"QCD and Standard Model"
Problem Set 5

Spontaneous Symmetry Breaking of SO(3)

In this exercise we continue the discussion on spontaneous symmetry breaking initiated in Problem Set 2.

Consider the following Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi^i \partial^\mu \phi^i - V(\phi)\,,$$

where ϕ^i are three real scalar fields in the fundamental of SO(3), and summation over $i = 1, 2, 3$ is assumed. The potential is given by

$$V(\phi) = -\frac{\mu^2}{2}\phi^i\phi^i + \frac{\lambda}{4}(\phi^i\phi^i)^2\,,$$

with $\mu^2, \lambda > 0$.

a) Show that $\mathcal{L}$ is invariant under a global SO(3) symmetry, i.e. rotations in the three dimensional field-space.

b) Minimize the potential and determine the ground state of the system. What is the vacuum manifold, i.e. the manifold of all values of the vacuum expectation value (vev) ϕ_0 of the field that minimize the potential ?

c) Does the vev break completely the SO(3) symmetry ? If not, what is the unbroken group ? How many Nambu-Goldstone bosons do you expect ? Compute the quadratic Lagrangian for the perturbations on top of the vacuum to verify your expectation.

d) Let us now gauge the theory by promoting the global SO(3) to a local symmetry. Write down the corresponding Lagrangian.
Hint : You should generalize the partial derivative to a covariant derivative one and add a kinetic term for the gauge fields.

e) Find the mass spectrum of the gauge fields by expanding the appropriate terms of the Lagrangian around the vacuum that you found before.

Q&A Session

Please note that on May 18 and 19 we will have a Q&A session in the tutorials. You are encouraged to ask questions about the previous four problem sets and we can also discuss certain concepts in more details. It would be helpful to send specific questions to all the tutors in the email addresses listed on the course homepage under the tab "Tutorials" until Monday, May 17 at the latest, and also indicate the tutorial session in which you would like your questions to be addressed (Tue/Wed). In this way, we can collect overlaps and discuss the topics that most of you are struggling with.