SPECIAL

W decay: $W^- \to e_L + \bar{\nu}_R$

(a) Precise conventions

$$\boxed{\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3}$$

$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \tag{1}$$

• helicity of massless left- (right) handed state

$$\psi_R = \begin{pmatrix} 0 \\ u_R \end{pmatrix}, \quad \psi_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix}, \quad \psi = \begin{pmatrix} u_L \\ u_R \end{pmatrix} \tag{2}$$

$$m = 0 \Rightarrow \not{p}\psi = 0$$

$$\Rightarrow \begin{pmatrix} 0 & p_0 - \vec{p}\cdot\vec{\sigma} \\ p_0 + \vec{p}\cdot\vec{\sigma} & 0 \end{pmatrix} \begin{pmatrix} u_L \\ u_R \end{pmatrix} = 0 \tag{3}$$

$$\Rightarrow (p_0 - \vec{p}\cdot\vec{\sigma})\, u_R = 0 \; ; \quad (p_0 + \vec{p}\cdot\vec{\sigma})\, u_L = 0 \tag{4}$$

$$\Downarrow$$

$$h\, u_L = -1/2\, u_L \qquad h\, u_R = 1/2\, u_R \tag{5}$$

$$h \equiv \vec{S}\cdot\hat{p} = \vec{S}\cdot\frac{\vec{p}}{|\vec{p}|} = \frac{\vec{\sigma}}{2}\cdot\frac{\vec{p}}{p_0} \quad (p^0 \equiv |\vec{p}|)$$

$$\Downarrow$$

left-handed fermions: $h = -1/2$ $(\vec{s} \uparrow\downarrow \vec{p})$

right – " – – " – : $h = 1/2$ $(\vec{s} \uparrow\uparrow \vec{p})$

• Polarization vector of W-boson:

$$\left.\begin{aligned} \varepsilon_{\mu\,T}^{(1)} &= \frac{1}{\sqrt{2}}\,(0;\,+1,\,+i,\,0)/\sqrt{2} \\ \varepsilon_{\mu\,T}^{(2)} &= \frac{1}{\sqrt{2}}\,(0;\,+1,\,-i,\,0)/\sqrt{2} \\ \varepsilon_{\mu\,L}^{(3)} &= (0;\,0,\,0,\,1) \end{aligned}\right\}\ \text{rest-frame} \qquad (6)$$

Notice: $$\sum_i \epsilon_\mu^{(i)}\,\varepsilon_\nu^{(i)*} = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2} \qquad (7)$$

$$(k^0 = m,\ k^i = 0)$$

• $$\underbrace{(T_i)_{jk} = -i\,\varepsilon_{ijk}}_{\text{generators of } SU(2) \text{ for the triplet (vector)}} \quad \text{give:} \quad \underbrace{[T_i, T_j] = i\,\varepsilon_{ijk}\,T_k}_{SO(3) = SU(2)} \qquad (8)$$

$$\Rightarrow \quad T_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad (9)$$

$$\Rightarrow \quad T_3 \, \vec{\varepsilon}_T^{\,(1)} = + \vec{\varepsilon}_T^{\,(1)}$$

$$T_3 \, \vec{\varepsilon}_T^{\,(2)} = - \vec{\varepsilon}_T^{\,(2)} \qquad (10)$$

$$T_3 \, \vec{\varepsilon}_L^{\,(3)} = 0$$

$\Rightarrow \quad \varepsilon_{\mu\,T}^{(1)}$: corresponds to spin +1 along the z-axis ($S_z = 1$)

$\varepsilon_{\mu\,T}^{(2)}$: $S_z = -1$

$\varepsilon_{\mu\,L}^{(3)}$: $S_z = 0$

• Boosting in z-direction:

$$\varepsilon_z{}' = \frac{\varepsilon_z + v\,\varepsilon_0}{\sqrt{1-v^2}}$$

$$\varepsilon_0{}' = \frac{\varepsilon_0 + v\,\varepsilon_z}{\sqrt{1-v^2}} \qquad (11)$$

$$p = \frac{mv}{\sqrt{1-v^2}} \; ; \quad E = \frac{m}{\sqrt{1-v^2}}$$

$$\Rightarrow \quad v = p/E \; ; \quad \frac{1}{\sqrt{1-v^2}} = E/m \qquad (12)$$

$$\Rightarrow \quad \varepsilon'^{(1)}_{\mu T} = \varepsilon^{(1)}_{\mu T} \quad ; \quad \varepsilon'^{(2)}_{\mu T} = \varepsilon^{(2)}_{\mu T}$$

– transverse polarization $(\perp \vec{p})$

$$\varepsilon'^{(3)}_{\mu L} = \left(\frac{|\vec{p}|}{m} \, ; \, 0, \, 0, \, E/m\right)$$ – longitudinal $(\parallel \vec{p})$

(13)

• Notice : $k_\mu \cdot \epsilon^{\mu(i)} = 0 \quad (\Leftrightarrow \partial_\mu A^\mu = 0)$

normalization : $\varepsilon^{(i)}_\mu \varepsilon^{*(i)\mu} = -1$

Important comment

I took $\epsilon^T = (0; 1, \pm i, 0)/\sqrt{2}$

$$\Leftrightarrow S_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Q. what dictates the form of S? why is $T_3 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}$ <u>not</u> good?

A. The choice of Pauli matrices.

<u>check</u>

$$A_i \equiv \bar{\psi} \gamma^i \psi = \psi^+ \gamma^0 \gamma^i \psi = \psi^+ \begin{pmatrix} -\sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} \psi$$

$$\Rightarrow A_i' = u^+ (1 - i\theta_k \sigma_k/2) \sigma_i (1 + i\theta_k \sigma_k/2) u$$

$$= u^+ (\sigma_i - i\theta_k/2 [\sigma_k, \sigma_i]) u =$$

$$= u^+ [\sigma_i + \theta_k \epsilon_{kij} \sigma_j] u = u^+ \sigma_i u + \epsilon_{ijk} \theta_j u^+ \sigma_k u$$

$$\Rightarrow A_i' = A_i + \epsilon_{ijk} \theta_j A_k = A_i + i(\theta_j T_j)_{ik} A_k$$

$$\Rightarrow \boxed{(T_i)_{jk} = -i\, \epsilon_{ijk}}$$

Q.E.D.

Trace formulas

$$\boxed{\bullet\quad Tr\,[\,\gamma_\mu \gamma_\nu \gamma_\alpha \gamma_\beta\,] = 4\left(g_{\mu\nu} g_{\alpha\beta} - g_{\mu\alpha} g_{\nu\beta} + g_{\nu\alpha} g_{\mu\beta}\right)} \qquad (14)$$

- $Tr\,[\,\gamma_\mu \gamma_\nu \gamma_\alpha \gamma_\beta \gamma_5\,] = c\,\epsilon_{\mu\nu\alpha\beta}$

use:

$$\gamma_5 = -i\,\gamma_0 \gamma^1 \gamma^2 \gamma^3 = i\,\gamma_0 \gamma_1 \gamma_2 \gamma_3$$

Check:

$$\gamma_0 \gamma^1 \gamma^2 \gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -\sigma_1 & 0 \\ 0 & \sigma_1 \end{pmatrix}\begin{pmatrix} -\sigma_2\sigma_3 & 0 \\ 0 & -\sigma_2\sigma_3 \end{pmatrix} = \begin{pmatrix} \sigma_1\sigma_2\sigma_3 & 0 \\ 0 & -\sigma_1\sigma_2\sigma_3 \end{pmatrix}$$

$$\Rightarrow \quad \gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -I \end{pmatrix} \checkmark \quad \text{ok}$$

$$\Rightarrow Tr\,[\,\gamma_0 \gamma_1 \gamma_2 \gamma_3 \gamma_5\,] = i\,Tr\,[\,\gamma_0 \gamma_1 \gamma_2 \gamma_3 \gamma_0 \gamma_1 \gamma_2 \gamma_3\,] = -4i$$

$\Rightarrow \quad c = -4i \quad$ or:

$$\boxed{Tr\,[\,\gamma_\mu \gamma_\nu \gamma_\alpha \gamma_\beta \gamma_5\,] = -4i\,\varepsilon_{\mu\nu\alpha\beta}} \qquad (15)$$

$\boxed{\text{W – decay}}$

$$-ig/\sqrt{2}\,\left[W_\mu^-\,\bar{e}_L\,\gamma^\mu\,\nu_L \;+\; h.c.\right]$$

$$\Downarrow$$

$$|m|^2 = \frac{g^2}{2}\,\epsilon_\mu\,\bar{u}(p)\,\gamma^\mu L\,v(q)\,\bar{v}(q)\,\gamma^\nu L\,u\,\epsilon_\nu^*$$

electron of momentum p — anti-neutrino (momentum q)

$$\Downarrow$$

$$|m|^2 = \frac{g^2}{2}\,\bar{u}(p)\,\not{\epsilon}\,L\,v(q)\,\bar{v}(q)\,\not{\epsilon}^*\,L\,u$$

next: $\sum_{\text{spin}} |m|^2 = \;?$

use: $\sum_s u\bar{u} = \not{p}$

$\sum_s v\bar{v} = \not{q}$

$$\Rightarrow \sum_s |m|^2 = g^2/2\;\mathrm{Tr}\left[\not{\epsilon}\,\not{q}\,\not{\epsilon}^*\,L\,\not{p}\right] = \frac{g^2}{2}\,\mathrm{Tr}\left[\not{\epsilon}\,\not{q}\,\not{\epsilon}^*\,\not{p}\,R\right] \qquad (16)$$

$$\Rightarrow \sum_{s} |m|^2 = \frac{g^2}{4} \mathrm{Tr} \left[\not{\varepsilon} \not{q} \not{\varepsilon}^* \not{p} (1 - \gamma_5) \right] \qquad (17)$$

- $$\mathrm{Tr}\, \not{\varepsilon} \not{q} \not{\varepsilon}^* \not{p} = 4 \left[\varepsilon \cdot q \; \varepsilon^* \cdot p - \varepsilon \cdot \varepsilon^* \; p \cdot q + \varepsilon \cdot p \; \varepsilon^* \cdot q \right]$$

$$\varepsilon = (0; +1, +i, 0)/\sqrt{2} \qquad (S_z^W = +1)$$

$$\varepsilon^* = (0; +1, -i, 0)/\sqrt{2}$$

$$p_\mu = (1; \sin\theta, 0, \cos\theta) \frac{M_W}{2} \qquad \text{limit: } (m_e = m_\nu = 0)$$

$$q_\mu = (1; -\sin\theta, 0, -\cos\theta) \frac{M_W}{2}$$

$$\Downarrow$$

$$\mathrm{Tr}\, \not{\varepsilon} \not{q} \not{\varepsilon}^* \not{p} = 4 \left(\frac{M_W}{2}\right)^2 \left[\frac{1}{2} \cdot 2 (-\sin^2\theta) + 2 \right]$$

$$\left(\text{use: } \begin{aligned} \varepsilon \cdot q &= \varepsilon^* \cdot q = -\frac{M_W}{2\sqrt{2}} \sin\theta \\ \varepsilon \cdot p &= \varepsilon^* \cdot p = \frac{M_W}{2\sqrt{2}} \sin\theta \\ p \cdot q &= \frac{M_W^2}{4} \cdot 2 \end{aligned} \right)$$

$$\boxed{\mathrm{Tr}\, \not{\varepsilon} \not{q} \not{\varepsilon}^* \not{p} = M_W^2 \left[1 + \cos^2\theta \right]} \qquad (18)$$

- $$\mathrm{Tr}\left[\not{\varepsilon}\,\not{q}\,\not{\varepsilon}^{*}\not{p}\,\gamma_5\right] = -4i\,\varepsilon_{\mu\nu\alpha\beta}\,\epsilon^{\mu} q^{\nu} \varepsilon^{*\alpha} p^{\beta}$$

$$= +\,2\,M_W^2 \cos\theta \qquad (19)$$

$$\Downarrow$$

$$\sum_s |m|^2 = \frac{g^2}{4} M_W^2 \left[\,\underbrace{1+\cos^2\theta}_{\text{parity conserving}} \;\underbrace{-\,2\cos\theta}_{\text{parity violation }(\gamma_5)}\,\right]$$

$$\Downarrow$$

$$\boxed{\sum_s |m|^2 = \frac{g^2}{4} M_W^2 (1-\cos\theta)^2} \qquad (20)$$

Differential cross section

$$\frac{d\Gamma}{d\Omega} = \frac{1}{(2\pi)^2} \int \frac{p^2\,dp}{2p_0} \int \frac{d^3q}{2q_0}\,\frac{1}{2k_0} \sum_s |M|^2\, \delta^{(4)}(k-p-q) \qquad (21)$$

$$k_0 = M_W, \quad \vec{k} = 0 \qquad \begin{matrix}(p \equiv |\vec{p}|\,; \\ q \equiv |\vec{q}|\,)\end{matrix}$$

$$\Downarrow$$

$$\frac{d\Gamma}{d\Omega} = \frac{1}{4\pi^2}\,\frac{g^2}{4}\,M_W^2\,(1-\cos\theta)^2\,\frac{1}{2M_W} \int \frac{\cancel{p^2}\,dp}{2\cancel{p_0}}\,\frac{1}{2\cancel{p_0}}\;\delta(M_W - 2p)$$

since : $\displaystyle \int \frac{d^3q}{2q_0}\;\delta^{(3)}(\vec{p}+\vec{q}) = \frac{1}{2|\vec{p}|} = \frac{1}{2p_0}$

$$\Downarrow$$

$$\frac{d\Gamma}{d\Omega} = \frac{g^2\,M_W}{4\pi^2\cdot 4\cdot 2\cdot 4}\,\frac{1}{2}\,(1-\cos\theta)^2$$

$$\boxed{\frac{d\Gamma}{d\Omega}(\theta) = \frac{g^2 M_W}{256\,\pi^2}\,(1-\cos\theta)^2} \qquad (22)$$

Total decay rate

$$\Gamma_W = \int \frac{d\Gamma}{d\Omega}\, d\Omega = 2\pi \int \frac{d\Gamma}{d\Omega} \sin\theta\, d\theta$$

$$\Downarrow$$

$$\Gamma_W = \frac{g^2 M_W}{256\pi^2} \cdot 2\pi \underbrace{\int_0^{\pi} \sin\theta\, d\theta\, (1-\cos\theta)^2}_{8/3}$$

$$\Rightarrow \boxed{\Gamma_W(+1) = \frac{g^2 M_W}{48\pi}} \quad \text{for } S_z^W = +1$$

similarly : $\boxed{\Gamma_W(-1) = \frac{g^2 M_W}{48\pi} = \Gamma_W(+1)}$

Must be equal = question of convention
what is up and what is down

- Exercise: $\Gamma(0) = ?$ $\quad (S_z^W = 0)$

$$\Rightarrow \quad \varepsilon_{\mu L} = (0;\, 0, 0, 1)$$

$$\Downarrow$$

$$\sum_s |m|^2 = \frac{g^2}{2} M_W^2 \sin^2\theta$$

must vanish for both theta=0 and theta=pi by the same helicity arguments as before for transverse W

$$\Downarrow$$

$$\frac{d\Gamma(0)}{d\Omega} = \frac{g^2 M_W}{16\pi^2} \cdot \frac{1}{8} \sin^2\theta \quad \leftarrow \quad \frac{\text{NO } P \text{ violation!}}{(\text{why???})}$$

answer: no preferred direction

$$\Gamma(0) = \frac{g^2 M_W \cdot 2\pi}{16\pi^2 \cdot 8} \underbrace{\int_0^\pi \sin\theta\, d\theta\, \sin^2\theta}_{4/3}$$

$$\Downarrow$$

$$\boxed{\Gamma(0) = \frac{g^2 M_W}{48\pi} = \Gamma(+1) = \Gamma(-1)}$$

must be same = rotational symmetry still present when W at rest

Excercise

Compute the decay rate for the ~~averaged~~ averaged spin of W

use:
$$\sum_i \epsilon_\mu^{(i)} \varepsilon_\nu^{*(i)} = \left[-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_W^2} \right]$$

and we need:

$$\sum \frac{1}{3} |M|^2 = \frac{g^2}{12} \mathrm{Tr} \Big[\overset{(a)}{- \gamma^\mu \not{l}\, \gamma_\mu \not{p}\, (1-\gamma_5)} + \overset{(b)}{\not{k}\, \not{l}\, \not{k} \not{p}\, (1-\gamma_5)} \Big]$$

$$= \frac{g^2}{12} \Big[\overset{(a)}{+ 8\, p\cdot l} + \overset{(b)}{\frac{4}{m_W^2} \left(2 k\cdot p\; k\cdot l - k^2 p\cdot l \right)} \Big]$$

$$k^2 = M_W^2 \; ; \qquad p\cdot l = M_W^2/2$$

$$k\cdot p = k\cdot l = M_W^2/2$$

$$= \frac{4 g^2}{12} \left[p q + \frac{2 k\cdot p\, k\cdot q}{m_W^2} \right]$$

$$\frac{M_W^2}{2} + \frac{M_W^2}{2} = M_W^2$$

$$= \frac{g^2}{3} M_W^2$$

$$\boxed{\sum \frac{1}{3} |M|^2 = \frac{g^2 M_W^2}{3}}$$

$$\Downarrow$$

$$\Gamma_W = \frac{1}{4\pi^2}\,\frac{g^2 M_W^2}{3}\int\frac{d^3p}{2p_0}\int\frac{d^3\ell}{2\ell_0}\,\frac{1}{2M_W}\,\delta^{(4)}(p+k-\ell)$$

$$= \frac{g^2 M_W}{48\pi} \quad \text{as expected}$$

$$\Gamma_W = \frac{1}{3}\Big[\underbrace{\Gamma(+1) + \Gamma(-1) + \Gamma(0)}_{\text{all equal } = \frac{g^2 M_W}{48\pi}}\Big]$$

Obviously, must be the same, since all polarisations give the same

$$\boxed{\Gamma_W = \frac{\alpha M_W}{12\,\sin^2\theta_W}}$$

Total W decay

$$\Gamma(W \to e\bar{\nu}) = \frac{\alpha M_W}{12 \sin^2\theta_W} \times 3 \quad / \text{ 3 generations of lepton}$$

$$\Gamma(W \to \bar{u} d) = \frac{\alpha M_W}{12 \sin^2\theta_W} \times 3 \text{ (color)} \quad (m_u = m_d = 0)$$

$$\times 2 \quad / \text{ 2 generations} \quad (m_t > M_W)$$

$$\Rightarrow \boxed{\Gamma_W = \frac{\alpha M_W}{12 \sin^2\theta_W}(3+6) = \frac{3}{4}\frac{\alpha M_W}{\sin^2\theta_W}}$$

Take : $\sin^2\theta_W \simeq 0.25$ (for simplicity)

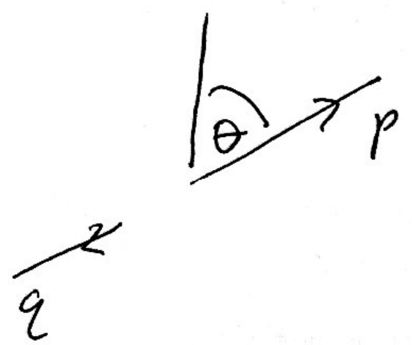
$\alpha(M_W) \simeq 1/120$

$$\left.\begin{array}{l} \Rightarrow \Gamma(W) \simeq \dfrac{M_W}{40} \simeq 2 \text{ GeV} \\ \\ \text{PDG} : 2.085 \pm 0.042 \end{array}\right\} \text{not bad for approximations we took}$$

W decay – explicit spinor formulas

$$g/\sqrt{2}\;\bar{u}_L \gamma^\mu d_L W_\mu^+ + h.c.$$

$$\Rightarrow \mathcal{M} = g/\sqrt{2}\;\bar{u}_L(p)\gamma^\mu v_L(q)\,\epsilon_\mu$$



we choose $\vec{p}$ in (x,z plane)

$$\vec{p} = p(\cos\theta, 0, \sin\theta)$$

$$= O(\theta)\begin{pmatrix} 0 \\ 0 \\ p \end{pmatrix}$$

where $O(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$

- take $\Uparrow$ $S_z^W = +1$ $\Rightarrow \epsilon_\mu = \frac{1}{\sqrt{2}}(0; 1, i, 0)$

The system is equivalent to fermions moving along z-axis and rotating ϵ_μ by $(-\theta)$

$$\Rightarrow \vec{\epsilon} \rightarrow \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}\begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} 1/\sqrt{2} = \begin{pmatrix} \cos\theta \\ i \\ \sin\theta \end{pmatrix}\frac{1}{\sqrt{2}}$$

i.e. $\epsilon_\mu = \frac{1}{\sqrt{2}}(0; \cos\theta, i, \sin\theta)$

Notes by Peskin, page 7

so we can take: $u(p) = \sqrt{2p}\, \xi_L = \sqrt{2p} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ - fermion ($h = -1/2$)

$v(q) = i\sigma_2 \xi_L^* = \sqrt{2q} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ - anti-fermion ($h = +1/2$)

$$\Rightarrow \mathcal{M} = \frac{g}{\sqrt{2}} u^+(p)\, \bar{\sigma}^\mu u(q)\, \epsilon_\mu =$$

$\boxed{p = q}$

$\parallel$

$M_W/2$

$$= g\sqrt{2pq}\, (0\ 1) \begin{bmatrix} \sin\theta & (1+\cos\theta) \\ (\cos\theta - 1) & -\sin\theta \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \frac{1}{\sqrt{2}}$$

$$= g\sqrt{2}\, p\, (\cos\theta - 1) \frac{1}{\sqrt{2}} = -g\, p\, (1-\cos\theta) = \frac{-g\, M_W}{2} (1-\cos\theta)$$

notice: $\mathcal{M} \to 0$, as $\theta \to 0$ as expected and computed by trace techniques

$$\Rightarrow \boxed{|M_T|^2_{(+)} = \frac{g^2 M_W^2}{4} (1-\cos\theta)^2}$$

Formula (20) in my notes on W decay

— no effort - only some thought

- $S_z^W = -1 \Rightarrow$ immediately $i \to -i \Rightarrow$

$$|M_T(-)|^2 = \frac{g^2 M_W^2}{4} (1+\cos\theta)^2$$

- <u>longitudinal</u>

$$\vec{\epsilon}_L = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\sin\theta \\ 0 \\ \cos\theta \end{pmatrix}$$

$$\Rightarrow \quad \epsilon_\mu(L) = (0; -\sin\theta, 0, \cos\theta)$$

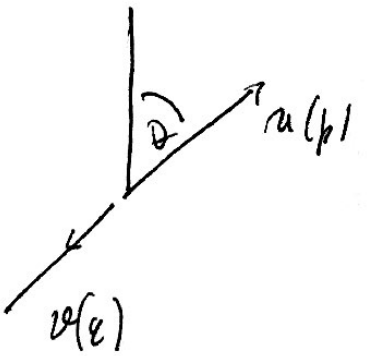
$$\mathcal{M}(L) = g/\sqrt{2} \cdot 2p \, (0\,1) \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= -g/\sqrt{2}\, M_W \sin\theta$$

$$\Rightarrow |\mathcal{M}(L)|^2 = \frac{g^2}{2} M_W^2 \sin^2\theta$$

page 12 of my notes on W decay

One can of course keep W-spin on z-axis and rotate the spinors (or solve for $h = \mp 1/2$ spinors in θ direction)



$$u = \sqrt{2E} \begin{pmatrix} -\sin\theta/2 \\ \cos\theta/2 \end{pmatrix}$$

$$v = \sqrt{2E} \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2 \end{pmatrix}$$

- $\epsilon_T(+) = 1/\sqrt{2}\,(0;\,1, i, 0)$

$$\Rightarrow \mathcal{M} = g/\sqrt{2} \cdot u^+ \bar{\sigma}^\mu v\, \epsilon_\mu = \frac{g}{2} \cdot 2E\, u^+ (\sigma^1 + i\sigma^2)\, v$$

$$= g\,\frac{M_W}{2} \cdot (-\sin\tfrac{\theta}{2}, \cos\theta/2) \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2 \end{pmatrix}$$

$$= +g\,\frac{M_W}{2}\,(-2\sin^2\theta/2) = -g\,\frac{M_W}{2}\,(1 - \cos\theta)$$

$$\Rightarrow |\mathcal{M}|^2 = g^2\,\frac{M_W^2}{4}\,(1-\cos\theta)^2 \;\checkmark$$