Neutrino Physics Course

Lecture XII

21/5/2021

LMU

Spring 2021

B and L in SM and Beyond

Summary of SM

$$m_f = y_f v \quad \Leftrightarrow \quad y_f h \bar{f} f$$

$$\Rightarrow \quad y_f = \frac{g}{2} \frac{m_f}{M_W}$$

$$\Downarrow$$

$$\Gamma(h \to f\bar{f}) \propto m_f^2$$

$$\boxed{SM \Rightarrow m_\nu = 0}$$

$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R$

$\Rightarrow$ (1) $\cancel{\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L} \Leftrightarrow \nexists\, \nu_R$

(2) $\nu_L^T C \nu_L \, m_M$?

$\exists$ Lorentz invariant

$\boxed{\text{not allowed}}$! $\cancel{SU(2)_L}$, $\cancel{U(1)_Y}$

$\boxed{\not{L} \text{ maximal}} \; \boxed{\Leftrightarrow \nexists\, \nu_R}$

$l_L \qquad \longleftrightarrow \qquad e_R$

Φ doublet

Imagine P symmetry

$$W_\mu^+ \left[\bar{\nu}_L \gamma^\mu e_L + \bar{\nu}_R \gamma^\mu e_R \right]$$

P

$$W_\mu^+ \left[\bar{u}_L \gamma^\mu d_L + \bar{u}_R \gamma^\mu e_R \right]$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L = q_L \qquad\qquad q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L = l_L \qquad\qquad l_R = \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$\nu_L \longleftrightarrow \nu_R$$

$$\Longrightarrow \boxed{m_\nu \neq 0}$$

$SU(2)_{L+R}$: $\bar{q}_L M_q q_R$ = inv.

$\bar{\ell}_L M_\ell \ell_R$ — " —

$$\Rightarrow \boxed{\begin{array}{c} m_u = m_d \\ m_\nu = m_e \end{array}}$$

What Higgs to use?

$\boxed{\text{A mess!}}$

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if: $\nu_L^T C \nu_L \Rightarrow 0\nu 2\beta$

Neutrinoless double beta

$\Delta L = 2$

Baryon and lepton numbers

$$W \quad [\bar{q} e + \bar{l} e] \Rightarrow \Delta B = 0 = \Delta L$$

$$Z, A \quad \text{—}\,\|\,\text{—} \qquad \qquad \text{—}\,\|\,\text{—}$$

$$h \quad \bar{f} f \qquad \qquad \qquad \text{—}\,\|\,\text{—}$$

$$\Big\Downarrow$$

$$\left(M_A = 0 \Leftrightarrow \frac{dQ}{dt} = 0 \right)$$

$$\boxed{\text{B and L are conserved}}$$

$\underline{\Delta B \neq 0}$ $\qquad$ p decay

$$\tau_p \gtrsim 10^{34}\ \text{yr}$$

$(\tau_\mu \simeq 10^{-6}\,\mathrm{sec})$

$\underline{\Delta L \neq 0}$ $\qquad 0\nu 2\beta$

$\tau_{0\nu 2\beta} \gtrsim 10^{25}\,\mathrm{yr}$

$\Downarrow$

1979 Weinberg
Wilczek, Zee

BSM $\quad \Delta B \neq 0 \neq \Delta L$

(Beyond Standard Model)

is unreachable

Fermi $\qquad$ WI

$\uparrow$ effective theory

$\downarrow$ $d = 6$ $\qquad$ lowest dimension

BSM $\boxed{\Lambda \gg M_W}$

$$\mathcal{H}_{eff} = \mathcal{H}_{eff}\,(\text{leading}) + O\left(M_W/\Lambda\right)$$

smallest dimension

$\boxed{\Delta B \neq 0}$ proton decay

$\Downarrow$

3 quarks

$$\Rightarrow \boxed{N_q \gtrsim 3}$$

(s+0) add

$$\mathcal{H}_{eff}^{\Delta B} = O^{\Delta B} = q\,q\,q\,(???)$$

$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad u_R,\ d_R$

1 fermion!!

$$\underbrace{u_R \quad d_R \quad d_R}$$

NOT Lorentz invariant

$SU(2)_L$ singlet

$U_Y(1)$ —"—

Quark or lepton?

$\Downarrow$

possible:

$u_R \; u_R \; d_R$

$u_R \; u_R \; u_R$

⋮

Lepton!!!

$\Uparrow$

Color

$\cancel{qq}$

$qqq \leftarrow$ color

$$\boxed{\epsilon_{\alpha\beta\gamma}\, q^\alpha q^\beta q^\gamma = \text{color invariant}}$$

$$\Downarrow$$

$$\underbrace{q\,q\,q\,l}$$

color invariant

Lorentz invariant

charge invariant

$SU(2)$ —"—

$d = 6$ 4 fermion

$$H_{eff}^{\Delta B \neq 0} = \frac{1}{\Lambda_B^2}\, q q q l$$

$\downarrow$

$$\Rightarrow p \to e^+ + \pi^0 (\gamma, h^0)$$

$$\tau_p \gtrsim 10^{34}\, yr$$

$$\mathcal{H}_{eff}^{\Delta B \neq 0} = \frac{1}{M_W^2} (\bar{e}\, \nu_e)(\bar{\nu}_\mu \mu)$$

$$\mu \to e + \bar{\nu}_e + \nu_\mu$$

$m_\mu = 100$ MeV

$m_e = 0.5$ MeV

$$\tau_\mu \simeq 10^{-6} \sec$$

<u>lazy person approach:</u>

$$\Gamma_\mu \simeq \frac{1}{M_W^4} m_\mu^5$$

$$\left(\tau = \frac{1}{\Gamma} \right)$$

$$\Gamma_p \simeq \frac{1}{\Lambda_B^4} m_p^5$$

$$\Downarrow \qquad \Downarrow$$

$$\tau_p / \tau_\mu \simeq \left(\frac{\Lambda_B}{M_W}\right)^4 \left(\frac{m_\mu}{m_p}\right)^5 = 10^{-5} \left(\frac{\Lambda_B}{M_W}\right)^4$$

$$\tau_p = 10^{-11} \sec \left(\frac{\Lambda_B}{M_W}\right)^4 \gtrsim 10^{41} \sec$$

$$\left(\Lambda_B / M_W\right)^4 \gtrsim 10^{52}$$

$$\Rightarrow \boxed{\Lambda_B \gtrsim 10^{15}\,\text{GeV}}$$

- $$\mathcal{H}_{eff}^{\Delta B \neq 0} \equiv \frac{1}{\Lambda_B^2} \mathcal{O}^{\Delta B \neq 0} = \frac{1}{\Lambda_B^2} \underbrace{qqql}$$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad u_R, \; d_R$$

SU(2) inv.

$$(i)\quad q_L^T C\, \underbrace{i\sigma_2}_{\text{SU(2) singlet}}\, q_L \propto u_L^T C\, d_L$$

SU(2) singlet

Lorentz singlet

$$(ii)\quad q_L^T C\, \underbrace{i\sigma_2}\, l_L \propto \left(u_L^T C\, e_L - d_L^T C\, \nu_L\right)$$

$$(iii)\quad u_R^T C\, u_R\,,\quad u_R^T C\, d_R\,,\quad d_R^T C\, d_R$$

$$u_R^T C\, e_R\,,\quad d_R^T C\, e_R$$

$$\Downarrow$$

$$O_1 = \left(q_L^{\alpha\,T} C\, i\sigma_2\, q_L^{\beta}\right)\left(q_L^{\gamma\,T} C\, i\sigma_2\, l_L\right)\epsilon_{\alpha\beta\gamma}$$

$$Y:\quad \tfrac{1}{3} + \tfrac{1}{3} + \tfrac{1}{3} + -1 = 0$$

$$= \left(u_L^T C\, d_L\right)\left(u_L^T C\, e_L - d_L^T C\, \nu_L\right)$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\underbrace{n\,u\,d\,e}_{\text{neutral}} \qquad\qquad \underbrace{(u\,d\,d\,\nu)}_{\text{neutral}}$$

$$O_2 = (u_L{}^T c\, d_L)(u_R{}^T c\, e_R)$$

$$O_3 = \underbrace{\underset{(q\ l)}{(u_L{}^T c\, e_L - d_L{}^T c\, \nu_L)}}_{Q_{em} = -1/3} \quad \begin{array}{l} \cancel{u_R{}^T c\, u_R} \\ \boxed{u_R{}^T c\, d_R} \\ \cancel{d_R{}^T c\, d_R} \end{array}$$

$$O_4 = (u_R{}^T c\, u_R)(d_R{}^T c\, e_R)$$

(symbolic) $\boxed{f\,f \equiv f^T \underset{(i\sigma_2)}{c} f}$

$$O_1 = (q_L q_L)(q_L l_L) \leftarrow qqql$$

$$O_2 = (q_L q_L)(u_R e_R) \leftarrow qqql$$

$$O_3 = (q_L l_L)(u_R d_R) \leftarrow qqql$$

$$O_4 = (u_R u_R)(d_R e_R) \leftarrow \cancel{(ud)(ue)?}$$

$$(qqql)$$

$\boxed{\text{not new}}$

$$\boxed{(\bar{l})_R \equiv (l^c)_R \equiv C \bar{l}_L^T \leftarrow \text{dublet}}$$

$$\text{all } O \sim \underbrace{qqql}_{\cancel{qqq\bar{l}}}$$

$$\Delta B = 1 = \Delta L \Rightarrow \boxed{\Delta(B-L) = 0}$$

$$\Rightarrow \boxed{\begin{array}{l} p \to e^+ + \text{boson} \\ \;\not\to e + \text{boson} \end{array}}$$

$$\boxed{\begin{array}{l} \underline{\text{exp.}} \quad \underbrace{p \to e + \pi^+ + \pi^+}_{} \\ \qquad\qquad\qquad \Downarrow \\ \qquad\qquad \Lambda_B \to M_W \end{array}}$$

$$\underbrace{\Delta L \neq 0} \quad (\Delta B \neq 0)$$

$$\mathcal{H}_{eff}^{\Delta L \neq 0} = \ell_L^T C i \sigma_2 \ell_L \,(\text{boson})$$

Lorentz SU(2)

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$\boxed{(d=5 \quad \Delta L \neq 0)}$$

$$\mathcal{L}_{eff}^{\Delta L \neq 0} = \underbrace{\left(l_L^T C \, i\sigma_2 \, l_L \right)} \underbrace{\left(\Phi^T i\sigma_2 \Phi \right)} \left(\frac{1}{\Lambda_L} \right)$$

$$y = -2$$

$$Q = T_3 + \frac{y}{2} \Rightarrow 0 = \frac{1}{2} - \frac{y}{2} = 0$$

but: $\Phi^T i\sigma_2 \Phi = 0 = \phi^u \phi^d - \phi^d \phi^u = 0$

$$l^T C \, i\sigma_2 \, l = 0$$

$$\left.\begin{array}{ll} q & qq \\ l & ql \end{array}\right\} \text{pairs}$$

$\ell \qquad \ell\,\ell\,,\ \Phi\,\Phi$

$\Phi \qquad \ell\,\Phi$

$\Downarrow$

$$\mathcal{L}_{eff}^{\Delta L \neq 0} = \frac{1}{\Lambda_L} \underbrace{\left(\ell_L^T i\sigma_2 \Phi \right)}_{\neq 0} C \underbrace{\left(\Phi^T i\sigma_2 \ell \right)}_{\neq 0}$$

$+$ (two other ways) $\boxed{???}$

$\Rightarrow$ $\boxed{\text{all equivalent!}}$

- $\ell = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad \Phi_{un} = \begin{pmatrix} 0 \\ v+h \end{pmatrix}$

$\Downarrow$

$$\Rightarrow \mathcal{L}_{eff} = \frac{1}{\Lambda_L}\left(\nu_L^T C \nu_L\right)(v+h)^2$$

$$= \frac{1}{\Lambda_L}\left(\nu_L^T C \nu_L\right)\Big(\underset{\substack{\uparrow \\ (a)}}{v^2} + \underset{\substack{\uparrow \\ (b)}}{2vh} + h^2\Big)$$

$$(a) \Rightarrow \quad m_\nu^M = \frac{v^2}{\Lambda_L} = v\left(\frac{v}{\Lambda_L}\right)$$

$$m_e = y_e v$$

$$\Rightarrow \quad \Lambda_L \gg v$$

$$v \simeq 100\ \text{GeV}$$

$$m_\nu \lesssim 1/10\ \text{eV} \quad \Rightarrow \quad \Lambda_L \gtrsim \frac{10^4}{10^{-10}}\ \text{GeV}$$

$$\boxed{\Lambda_L \gtrsim 10^{14}\,\mathrm{GeV}}$$

$$\boxed{\text{Probe of neutrino mass} \\ = \text{hopeless}}$$

$$(b) \quad \frac{v}{\Lambda_L}\, \nu_L^T C \nu_L\, h \equiv g_\nu\, h\, \nu_L^T C \nu_L$$

$$g_\nu = \frac{v}{\Lambda_L} \lesssim \frac{100}{10^{14}} \simeq 10^{-12}$$

$$\boxed{B(h \to \nu\nu) \simeq 10^{-24}} \quad (\text{tragic})$$

$\Downarrow$ summarize

- $\boxed{d=6 \quad \Delta B \neq 0 \Leftrightarrow p \text{ decay}}$
- $d=5 \quad \Delta L \neq 0 \Leftrightarrow$ neutrino mass parameter
- $0\nu 2\beta \quad \Delta L = 2$ process

$$\underbrace{\begin{matrix}(d+d \to u+u+e+e) \\ n+n \to p+p+e+e\end{matrix}}_{6 \text{ fermion}}$$

$$\mathcal{H}_{eff}^{0\nu 2\beta} = \frac{1}{\Lambda_{0\nu 2\beta}^5} \underbrace{\bar{u}\bar{u}\,\bar{e}\bar{e}\,dd}_{d=9}$$

$$\boxed{\begin{matrix}\tau_{0\nu 2\beta} \gtrsim 10^{25}\,\mathrm{yr} \\ \Lambda_{0\nu 2\beta} \gtrsim 3\,\mathrm{TeV}\end{matrix}}$$