1. Explicit symmetry breaking and pseudo-Goldstone bosons

a) $\varepsilon = 0$:

$$\mathcal{L} = \mathcal{L}_0 = \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 + \frac{\mu^2}{2}\left((\phi_1)^2 + (\phi_2)^2\right) - \frac{\lambda}{4}\left((\phi_1)^2+(\phi_2)^2\right)^2$$

$$= \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i + \frac{\mu^2}{2}\phi_i\phi_i - \frac{\lambda}{4}(\phi_i\phi_i)^2; \quad i=1,2$$

•) The symmetry group of the Lagrangian is $SO(2)$ (If we consider discrete symmetries). The action of $SO(2)$ over the fields ϕ_i is given by

$$\phi_i \longrightarrow R_{ij}\phi_j,$$

where $R_{ij} \in SO(2)$. Rewritting ϕ_i as $\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}$, the the transformation is

$$\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} \longrightarrow \begin{pmatrix}\phi_1'\\ \phi_2'\end{pmatrix} = \begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}$$

for $\alpha \in \mathbb{R}$. Notice that $\boxed{\phi_i\phi_i = \phi_i'\phi_i'} \Rightarrow \mathcal{L} = \mathcal{L}'$

•) Ground states: we now proceed to minimize the potential.

$$V(\phi_i) = -\frac{\mu^2}{2}\phi_i\phi_i + \frac{\lambda}{4}(\phi_i\phi_i)^2.$$

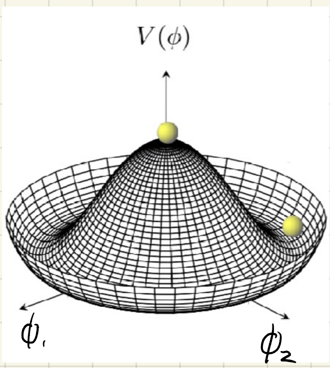
$$\frac{\partial V}{\partial\phi_i} = 0 \rightarrow -\mu^2\phi_i + \lambda(\phi_j\phi_j)\phi_i = 0$$

$$(-\mu^2 + \lambda\phi_j\phi_j)\phi_i = 0,$$

Note: $\phi_i = 0$ is a local maximum of V.

$$\lambda\phi_j\phi_j = \mu^2$$

$$\phi_j\phi_j = \frac{\mu^2}{\lambda} \equiv v^2$$



The ground states correspond to constant field configurations $\phi_i(x) = \phi_i$ such that $\phi_i\phi_i{}^2 = v^2$, where $v = \frac{\mu}{\sqrt{\lambda}}$

•) Noether currents:

Lets choose a ground state ϕ_i and perform an intenitesimal transformation, (i.e. $\alpha \ll 1$)

$$\phi_1' = \cos\alpha\, \phi_1 + \sin\alpha\, \phi_2 = \phi_1 + \alpha\phi_2 + O(\alpha^2)$$
$$\phi_2' = -\sin\alpha\, \phi_1 + \cos\alpha\, \phi_2 = \phi_2 - \alpha\phi_1 + O(\alpha^2)$$
$$\downarrow$$
$$\delta\phi_1 = \alpha\,\phi_2$$
$$\delta\phi_2 = -\alpha\,\phi_1$$

The Noether currents, j^μ, is given by:

$$\alpha j^\mu = \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi_i}\, \delta\phi_i$$

$$\rightarrow \alpha j_\mu = (\partial_\mu \phi_1)(\alpha\phi_2) + (\partial_\mu \phi_1)(-\alpha\phi_2)$$

$$\boxed{j_\mu = \phi_2 \partial_\mu \phi_1 - \phi_1 \partial_\mu \phi_2} = -\phi_1 \overleftrightarrow{\partial_\mu} \phi_2$$

•) Nambu-Goldstone boson:

SO(2) has one generator and the condition $\phi_i\phi_i = v^2$ breaks SO(2) invariance, thus there is one Nambu-Goldstone boson. Lets consider the ground state $\phi_i = \delta_{i2} v$, and expand ϕ_i around it: $\phi_1 = h_1$ and $\phi_2 = v + h_2$. The Lagrangian, after this field redefinition, becomes:

$$\mathcal{L} = \frac{1}{2}\partial_\mu h_i \partial_\mu h_i + \frac{\mu^2}{2}(h_1^2 + h_2^2 + 2h_2 v + v^2) - \frac{\lambda}{4}(h_1^2 + h_2^2 + 2h_2 v + v^2)^2$$
$$\mathcal{L} = \frac{1}{2}\partial_\mu h_i \partial_\mu h_i + (h_1^2 + h_2^2 + 2h_2 v + v^2)\Big(\underbrace{\frac{\mu^2}{2}}_{=\lambda v^2/2} - \frac{\lambda}{4}(h_1^2 + h_2^2 + 2h_2 v) - \frac{\lambda}{4}v^2\Big)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu h_i \partial_\mu h_i + (h_1^2 + h_2^2 + 2h_2 v + v^2)\frac{\lambda}{4}\left(v^2 - (h_1^2 + h_2^2 + 2h_2 v)\right)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu h_i \partial_\mu h_i - \frac{\lambda}{4}(h_1^2 + h_2^2 + 2h_2 v)^2 + \frac{\lambda}{4} v^4$$

Notice h_1 is massless. It corresponds to the Goldstone mode (around $\phi_i = \delta_{i2}\,\phi_0$, i.e. up to linear order)

Remark: The solution above is valid for any other ground state we choose, e.g. $\phi_i = R_{i2}\, v$, with $R_{ij} \in SO(2)$, and the expansion around the ground state given by

$$\phi_i = R_{ij}(h_j + \delta_{j2}\, v) = R_{ij} h_j + R_{i2}\, v.$$

This parametrization is useful to study the spectrum of perturbations around a given ground State.

However, a more general parametrization, similar to the one discussed in PS2, can be used in this problem.:

Lets define the following complex field:

$$\Phi = \frac{1}{\sqrt{2}}(\phi_1 + i\,\phi_2)$$

$$\Rightarrow \Phi^*\Phi = \frac{1}{2}(\phi_1^2 + \phi_2^2) = \frac{1}{2}\phi_i\phi_i$$

The Lagrangian $\mathcal{L}_0$ is rewritten in terms of Φ as:

$$\mathcal{L}_0 = \partial_\mu \Phi^* \partial_\mu \Phi + \mu^2 \Phi^*\Phi - \lambda(\Phi^*\Phi)^2,$$

and rewriting Φ as $\Phi(x) \equiv \left(v + \frac{1}{\sqrt{2}} h(x)\right) e^{i\theta(x)/v}$,

the lagrangian becomes:

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu h\, \partial_\mu h + \left(1 + \frac{1}{\sqrt{2}}\frac{h}{v}\right)^2 \partial_\mu \theta\, \partial_\mu \theta$$
$$+ \mu^2\left(v + \frac{h}{\sqrt{2}}\right)^2 - \lambda\left(v + \frac{h}{\sqrt{2}}\right)^4$$

$\hookrightarrow$ θ is massless and corresponds to the Nambu-Goldstone mode. This parametrization is independent of the ground state we choose, and is useful for discussing the Higgs phenomenon, as we saw in PS2.

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b) We consider now $\varepsilon \neq 0$.

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi_i \partial_\mu \phi_i + \frac{\mu^2}{2}\phi_i\phi_i - \frac{\lambda}{4}(\phi_i\phi_i)^2 + \varepsilon\, \mathcal{U}(\phi_1)$$

The $SO(2)$-symmetry is explicitly broken, as $\mathcal{U}$ depends non-trivially on the field ϕ_1.

Lets first determine the ground state(s). We consider constant field configuration ϕ_1 and ϕ_2 which minimize now the potential

$$V' \equiv V(\phi_i) - \varepsilon\, \mathcal{U}(\phi_1)$$

$$\Rightarrow \quad 0 = \frac{\partial}{\partial \phi_i}\left(V(\phi_i) - \varepsilon\, \mathcal{U}(\phi_1)\right)$$

$$0 = -\mu^2\phi_i + \lambda(\phi_j\phi_j)\phi_i - \varepsilon\, \delta_{1i}\, \mathcal{U}'$$

$$0 = \left(-\mu^2 + \lambda\, \phi_j\phi_j\right)\phi_i - \varepsilon\, \mathcal{U}'\, \delta_{1i}$$

We get two equations:
$$\begin{cases} 0 = (-\mu^2 + \lambda\,\phi_j\phi_j)\,\phi_1 - \varepsilon\,\mathcal{U}'(\phi_1) & (i) \\ 0 = (-\mu^2 + \lambda\,\phi_j\phi_j)\,\phi_2 & (ii) \end{cases}$$

Solutions to (i) and (ii) will minimize V' if the mass matrix $M^2_{2\times 2}$ is positive definite. The components M^2_{ij} are given by:

$$M^2_{ij} = \frac{\partial^2 (V - \varepsilon\,\mathcal{U})}{\partial \phi_i\, \partial \phi_j}$$

$$= \frac{\partial}{\partial \phi_i}\left(-\mu^2\phi_j + \lambda(\phi_k\phi_k)\phi_j - \varepsilon\,\delta_{1j}\,\mathcal{U}'\right)$$

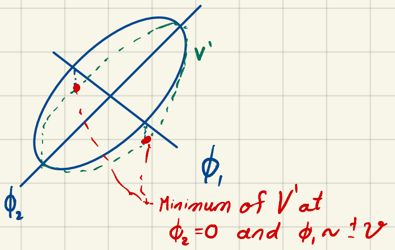
$$= -\mu^2\delta_{ij} + 2\lambda\,\phi_i\phi_j + \lambda(\phi_k\phi_k)\delta_{ij} - \varepsilon\,\delta_{1i}\delta_{1j}\,\mathcal{U}''$$

From (ii), and depending on $\mathcal{U}$, there are two cases: $\phi_2 = 0$ or $\phi_i \phi_i = v^2$

<u>Case 1:</u> $\phi_2 = 0 \Rightarrow$ Then (i) becomes an equation for ϕ_1

E.g.

$$\mathcal{U}(\phi_1) = \frac{\mu'^2 \phi_1^2}{2}$$



$$0 = (-\mu^2 + \lambda \phi_1^2)\phi_1 - \varepsilon\, \mathcal{U}'(\phi_1) \quad (*)$$

and m^2 becomes:

$$m^2 = \begin{pmatrix} -\mu^2 + 3\lambda\phi_1^2 - \varepsilon\mathcal{U}'' & 0 \\ 0 & -\mu^2 + \lambda\phi_1^2 \end{pmatrix} = \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}$$

Now for $|\varepsilon| \ll 1$, we notice that the ground state is at $\phi_1 \sim \pm v$. Lets expand $(*)$ around $\pm v$, with $\phi_1 \equiv \phi_\pm \equiv \pm v + \varepsilon h_\pm$

$$\Leftrightarrow \quad 0 = (-\mu^2 + \lambda(\pm v + \varepsilon h_\pm)^2)(\pm v + \varepsilon h_\pm) - \varepsilon\,\mathcal{U}'(\pm v + \varepsilon h_\pm)$$

$$0 = \lambda(\pm 2 v \varepsilon h_\pm)(\pm v) - \varepsilon\,\mathcal{U}'(\pm v) + \mathcal{O}(\varepsilon^2)$$

$$2\lambda v^2 h_\pm = \mathcal{U}'(\pm v) + \mathcal{O}(\varepsilon) \Rightarrow h_\pm = \frac{\mathcal{U}'(\pm v)}{2\mu^2}$$

$$\boxed{\phi_\pm = \pm v + \frac{\varepsilon}{2\mu^2}\,\mathcal{U}'(\pm v) + \mathcal{O}(\varepsilon^2)}$$

Also:

$$\begin{cases} \phi_\pm^2 = v^2 + \frac{\varepsilon v}{\mu^2}\,\mathcal{U}'(v_\pm) + \mathcal{O}(\varepsilon^2) \\ \varepsilon\,\mathcal{U}''(\phi_\pm) = \varepsilon\,\mathcal{U}''(v) + \mathcal{O}(\varepsilon^2) \end{cases}$$

Thus, up to $\mathcal{O}(\varepsilon)$, we get

$$m_1^2 = -\mu^2 + 3\lambda v^2 + \frac{3\lambda}{2}\varepsilon\frac{v}{\mu^2}\,\mathcal{U}'(v_\pm) - \varepsilon\,\mathcal{U}''(v_\pm)$$

$$= 2\mu^2 + \varepsilon\left(\frac{3}{2}\frac{\sqrt{\lambda}}{\mu}\,\mathcal{U}'(v_\pm) - \mathcal{U}''(v_\pm)\right)$$

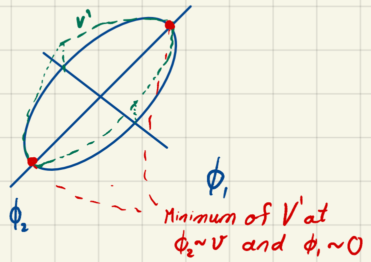
$$m_2^2 = -\mu^2 + \lambda v^2 + \lambda\frac{\varepsilon v}{2\mu^2}\,\mathcal{U}'(v)$$

$$m_2^2 = \varepsilon\frac{\sqrt{\lambda}}{2}\mu\,\mathcal{U}'(v) \rightarrow \phi_2 \text{ is the "pseudo-Goldstone boson"}$$

Case 2: $\phi_2 \neq 0 \Rightarrow \mu^2 = \lambda \phi_i \phi_i$

$$\phi_i \phi_i = v^2 = \frac{\mu^2}{\lambda}$$

e.g. $\mathcal{U}(\phi_1) = -\frac{\mu^2}{2}\phi_1^2$



$$(i) \Rightarrow \quad 0 = -\varepsilon\, \mathcal{U}'(\phi_1)$$

$$0 = -\varepsilon\, \mathcal{U}'(\phi_1)$$

$$\Rightarrow \quad \phi_2 \sim v$$

Note: In general one can perform a rotation $\phi_i' = R_{ij}\phi_j$, s.t. a minimim of V' is near $\phi_i = \delta_{i2} v$

The mass matrix in this case is

$$m_{ij}^2 = -\mu^2 \delta_{ij} + 2\lambda \phi_i \phi_j + \lambda \underbrace{(\phi_K \phi_K)}_{=\mu^2} \delta_{ij} - \varepsilon \delta_{1i} \delta_{1j} \mathcal{U}''$$

$$m_{ij}^2 = 2\lambda \phi_i \phi_j - \varepsilon \delta_{1i} \delta_{1j} \mathcal{U}''$$

Similar to

$$m^2 = \begin{pmatrix} 2\lambda \phi_1 \phi_1 - \varepsilon \mathcal{U}'' & 2\lambda \phi_1 \phi_2 \\ 2\lambda \phi_1 \phi_2 & 2\lambda \phi_2 \phi_2 \end{pmatrix} \sim \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}$$

$$\begin{cases} \det m = \cancel{4\lambda^2 \phi_1^2 \phi_2^2} - 2\lambda \phi_2 \phi_2 \varepsilon \mathcal{U}'' \cancel{- 4\lambda^2 \phi_1^2 \phi_2^2} = m_1^2 m_2^2 \\ \operatorname{tr} m = \underbrace{4\lambda \phi_K \phi_K}_{=4\lambda v^2} - \varepsilon \mathcal{U}'' = 4\mu^2 - \varepsilon \mathcal{U}'' = m_1^2 + m_2^2 \end{cases}$$

$$\Rightarrow \begin{cases} -2\lambda \phi_2^2 \varepsilon \mathcal{U}'' = m_1^2 m_2^2 \\ 4\mu^2 - \varepsilon \mathcal{U}'' = m_1^2 + m_2^2 \end{cases}$$

$$\left.\begin{matrix} \phi_2 = v + O(\varepsilon) \\ \phi_1 = O(\varepsilon) \end{matrix}\right\} \Rightarrow \begin{cases} -2\mu^2 \varepsilon \mathcal{U}'' = m_1^2 m_2^2 + O(\varepsilon^2) \\ 4\mu^2 - \varepsilon \mathcal{U}'' = m_1^2 + m_2^2 + O(\varepsilon^2) \end{cases}$$

Up to $O(\varepsilon)$, we get:

$$4\mu^2 - \varepsilon \mathcal{U}'' = m_1^2 - \frac{2\varepsilon \mu^2 \mathcal{U}''}{m_1^2}$$

$$0 = m_1^4 - m_1^2 (4\mu^2 - \varepsilon \mathcal{U}'') - 2\varepsilon \mu^2 \mathcal{U}''$$

$$m_\pm^2 = \frac{1}{2}\left[(4\mu^2 - \varepsilon\mathcal{U}'') \pm \sqrt{(4\mu^2 - \varepsilon\mathcal{U}'')^2 + 8\varepsilon\mu^2\mathcal{U}''}\right)$$

We get the masses:

$$m_1^2 = \frac{1}{2}\left(4\mu^2 - \varepsilon\mathcal{U}'' - \sqrt{16\mu^4 + \varepsilon^2\mathcal{U}''^2}\right)$$

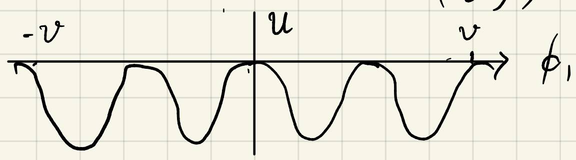
$$m_1^2 = -\frac{1}{2}\varepsilon\mathcal{U}'' + \mathcal{O}(\varepsilon^2),$$ Note: $\mathcal{U}'' < 0$ at the minimum of V'.

$$m_2^2 = \frac{1}{2}\left(4\mu^2 - \varepsilon\mathcal{U}'' + \sqrt{16\mu^4 + \varepsilon^2\mathcal{U}''^2}\right)$$

$$m_2^2 = 2\mu^2 - \frac{1}{2}\varepsilon\mathcal{U}'' + \mathcal{O}(\varepsilon^2)$$

In this case, ϕ_1 corresponds to the "pseudo Goldstone mode"

Conclusion: Which case, (i) or (ii), one should consider, depends on the explicit form of $\mathcal{U}$, and it might happen that it has several minima and both cases are relevant. E.g. $\mathcal{U}(\phi_1) = -\mu'^2 \sin^2\left(n\pi\left(\frac{\phi_1}{v}\right)\right)$.



In any case, there are two massive modes, of masses

$$m_s = 2\mu^2 + \mathcal{O}(\varepsilon),$$
$$m_p = \mathcal{O}(\varepsilon),$$

corresponding to a scalar and a pseudo Goldston boson, respectively.

2. Higgs phenomenon in $SU(2) \times U(1)$

a) Vaccum Manifold:

Lets Minimize $V(H) = \lambda\left(H^\dagger H - \frac{v^2}{2}\right)^2$

Note that $V(H) \geq 0$, thus if $V(H)=0$, then H is at the minimum of V.

$$V(H)=0 \Rightarrow H^\dagger H - \frac{v^2}{2} = 0$$
$$H^\dagger H = \frac{v^2}{2}$$

Then, the constant field configurations $H(x)=H$, such that $H^\dagger H = \frac{v^2}{2}$, minimize the potential. The set of all such configurations, up to gauge transformations, is the Vaccum manifold:

$$\mathcal{M} = \left\{ H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \middle| \, H_i \in \mathbb{C},\ H^\dagger H = \frac{v^2}{2} \right\} \Big/ G.$$

where $G = SU(2) \times U(1)$. (Note: H and H' are equivalent if there is a gauge transformation $g \in G$, such that $H' = gH$.)

Remark: Since we want to minimize the total energy, W_μ^a and B_μ are pure gauge configurations. For simplicity we set them $W_\mu^{a(v)} = B_\mu^{(v)} = 0$.

Lets choose a ground state, namely $H^{(v)} = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$ (Known as unitary gauge). An unbroken generator, Q, is a Hermitian matrix such that

$$QH^{(v)} = 0 \quad \left(\text{equivalently } e^{i\theta Q} H^{(v)} = H^{(v)}\right)$$

For our specific choice (unitary gauge), with $Q = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\Rightarrow c = d = 0$, and from Hermiticity $a = 1$, $c = 0$ (setting $\mathrm{Tr}[Q,Q] = 2$) $\Rightarrow Q = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = T^3 + Y$

where $T^3 = \frac{\sigma^3}{2} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $Y = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Thus, we see there is only one unbroken generator and correspondingly an unbroken subgroup $U(1)_Q$

b) Lets now write the potential around $H^{(v)}$:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$H^\dagger H = \frac{1}{2}(v+h)^2$$

$$\Rightarrow V(H) = \lambda\left(H^\dagger H - \frac{v^2}{2}\right)^2$$

$$= \lambda\left(vh + \frac{h^2}{2}\right)^2$$

$$= \lambda v^2 h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4$$

$$= \frac{m_h^2}{2} h^2 \Rightarrow \boxed{m_h = \sqrt{2\lambda}\, v}$$

$$\boxed{V(h) = \frac{m_h^2}{2} h^2 + \frac{m_h^2}{2v} h^3 + \frac{m_h^2}{8v^2} h^4}$$

c) $$D_\mu H = \partial_\mu H + \Big[-i\frac{g}{2} W_\mu^1 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - i\frac{g}{2} W_\mu^2 \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$
$$- \frac{ig}{2} W_\mu^3 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - i\frac{g'}{2} B_\mu \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\Big] H$$

$$\hookrightarrow D_\mu H = \begin{pmatrix} -\frac{ig}{2\sqrt{2}}(W_\mu^1 - iW_\mu^2)(v+h) \\ -\frac{i}{2\sqrt{2}}(g' B_\mu - g W_\mu^3)(v+h) + \frac{1}{\sqrt{2}}\partial_\mu h \end{pmatrix}$$

Lets introduce $W_\mu^\pm$, Z_μ and A_μ:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right)$$

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g W_\mu^3 - g' B_\mu\right)$$

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g B_\mu + g' A_\mu^3\right)$$

In terms of $W^{\pm}_\mu$, Z^μ and A_μ, the covariant derivative is now

$$\hookrightarrow D_\mu H = \begin{pmatrix} -\frac{igv}{2} W_\mu^+ \\ \frac{1}{\sqrt{2}} \partial_\mu h + \frac{i\sqrt{g^2+g'^2}}{2\sqrt{2}} v Z \end{pmatrix} + \begin{pmatrix} -i g W_\mu^+ h \\ i \frac{\sqrt{g^2+g'^2}}{2\sqrt{2}} Z_\mu h \end{pmatrix}$$

and the kinetic term becomes (to the quadratic part)

$$\left[(D_\mu H)^\dagger (D^\mu H)\right]^{(2)} = \frac{1}{2} \partial_\mu h \partial^\mu h + \frac{g^2 v^2}{2} W_\mu^+ W^{\mu -} + \frac{1}{2}\left(\frac{(g^2+g'^2)v^2}{4}\right) Z_\mu^2.$$

From the above, we conclude that W^+_μ, W^-_μ and Z acquire masses $m_{W^\pm}$ and m_Z, respectively.

d)

$$\boxed{\begin{aligned} m_h &= \sqrt{2\lambda}\; v \\ m_{W^\pm} &= \frac{g}{2}\; v \\ m_Z &= \frac{\sqrt{g^2+g'^2}}{2}\; v \\ m_A &= 0 \end{aligned}}$$

We conclude by summarizing the symmetry breaking pattern:

$SU(2) \times U(1) \longrightarrow U(1)_Q$

(3 generators) W^a_μ (1 generator) B_μ $\longrightarrow$ 1 generator A_μ → It remains massless

+3 would be Nambu-Goldstone Bosons.

↳ They end up being eaten by W^+_μ, W^-_μ and Z_μ