Solutions to problem set 5

Problem 1

Mg^{2+} induced RNA folding.

a) The radius of gyration R_g decreases with increasing salt concentration as the *Tetrahymena* ribozyme RNA undergoes a folding transition. At low salt, the electrostatic repulsion is too strong and the RNA is (partially) unfolded. At $\geq$ mM concentrations of Mg^{2+}, the electrostatic repulsion is sufficiently screened such that tertiary interactions can form and the molecules folds into a 3D structure. Folding transitions for complex RNA often involve multiple tertiary interactions and are, therefore, cooperative. The cooperative transition leads to a sharp transition over a narrow Mg^{2+} concentration range.

b) The classical interpretation of the Hill model is that upon folding the ribozyme binds 2.8 Mg^{2+} ions cooperatively.

c) General limitations: 1) "All-or-nothing" binding, which is only an approximation. 2) Allows non-integer n, which microscopically is difficult to interpret, since there is no "0.8 magnesium ion".
For ions binding to RNA and additional complication is that there will always be overall charge neutrality, not just mass action. Thus, if Mg^{2+} binds, other ions must be displaced from the ion atmosphere.

Problem Packing viral DNA.

Foo.

a) Estimate total electrostatic energy without ions:
Total charge $Q = 2$ e/bp $\cdot$ 20,000 bp = 40,000 e
Electrostatic energy of charged sphere:
$E_{elec} = \frac{1}{4\pi\epsilon\epsilon_0}\frac{3}{5}\frac{Q^2}{R}$
with $\epsilon = 80$, e $= 1.6 \cdot 10^{-19}$ C and $R = 20 \cdot 10^{-9}$ m
$E_{elec} = 1.4 \cdot 10^{-13}$ J $\approx 3 \cdot 10^7\ k_B T$

b) Total length of the genome: 20 kbp $\cdot$ 0.34 nm/bp = 6800 nm
Work done: $W = F \cdot \Delta x \approx 0.5$ 60 pN 6800 nm $= 2 \cdot 10^5$ pN$\cdot$nm $= 5 \cdot 10^4\ k_B T$

c) The electrostatic energy in part a) is much larger than the work done in part b). Therefore, the motor could never pack the DNA under these assumptions. Counterions apparently screen the electrostatic repulsion and reduce the energy required for packing by about 3 orders of magnitude.

Problem 3

Estimates of molecular forces.

a) C-O bond:
$E = 84$ kcal/mol $= 84$ kcal/mol $\cdot$ 4.184 kJ/kcal $\cdot$ 1000 J/kJ / $(6 \cdot 10^{23}$/mol)
$= 5.9 \cdot 10^{-19}$ J
$F = E \,/\, \Delta x = 5.9 \cdot 10^{-19}$ J / $(10^{-10}$ m$) = 5.9 \cdot 10^{-9}$ N $= 5.9$ nN

S-S bond:
$E = 3.6 \cdot 10^{-19}$ J
$F = 3.6$ nN
i.e. the rupture forces for covalent bond are in the nN range. For more information, see Michel Crandbois, Martin Beyer, Matthias Hauke Clausen-Schaumann, Hermann E. Gaub, *How Strong Is a Covalent Bond?*, *Science* (1999)

b) Non-covalent bonds in biological systems have to be stronger than $E = 4$ pN·nm $= 10^{-21}$ J and have to withstand forces larger than ≈ 4 pN·nm/1 nm $= 4$ pN, otherwise thermal fluctuations would constantly break them. At the same time, they are considerably weaker than covalent bonds with energies in the range of $E \approx 10^{-19}$ J and forces ≈ 1 nN. Therefore, typical rupture forces for non-covalent bonds are 10-100 pN and typical energies 10-100 pN·nm $\approx$ 2-20 times $k_B\,T$.