

Parton wave functions for 1D Haldane-Shastry model

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We demonstrate the MPO-MPS approach to construct the parton wave functions for the one-dimensional Haldane-Shastry model,

$$H_{\text{HS}} = \sum_{i < j} \frac{\hat{\vec{S}}_i \cdot \hat{\vec{S}}_j}{(\pi/N)^2 \sin^2[\pi(i-j)/N]}.$$

Following [Phys. Rev. Lett **124**, 246401 \(2020\)](#), the ground state for even number of spins N can be obtained by Gutzwiller projection of half-filled bands of free fermions

$$|\Phi_{\text{HS}}\rangle = P_G \prod_m \prod_{\alpha=\uparrow, \downarrow} d_{m\alpha}^\dagger |0\rangle,$$

where

$$d_{m\alpha}^\dagger = \frac{1}{N^{1/2}} \sum_{j=1}^N e^{-i(jm)} c_{j\alpha}^\dagger$$

is the creation operator in the momentum space and the occupied momenta are $m = \frac{2\pi}{N}s$ with

$$s = \begin{cases} 0, \pm 1, \dots, \pm \left(\frac{N}{4} - 1\right), \frac{N}{4} & \text{if } N \bmod 4 = 0 \\ 0, \pm 1, \dots, \pm \frac{N-2}{4} & \text{if } N \bmod 4 = 2 \end{cases}$$

and $P_G = \prod_{i=1}^N P_j$ is the Gutzwiller projection operator, which removes doubly occupied states on each site.

Exercise (a): Complete MPOMPS_ex.m

Complete the function `MPOMPS_ex.m` which is zipped together with this tutorial material.

This function constructs the parton wave functions via the MPO-MPS approach. Your task is to build

the $D = 2$ MPO for single particle creation operators $d_m^\dagger = [0 \ 1] \prod_{\ell=1}^{2N} \begin{bmatrix} 1 & 0 \\ A_{m\ell} c_\ell^\dagger & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Here we test on $N = 32$ with bond dimension $D = 100$:

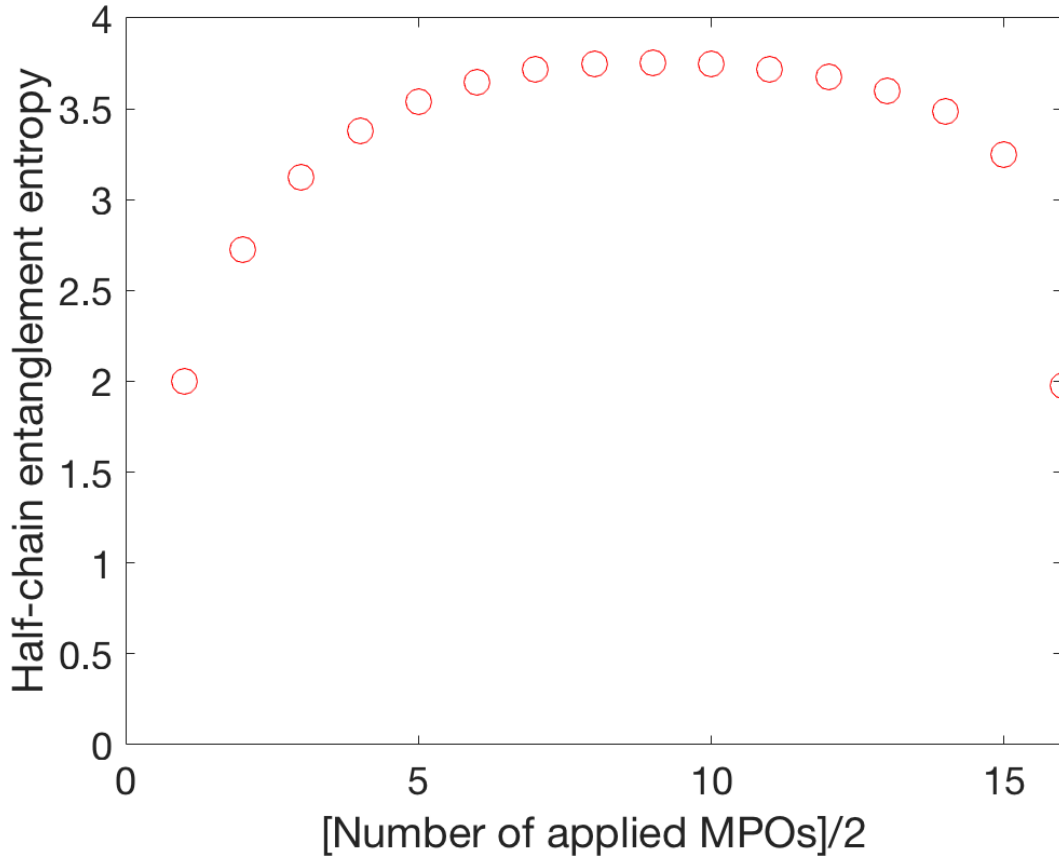
```
clear;
nsite = 32; % # of spins
dkeep = 100; % maximum bond dimension to keep
% define the filled orbitals for the 1D Haldane-Shastry model
% orb0: the original modes;
% orb1: the Wannier transformed modes;
% orb2: the Wannier transformed modes + the left-meet-right strategy
```

```
[orb0, orb1, orb2] = HS1D_ex(nsite);
% MPO-MPS method to build parton wave functions
[mps, es, dw] = MPOMPS_ex(orb0, dkeep);
```

# of applied MPOs	Entanglement Entropy	Truncation Error
2	1.99924925	0.000e+00
4	2.72802307	0.000e+00
6	3.12452437	0.000e+00
8	3.38122591	1.288e-04
10	3.54105778	6.471e-04
12	3.64893046	9.542e-04
14	3.71632110	1.103e-03
16	3.74632093	1.293e-03
18	3.75541334	1.239e-03
20	3.74903499	1.243e-03
22	3.71894382	1.172e-03
24	3.67383379	1.086e-03
26	3.59605228	8.492e-04
28	3.48829077	5.203e-04
30	3.24657328	5.940e-05
32	1.97742082	2.867e-08

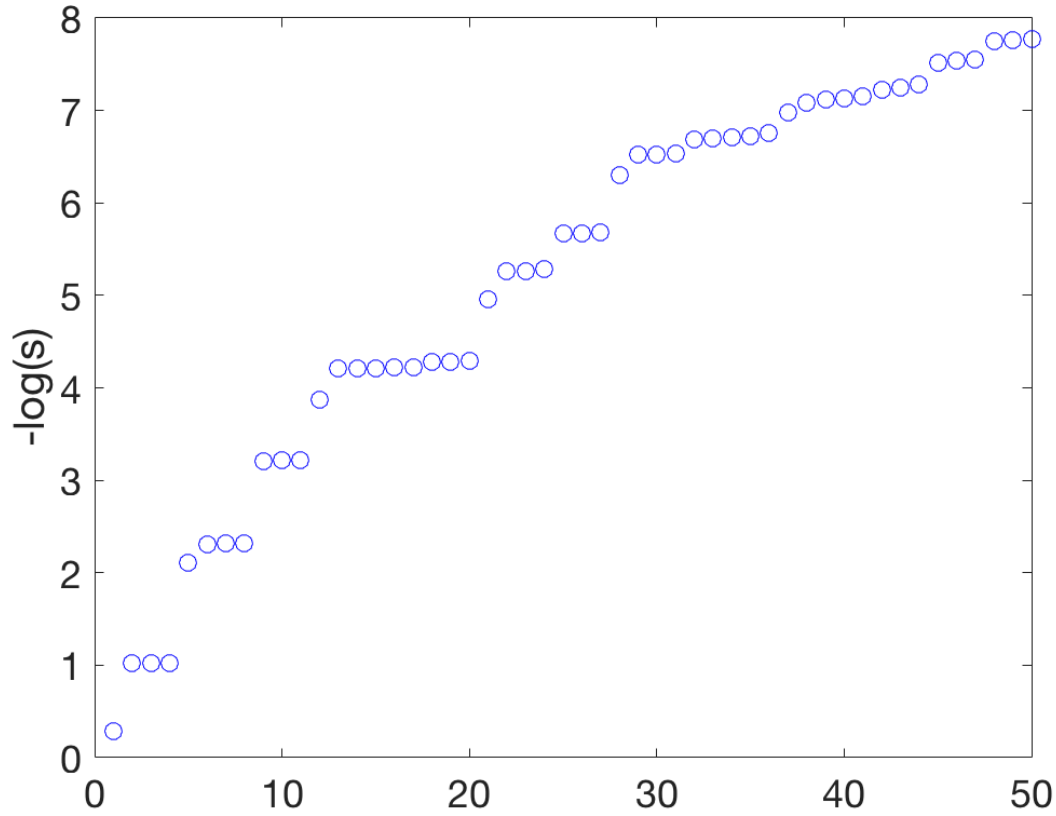
We track the evolution of the von Neumann entanglement entropy at the center of the spin chain upon applying the single particle creation operators $d_{m\alpha}^\dagger$. In the Exercise (c), we will inspect how the use of the maximally localized Wannier orbitals and "left-meet-right" strategy could possibly reduce the growth of the entanglement entropy and hence the truncation error.

```
% plot half-chain entanglement entropy
figure;
plot(es, 'ro', 'MarkerSize', 12);
axis([0, 16, 0, 4]);
ylabel('Half-chain entanglement entropy');
xlabel('[Number of applied MPOs]/2');
set(gca, 'FontSize', 18);
```



Here, we plot the entanglement spectrum at the center of the final state $|\Phi_{\text{HS}}\rangle$, the degeneracy structure of the lowest few entanglement eigenvalues agrees with the counting rules of $SU(2)_1$ Wess-Zumino-Witten model (the higher ones are not due to the truncation errors and the finite size effect.).

```
% plot entanglement spectrum of half-chain
figure;
[~,sv] = canonForm(mps,nsite/2);
plot(-log(sv(1:min(50,numel(sv)))),'bo','MarkerSize',8);
axis([0, 50, 0, 8]);
ylabel('-log(s)');
set(gca,'FontSize',18);
```



Exercise (b): Complete HS1D_ex.m

Complete the function `HS1D_ex.m` which is zipped together with this tutorial material.

This function generates the coefficients of the single particle operators for the parton wave functions.

Your task is to construct (i) the Wannier orbitals $\zeta_r^\dagger = \sum_{m=1}^N B_{mr} d_m^\dagger$ via the position operator, which helps to minimize entanglement growth (ii) To further minimize the truncation error, we reorder the Wannier orbitals such that the operators localized at the left or right edge are applied first and gradually move toward the center.

Exercise (c): Compare three different methods

(1) Compare the growth of the half-chain entanglement entropy and truncation errors during the MPO-MPS operation using (i) the original single particle orbitals (ii) the Wannier orbitals (iii) the Wannier orbitals with the left-meet-right strategy.

(2) Compute the nearest-neighbor and next-nearest-neighbor spin-spin correlations. Do they agree with the

analytical result $\langle \mathbf{S}_p \cdot \mathbf{S}_{p+q} \rangle = \frac{\sum_{a=1}^{N/2} \frac{3(-1)^q}{2a-1} \sin \left[\frac{\pi}{N} (2a-1)q \right]}{2N \sin \frac{\pi}{N} q} ?$