“QCD and Standard Model”
Problem Set 8

1. The low-energy description of weak interactions. II. The neutral sector of the Fermi theory

In this exercise we will complete our overview of the effective description of electroweak interactions, with a particular focus on the neutral sector. The absence of the so-called *Flavour-Changing Neutral Currents* (FCNCs) — currents that do not conserve flavor— is not at all obvious from the bottom-up construction of the EW theory and had to be addressed in a very careful manner. In the top-down derivation of this exercise you will see how this happens naturally.

a) As the $W^{\pm}_{\mu}$ bosons, also the Z_{μ} and the photon couple effectively to currents. Write down these interactions after the Higgs acquires a VEV in the neutral sector of the electroweak Lagrangian. Introduce the Weinberg angle to rotate W^{3}_{μ} and B_{μ} appropriately.

b) Integrate out Z_{μ} classically, as you did for $W^{\pm}_{\mu}$ in the previous problem set, in order to obtain the neutral sector of the $4-$Fermi theory.

c) Explain why it is not possible to integrate out the photon in the same way. This corresponds to the fact that $4-$Fermi theory disregards EM interactions, for they are described by QED.

d) Is the neutral current flavour-diagonal ?

2. W boson decay

In this exercise you will compute the decay rate for the process $W \to e\bar{\nu}$. The relevant interaction term is

$$\frac{g}{\sqrt{2}} W_\mu^- \bar{e}_L \gamma^\mu \nu_L = \frac{g}{\sqrt{2}} W_\mu^- \, \bar{e} \gamma^\mu \frac{1}{2}(1+\gamma_5)\nu \; .$$

In what follows, the electron and neutrino should be taken as unpolarized, i.e. you should sum over all polarizations. For simplicity, take the W at rest. You can assume that the electron and neutrino are massless (their masses are negligible compared to their energies since $M_W \simeq 80$ GeV).

W boson polarization vectors :

— *positive z direction* $\epsilon_T^\mu(+) = \frac{1}{\sqrt{2}}(0,1,i,0)$
— *negative z direction :* $\epsilon_T^\mu(-) = \frac{1}{\sqrt{2}}(0,1,-i,0)$
— *Longitudinally polarized :* $\epsilon_L^\mu(0) = (0,0,0,1)$

a) Compute the partial differential decay rate and the total decay rate for the $W \to e\bar{\nu}$ process, with W polarized in the positive z direction.

b) Repeat the same for the W boson polarized in the negative z direction.
Is the total decay rate the same as before ? Why ?

c) Now, compute the same for the longitudinally polarized W boson (zero spin component in the z direction). Is the total rate the same as before ? Why ?

d) Compute the total decay rate for an unpolarized W. Is it the same as before ? Why ?

e) Compute the total decay rate of W into all SM quarks and leptons (assume that all are massless for simplicity). Take three generations of fermions and recall that the top quark is very heavy - heavier than the W boson. Take $g^2/4\pi = 1/30$ and $M_W = 80\,GeV$.
Take into account the difference between quarks and leptons (quarks carry three colors). Compare with the PDG experimental result :
http ://pdg.lbl.gov/2017/tables/rpp2017-sum-gauge-higgs-bosons.pdf.
We expect a small error since the τ lepton and b quark masses are not completely negligible.