"QCD and Standard Model"
Problem Set 9

1. Flavor parameters and CKM matrix

The Yukawa sector of the Standard model reads

$$\mathcal{L}_Y \;=\; -\Lambda^{(e)}_{ij}\,\bar{E}^i_L H e^j_R - \Lambda^{(d)}_{ij}\bar{Q}^i_L H d^j_R - \Lambda^{(u)}_{ij}\bar{Q}^i_L \tilde{H} u^j_R + \text{h.c.}\ ,$$

where the Λ's are Yukawa matrices, and i, j family indices.

a) Rotate the quark fields in order to diagonalize the Yukawa interactions. Show that the transformations on the right-handed quarks are unphysical but the ones on the left-handed quarks cannot be rotated away : in the diagonal (weak) basis they give rise to a mixing matrix (the CKM matrix) in the charged-current sector of the theory.

b) Given an arbitrary number N of quark families, determine the number of independent real parameters (mixing angles) and imaginary parameters (complex phases) of the CKM matrix. Make sure that the complex phases are really independent, *i.e.*, they cannot be reabsorbed into quark field redefinitions. Apply the formula when $N = 2$ and $N = 3$.

c) Make a CP transformation of the Yukawa term and convince yourselves that invariance under it implies that the Yukawa matrices must satisfy $\Lambda = \Lambda^*$. The existence of complex phases therefore points at CP violation. Using the results of point (b), justify why the experimental evidence of CP violation was a strong indication of the existence of a third generation of quarks.

d) Show that if neutrinos are massless, as the Standard Model assumes, there is no mixing matrix in the lepton sector.

2. Higgs boson decay to fermions

Consider the following Yukawa coupling between the Higgs and fermions

$$\mathcal{L}_Y = \frac{g}{2}\frac{m_f}{M_W} h \bar{f} f\ .$$

Find the decay rate for the Higgs into a fermion-antifermion pair. Account for the situation $f =$ quark.