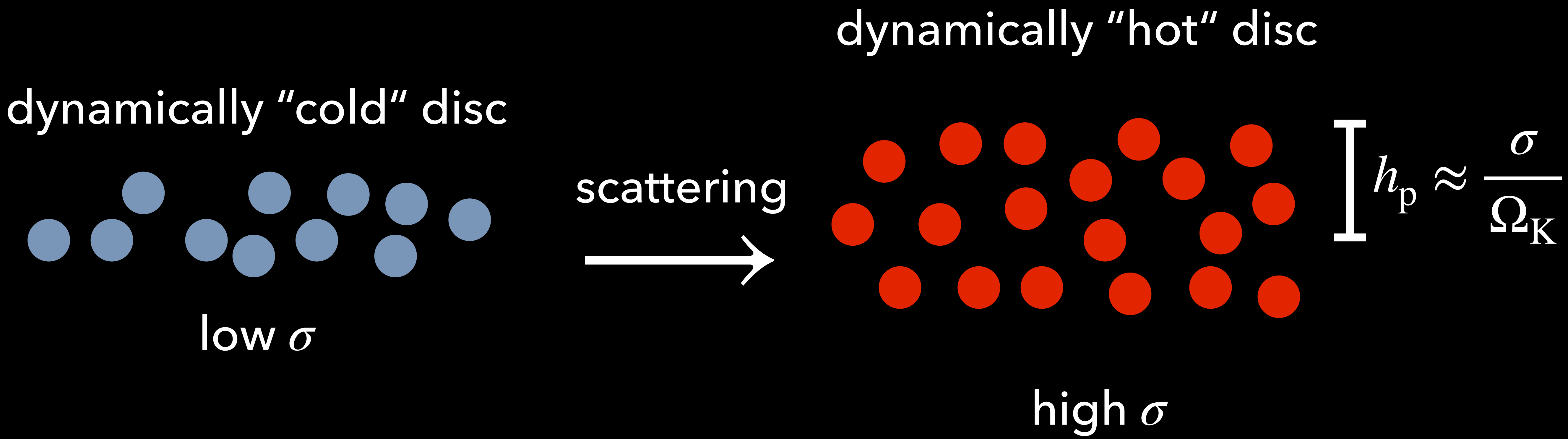
swarm number density: $n_{\rm sw} \approx \dfrac{\Sigma_{\rm sw}\Omega_{\rm K}}{2m\sigma}$

$$\frac{dE}{dt} = \frac{4\pi G^2 m^3 n_{\rm sw}}{\sigma} \ln\Lambda$$

coulomb logarithm: factor by which small-angle collisions are more effective than large-angle collisions

DYNAMICAL FRICTION

- Same principle as before, but now with different masses.



dynamically “cold” disc

low σ

scattering

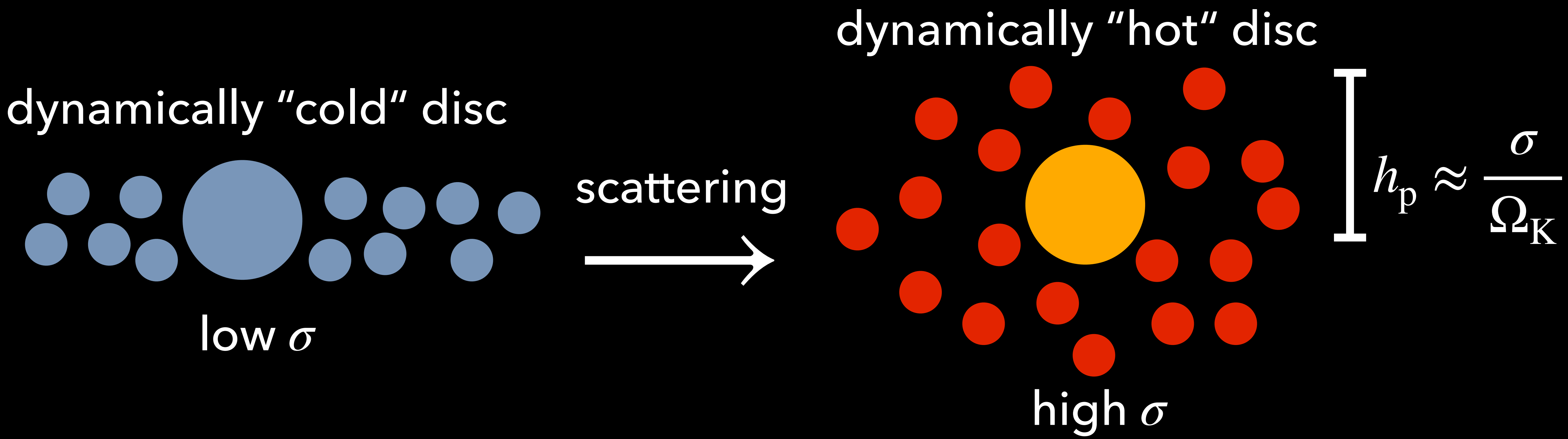
dynamically “hot” disc

$$h_\mathrm{p} \approx \frac{\sigma}{\Omega_\mathrm{K}}$$

high σ

DYNAMICAL FRICTION

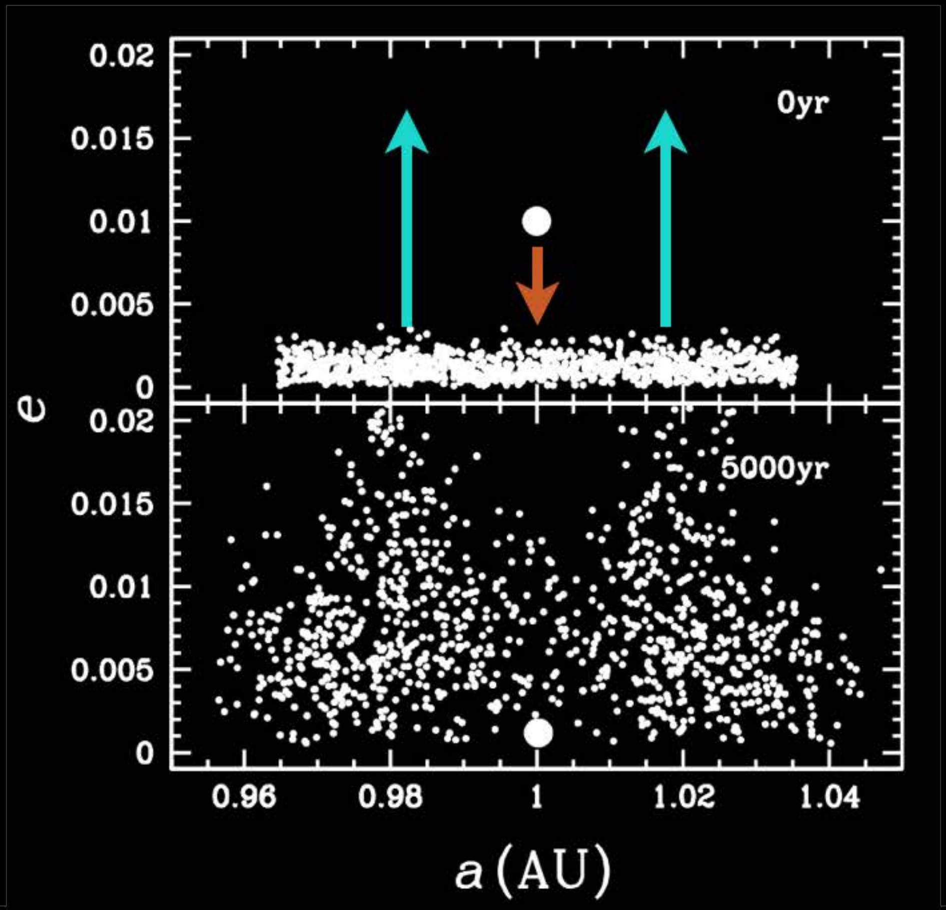
- Same principle as before, but now with different masses.
- Massive body heats up its environment



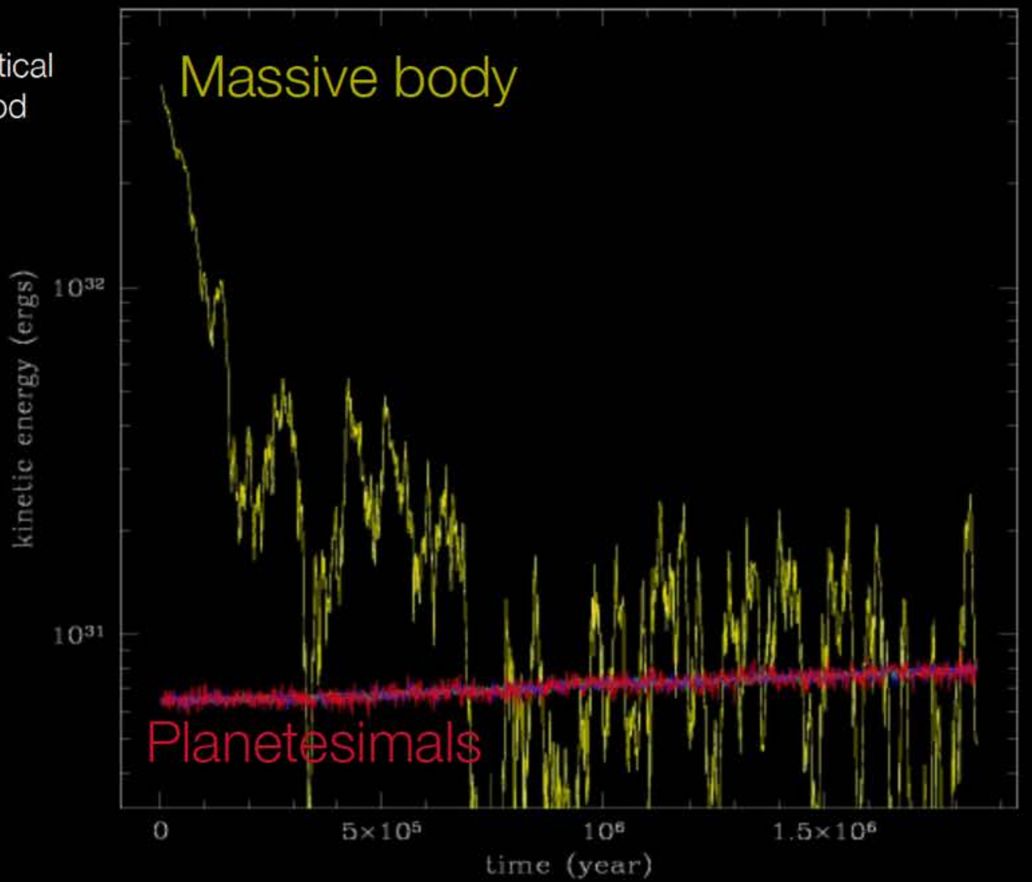
Massive body is scattered less, smaller masses are scattered more. Many encounters lead to energy equipartition:

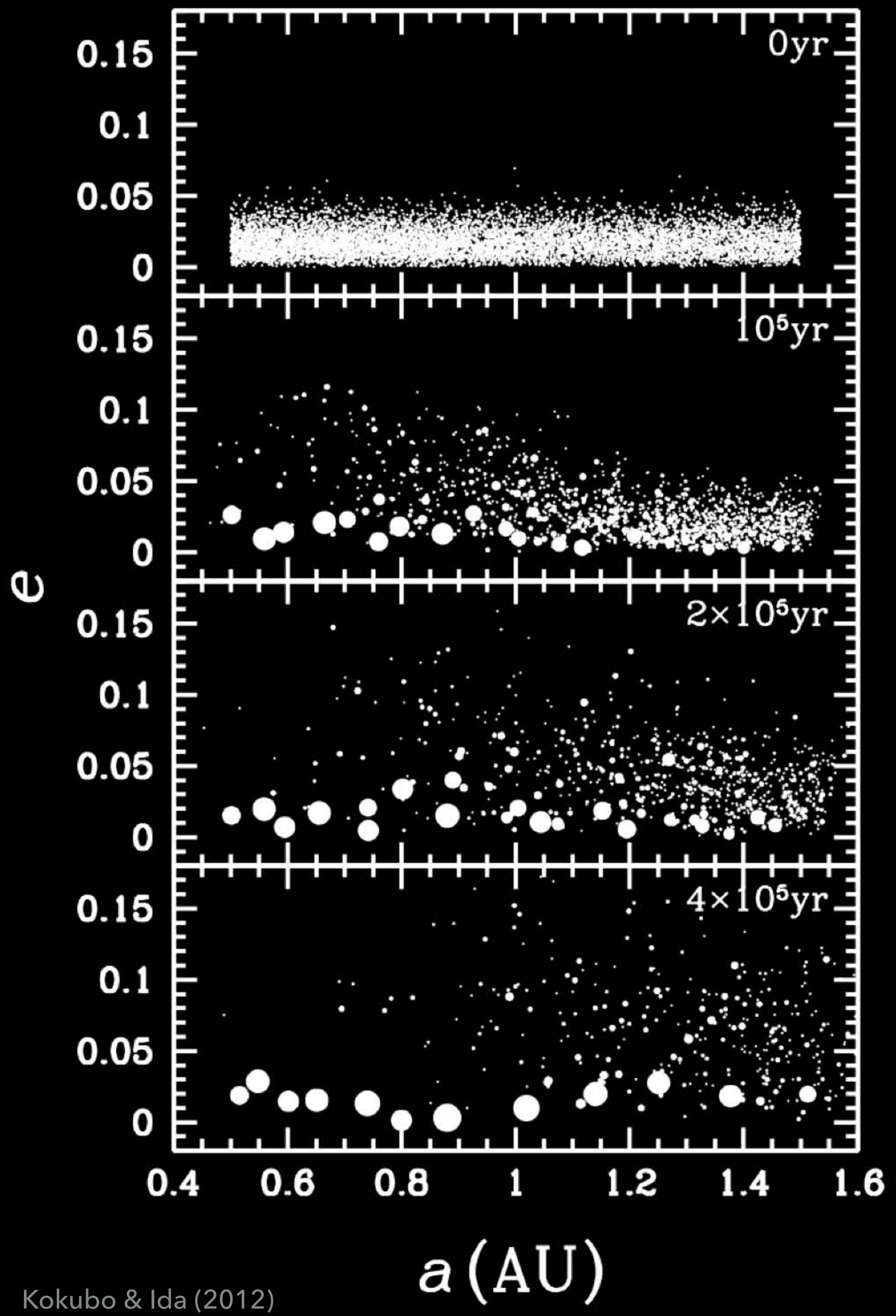
$$\frac{1}{2}m\sigma_m^2 = \frac{1}{2}M\sigma_m^2$$

DYNAMICAL FRICTION



Benz
Statistical
method

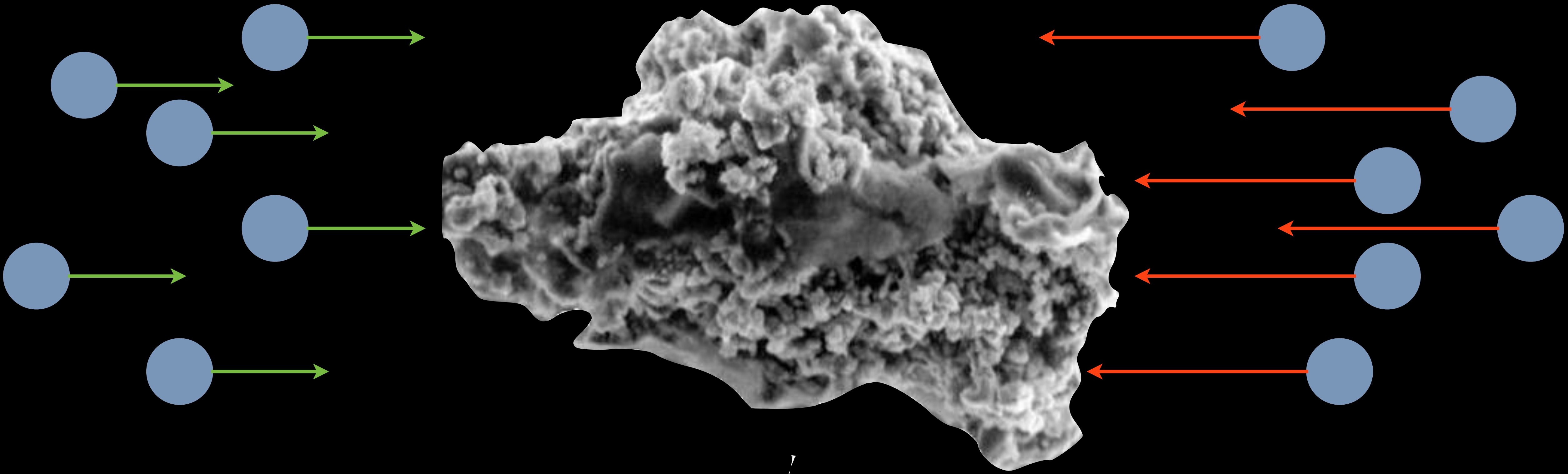




Kokubo & Ida (2012)

GAS DAMPING

- Gas drag is similar to that of smaller particles, but sizes are large enough to put them in the hydrodynamic (Stokes) regime.



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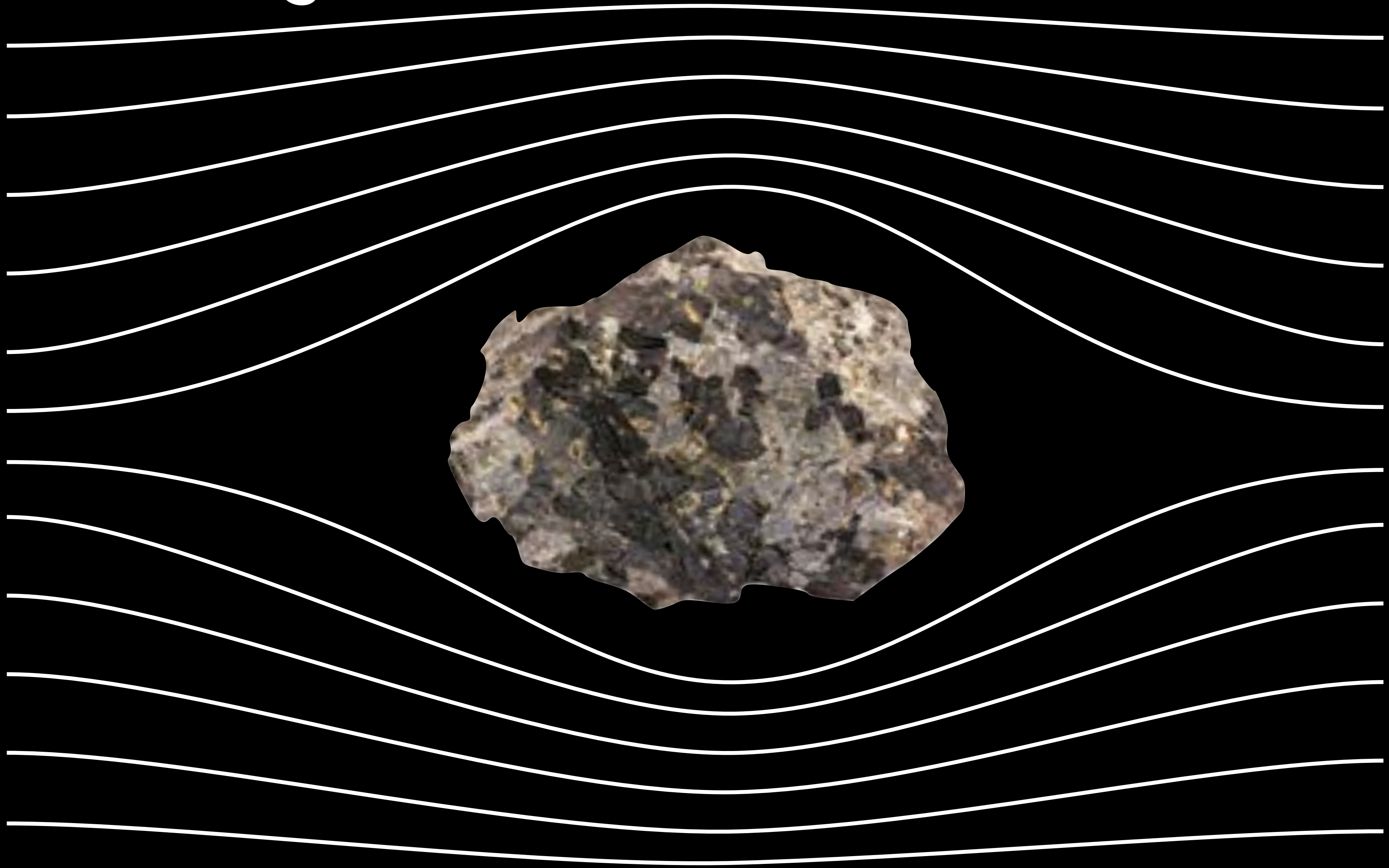
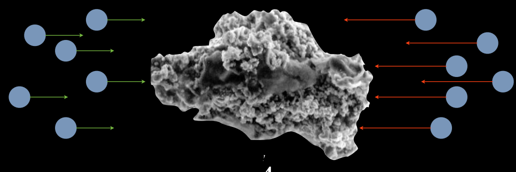


$$F_{\text{Stokes}} = -\frac{C_{\text{D}}}{2}\pi a^2 \rho_{\text{gas}} v \mathbf{v}$$



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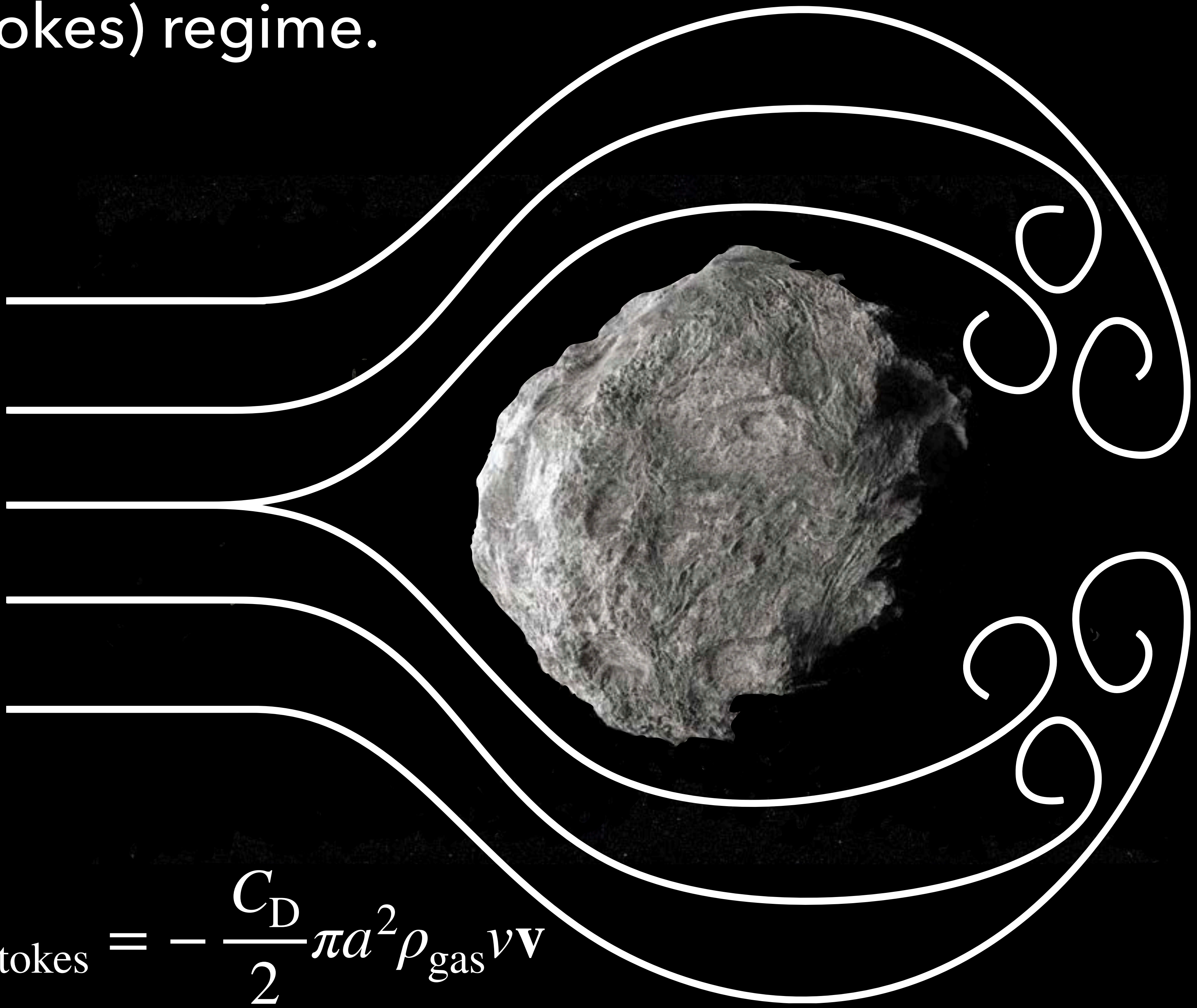

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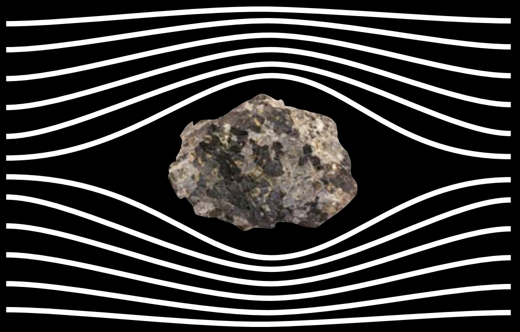
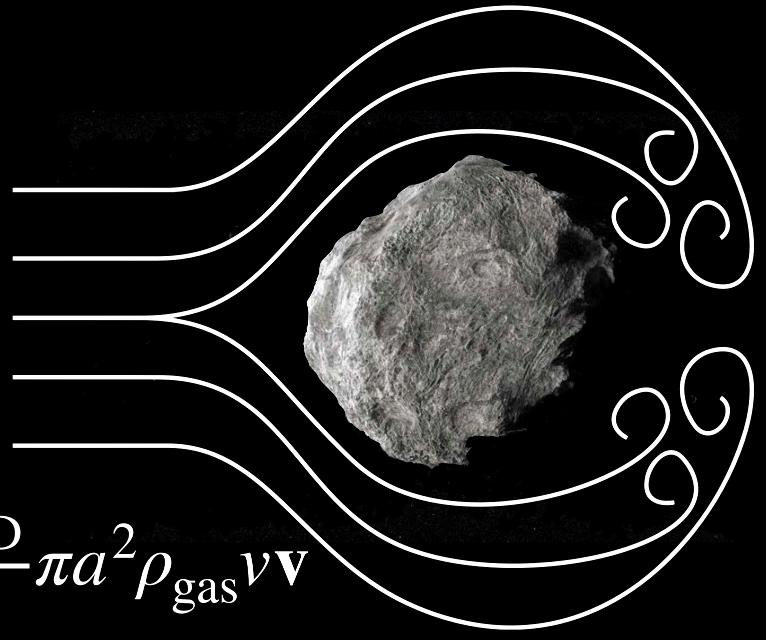


GAS DAMPING

- Gas drag is similar to that of smaller particles, but sizes are large enough to put them in the hydrodynamic (Stokes) regime.
- Damping acts against viscous stirring, thereby facilitating growth. For strong damping, the system evolves into a shear dominated regime (thin planetesimal disc). The 2D dynamics lead to high collisional probabilities and large focussing factors ⟶ large growth rates.
- Assuming the random velocity is $v \sim e v_\mathrm{K}$, the damping timescales is approximately:

$$\tau_\mathrm{damp,gas} \approx \frac{mv}{F_\mathrm{drag}} = \frac{2m}{C_\mathrm{D} \pi a^2 \rho_\mathrm{gas} e v_\mathrm{K}} \propto a$$



$$F_\mathrm{Stokes} = -\frac{C_\mathrm{D}}{2} \pi a^2 \rho_\mathrm{gas} v \mathbf{v}$$

ORDERLY AND RUNAWAY GROWTH

- Recall our mass accretion rate including the focusing factor:

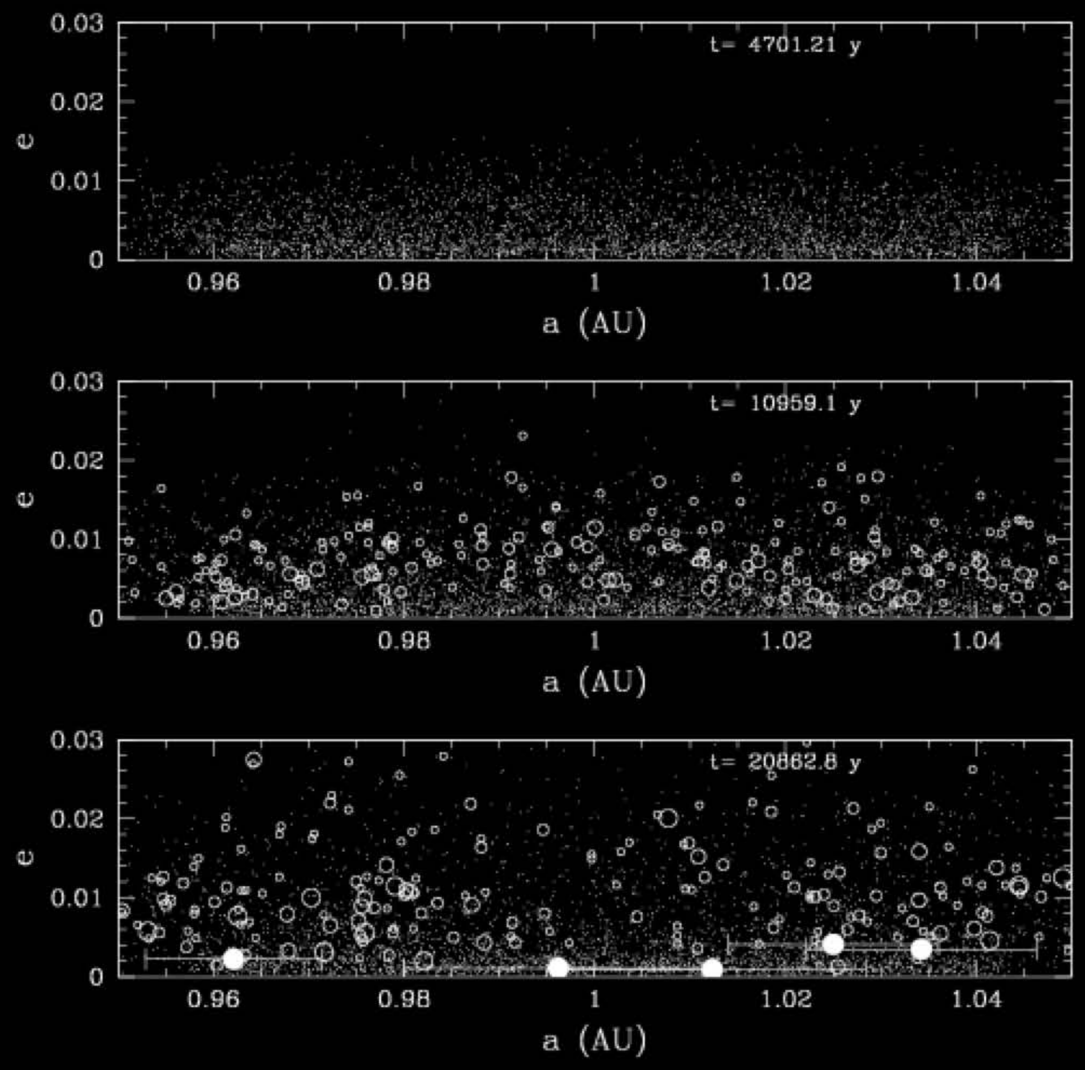
$$\frac{dM_{\rm p}}{dt} = \frac{\sqrt{3}}{2}\Sigma_{\rm p}\Omega_{\rm K}\,\pi R_{\rm s}^2 \overbrace{\left(1+\frac{v_{\rm esc}^2}{\sigma^2}\right)}^{\Gamma=\text{focusing factor}}$$

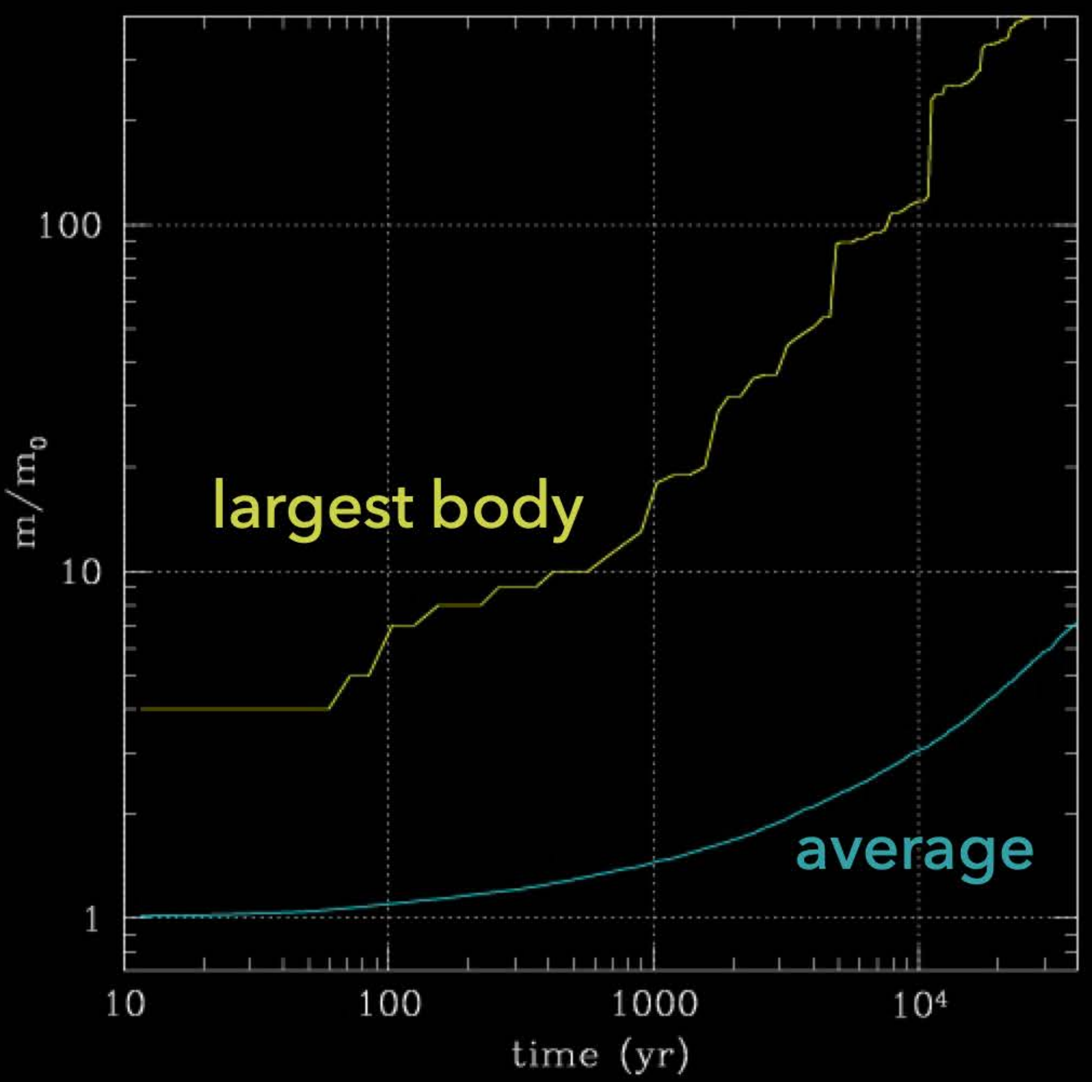
- Depending on the velocity dispersion, we get two different growth regimes:

$$\frac{1}{M_{\rm p}}\frac{dM_{\rm p}}{dt} \propto \begin{cases} M_{\rm p}^{-1/3}, & \sigma \gg v_{\rm esc}\ \text{(orderly)} \\ M_{\rm p}^{1/3}, & \sigma \ll v_{\rm esc}\ \text{(runaway)} \end{cases}$$

- Viscous stirring increases σ, leading to orderly growth, with a power-law size distribution having most of the mass in the largest bodies
- Addition of Dynamical friction and gas drag tends to equalise kinetic energies and damp σ of the more massive bodies, leading to runaway growth of a small number of embryos

ORDERLY AND RUNAWAY GROWTH





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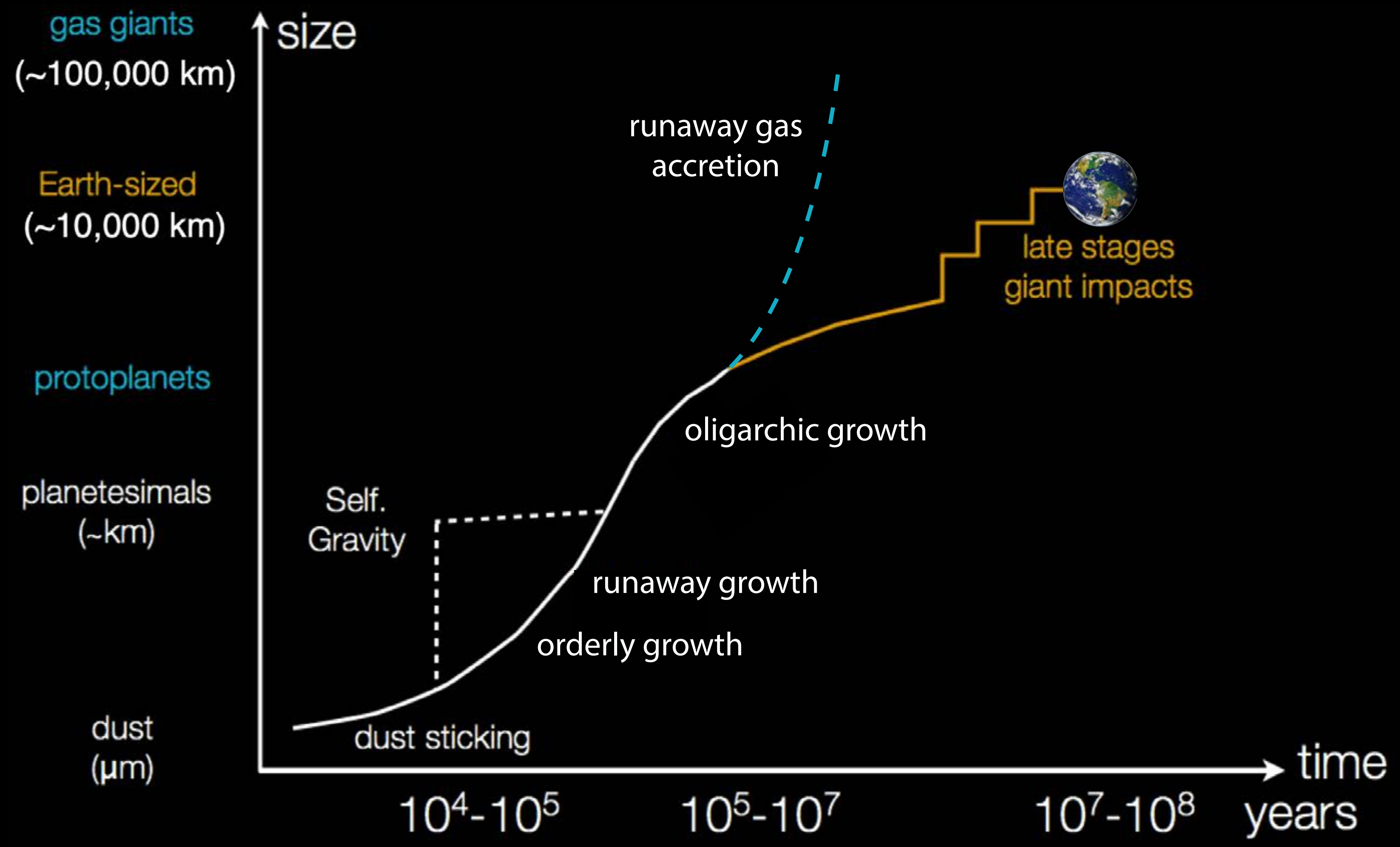
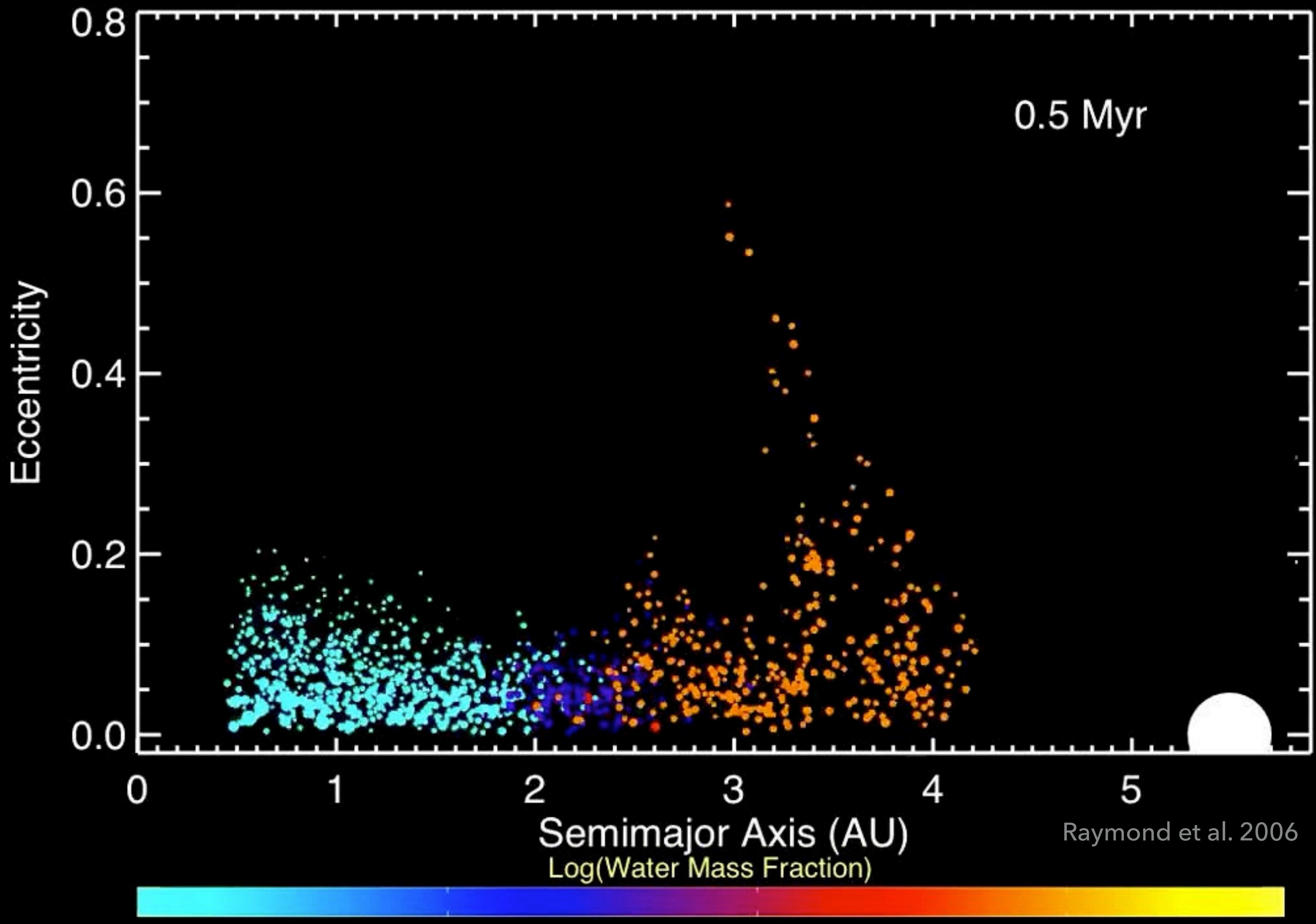
OLIGARCHIC GROWTH

- Runaway growth continues until the feedback from the big bodies stirs up the neighbouring planetesimals again
- Growth rates slows (similar to orderly growth), but are still faster than planetesimals in their surroundings (similar to runaway growth)
- Transition from runaway to oligarchic growth occurs between 10^{-3}–$10^{-2}\ M_\oplus$



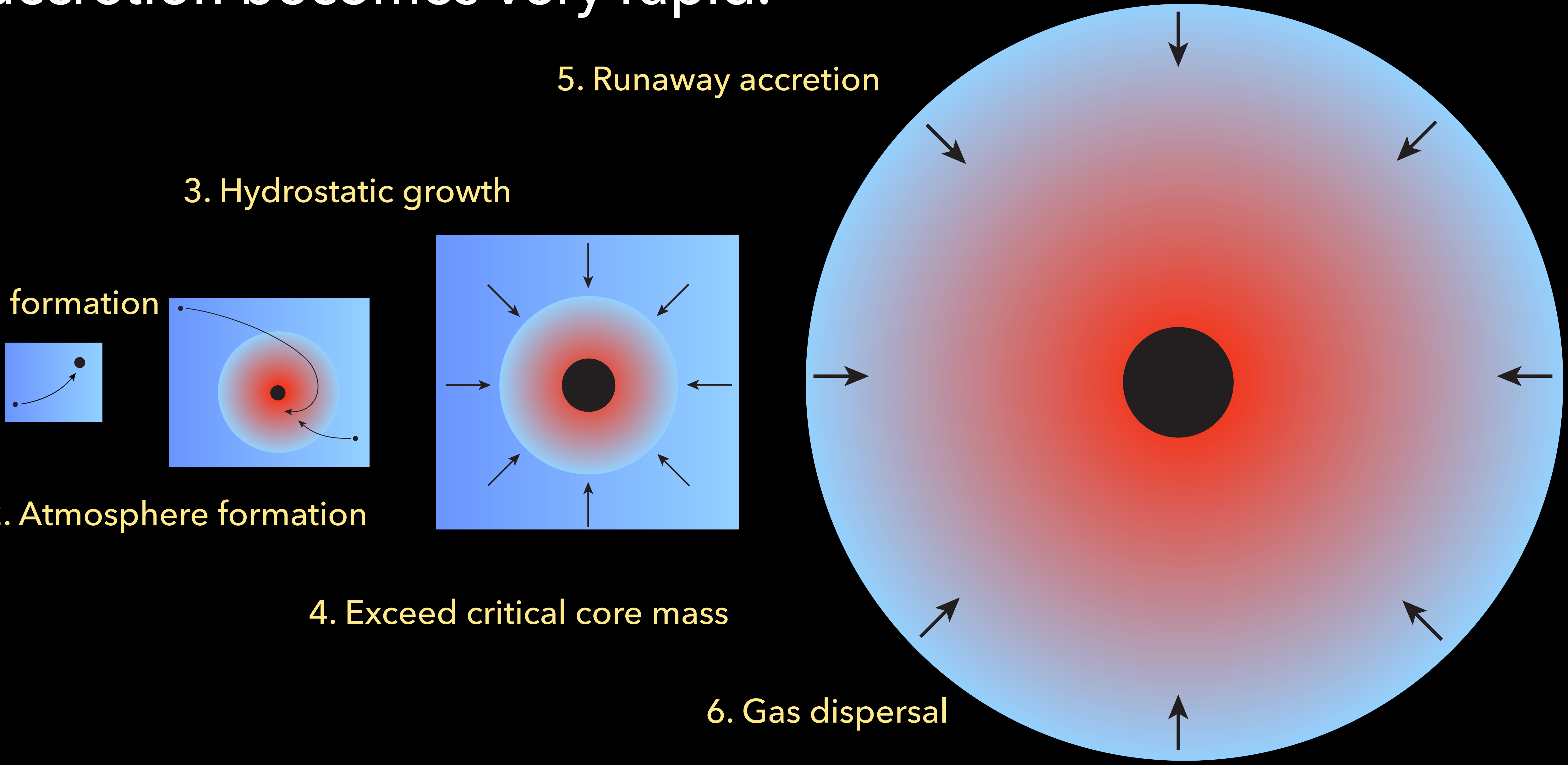
OLIGARCHIC GROWTH



Raymond et al. 2006

GAS ACCRETION

- Gas accretion begins very slowly because it is pressure supported. Prevents new gas from being accreted.
- Once a critical core mass is achieved, runaway gas accretion becomes very rapid.



1. Core formation

2. Atmosphere formation

3. Hydrostatic growth

4. Exceed critical core mass

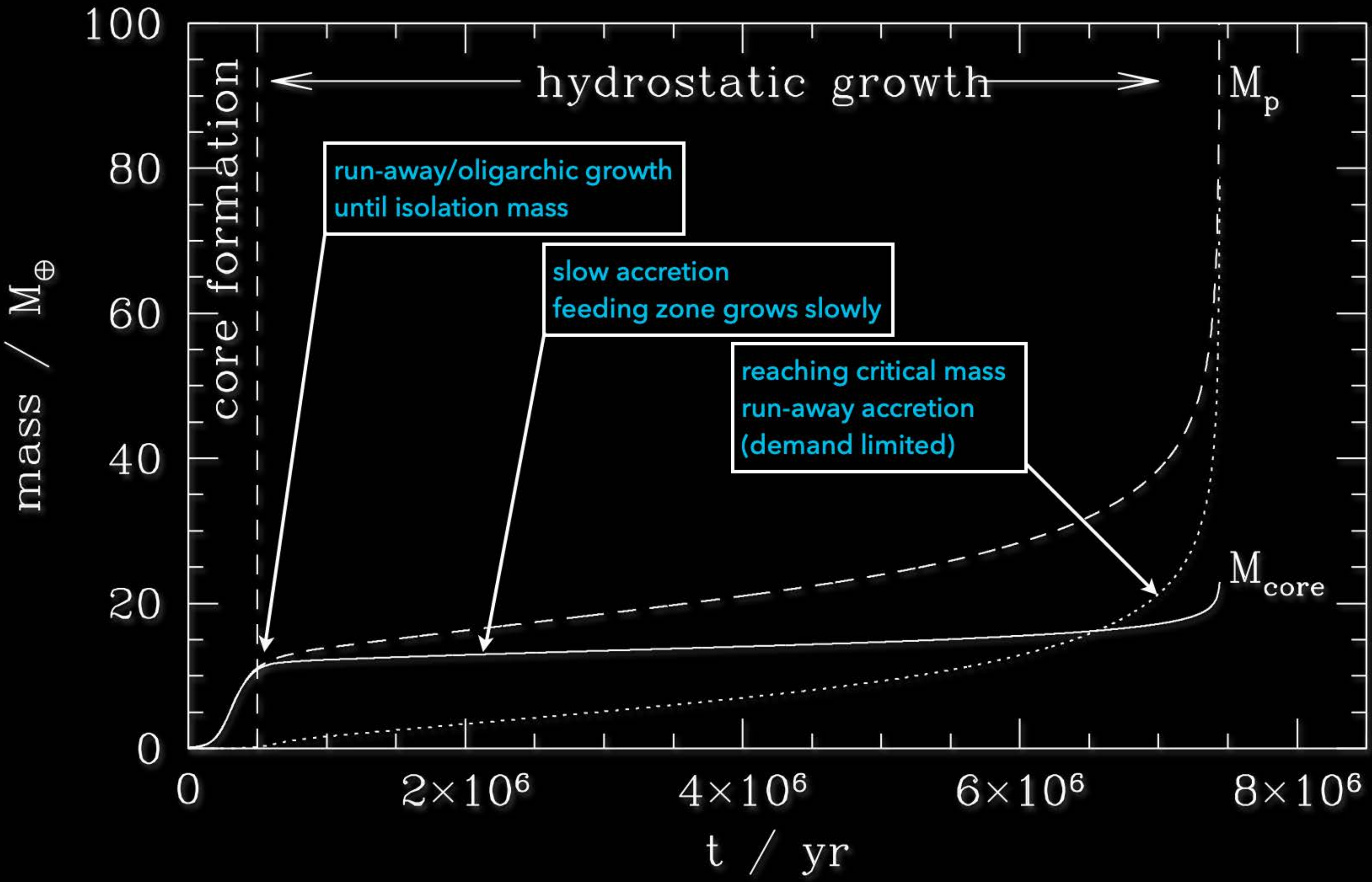
5. Runaway accretion

6. Gas dispersal

7. Kelvin-Helmholtz contraction

GAS ACCRETION: RUNAWAY GROWTH

- Accounting for boundary conditions (attached vs detached) leads to a step-like growth behaviour:

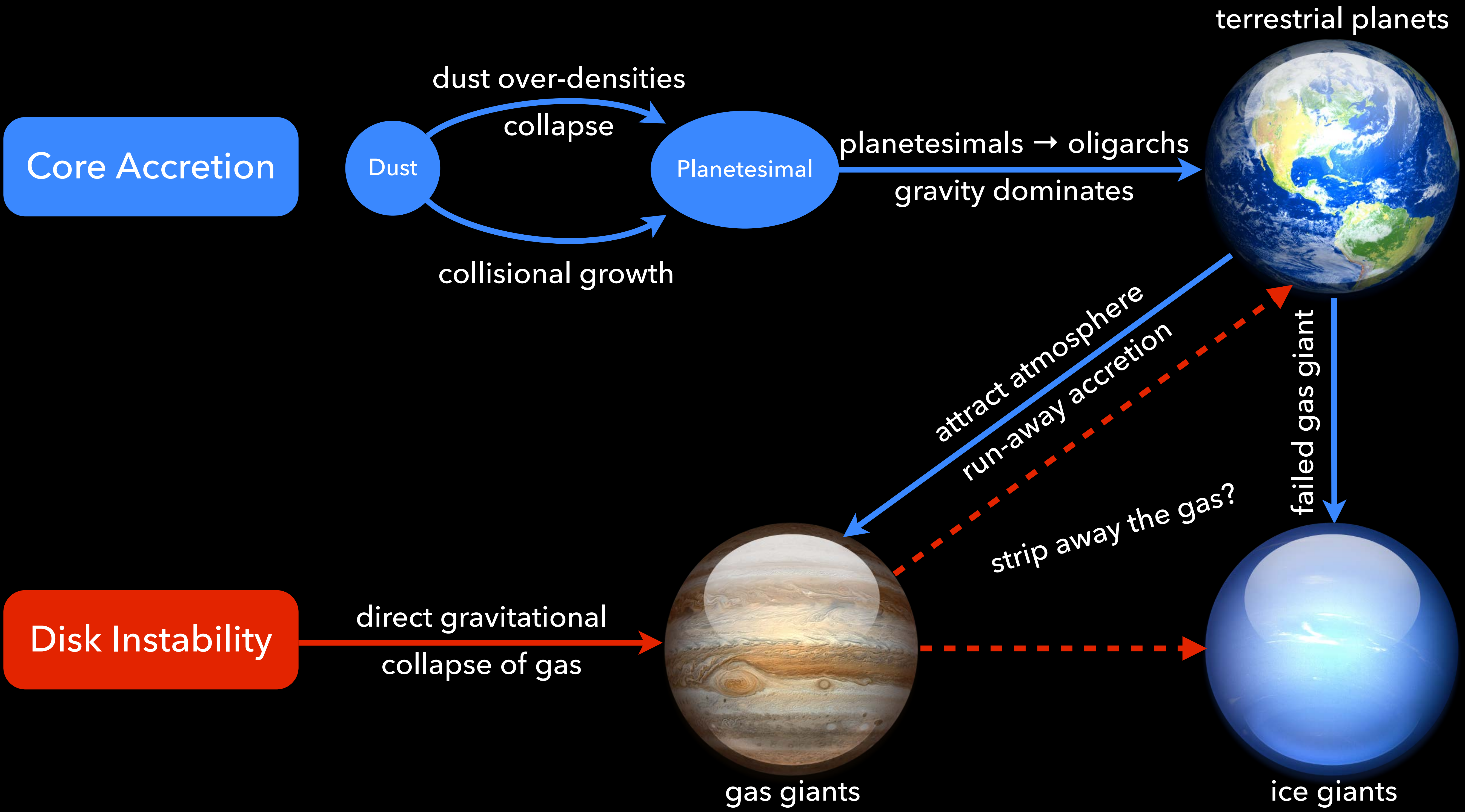


run-away/oligarchic growth
until isolation mass

slow accretion
feeding zone grows slowly

reaching critical mass
run-away accretion
(demand limited)

GAS ACCRETION: RUNAWAY GROWTH



Core Accretion

Dust

dust over-densities

collapse

collisional growth

Planetesimal

planetesimals → oligarchs

gravity dominates

terrestrial planets

attract atmosphere

run-away accretion

strip away the gas?

failed gas giant

Disk Instability

direct gravitational

collapse of gas

gas giants

ice giants

SUMMARY 1/3

- Disc temperature is important for determining the condensation sequence, which affects the chemistry of solids in the disc.
    - CI-chondrites show the least processing and closely match the abundances in the Sun. Give a good window on the chemical composition of the solar nebula.
- Growth of small grains initially occurs through collisions, but the growth efficiency drops near cm sizes due to bouncing, fragmentation, and radial drift.
    - Dust traps are essential to prevent the solid material from draining onto the star.
    - Likely need another mechanism to make the jump to planetesimal sizes.

SUMMARY 2/3

- Planetesimals again grow through collisions, but are now large enough for self-gravity to play an important role.
    - Gravitational focusing and internal structure.
- Once planets get too large, they reach an isolation mass, where the growth due to planetesimal accretion slows down dramatically.
    - Snow lines play an important role in accelerating core formation and allowing cores to reach the runaway gas accretion phase before the gas in the disc is dispersed.

SUMMARY 3/3

- Velocity dispersion and growth rates are co-dependent:
    - Velocity dispersion:
        - Viscous stirring
        - Dynamical friction
        - Gas damping
    - Different growth rates:
        - Orderly growth
        - Runaway solid accretion
        - Oligarchic growth
- Runaway gas accretion
    - Core mass is large enough ($\gtrsim 10\ \mathrm{M}_{\oplus}$)
    - Gas is still present and is able to cool efficiently