

“QCD AND STANDARD MODEL”
Problem Set 7

The low-energy description of weak interactions. I. The charged sector of the Fermi theory

In the previous Exercise sheet we discussed the Higgs phenomenon in a gauged $SU(2)\times U(1)$ theory. Actually, this is the part of the Standard Model (SM) describing the dynamics of the electroweak interactions. Yet, the way for SM has been paved by an effective description, called *Fermi theory*, whose phenomenology of weak decays gives very accurate insights. In this exercise, we will show how to derive the model and to estimate its relevant parameters.

- a) Add the fermionic sector to the $SU(2)\times U(1)$ theory from the previous Exercise sheet. You can ignore the fermion masses and their interactions with the Higgs field for now (don't add the Yukawa sector).
- b) Experiments show that the theory involves vector and axial currents. They are related to global $U(1)$ symmetries, which act on a Dirac bispinor as

$$\psi \rightarrow \psi' = e^{i\alpha}\psi , \tag{1}$$

$$\psi \rightarrow \psi' = e^{i\gamma_5\beta}\psi , \tag{2}$$

respectively. Construct a combination of the resulting Noether currents (take the Dirac field to be free and massless), in such a way that the low-energy theory reflects the coupling of weak interactions only to left-handed fermions at high energies.

- c) When the Higgs field acquires a VEV, the charged current sector of the electroweak Lagrangian can be written as

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} (W^{+\mu} J_{\mu}^{+} + W^{-\mu} J_{\mu}^{-}) . \tag{3}$$

What are the currents J_{μ}^{\pm} , respecting the prescription of the previous point?

- d) Consider the vector bosons W^{\pm} only as internal states and integrate them out, requiring that $E \ll m_W$, where E is the energy scale of the system. Write down the local interaction term of the 4-Fermi Lagrangian density.
- e) Perform naïve power counting on the charged current interactions. Are they renormalizable?
- f) Compute the decay rate for the muon

$$\mu \longrightarrow e^{-} + \nu_{\mu} + \bar{\nu}_e \tag{4}$$

in this effective theory, considering the electron ultra-relativistic. Given the measured decay time, $\tau_{\mu} = 2.197 \mu s$, estimate the value of the VEV of the electroweak sector, v . Is it in agreement with its experimental measure?