“QCD AND STANDARD MODEL”
Problem Set 6

1. Explicit symmetry breaking and pseudo-Goldstone bosons

Consider the following Lagrangian capturing the dynamics of two real scalar fields

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 \ ,$$

where

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 + \frac{\mu^2}{2}\Big((\phi_1)^2 + (\phi_2)^2\Big) - \frac{\lambda}{4}\Big((\phi_1)^2 + (\phi_2)^2\Big)^2 \ ,$$

and

$$\mathcal{L}_1 = \epsilon\, U(\phi_1) \ ,$$

where ϵ is a small parameter and <u>U depends non-trivially on the field ϕ_1 only</u>.

a) Take $\epsilon = 0$. What is the symmetry group of the Lagrangian ? Find the ground state(s), the Noether current and the Nambu-Goldstone boson(s).

b) Take now $\epsilon \neq 0$. Find the lightest mode and its mass to the leading order in ϵ. This mode is called “pseudo-Goldstone mode.”

2. Higgs phenomenon in SU(2)×U(1)

Consider the following Lagrangian invariant under a gauged SU(2)×U(1) symmetry

$$\mathcal{L} = -\frac{1}{4}W^a_{\mu\nu}W^{\mu\nu\,a} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu H)^\dagger D^\mu H - \lambda\left(H^\dagger H - \frac{v^2}{2}\right)^2 \ ,$$

where

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc}W^b_\mu W^c_\nu \ ,$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \ ,$$

and the covariant derivative of the complex doublet field $H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$, $H_{1,2} \in \mathbb{C}$, is given by

$$D_\mu H = \partial_\mu H - igW^a_\mu\tau^a H - i\frac{g'}{2}B_\mu H \ .$$

In the above τ^a, $a = 1, 2, 3$, are the SU(2) generators and g, g' are the gauge couplings associated with the SU(2) and U(1) groups, respectively.

a) Minimize the potential and identify the vacuum manifold. Write down the unbroken generators, if there are any. What is the unbroken subgroup ?

b) Write the potential around the minimum, identify the Higgs mass m_h and write the terms in the potential (quadratic, cubic and quartic) as functions of m_h and the vacuum expectation value (VEV) v.

Hint : Work in the unitary gauge, meaning that you use the gauge redundancy to absorb the would-be Nambu-Goldstone bosons in the gauge fields, and use the convention

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix} ,$$

with h a real scalar field.

c) Expand the kinetic term of H around the vacuum and determine how many gauge bosons acquire masses and how many remain massless. Does that agree with your expectations from point a) ? Explain.

d) Find the masses of the physical gauge bosons

$$W^{\pm}_{\mu} = \frac{W^1_{\mu} \mp i W^2_{\mu}}{\sqrt{2}} , \quad Z_{\mu} = \frac{g W^3_{\mu} - g' B_{\mu}}{\sqrt{g^2 + g'^2}} , \quad A_{\mu} = \frac{g B_{\mu} + g' W^3_{\mu}}{\sqrt{g^2 + g'^2}} .$$