Neutrino Physics Course

Lecture X

14/5/2021

LMU

Spring 2021

Higgs mechanism

$\Rightarrow$ Standard Model

- <u>$U(1)$ Higgs summary</u>

$$\mathcal{L} = \frac{1}{2}|D_\mu \phi|^2 - V(\phi)$$

$$- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \mathcal{L}_y$$

$$\phi \to e^{i\alpha(x) Q} \phi = e^{i\alpha(x)} \phi$$

$$V(\phi) = \frac{\lambda}{4} \left(|\phi|^2 - v^2 \right)^2$$

$$M_0 = \{ \phi_0 : |\phi_0|^2 = v^2 \} = S_1$$

$$\Downarrow$$

$$\boxed{\phi_{un} = (v+h)} \text{ Higgs boson}$$

$$\frac{1}{2}|D_\mu \phi|^2 = \frac{1}{2}(\partial_\mu h)^2 + \underline{\underline{\frac{1}{2} g^2 A_\mu A^\mu (v+h)^2}}$$

$$D_\mu = \partial_\mu - i g A_\mu$$

$$\Rightarrow \underbrace{\frac{1}{2} m_A^2 A_\mu A^\mu}_{} + \boxed{g\, m_A A_\mu A^\mu h} + O(h^2)$$

$$m_A = g v$$

$$\left\{ \begin{array}{lll} A^\mu (d=3) & \phi_{un} \; (d=1) & \\ A_\mu (d=2) & \phi \; (d=2) & \phi \in \mathbb{C} \end{array} \right\}$$

$\boxed{\text{Higgs couples to mass}}$

- $m_A = g v$, $g\, m_A A A h$

- $m_h^2 = 2\lambda v^2$, $h^4 \frac{\lambda}{4}$

$\boxed{\phi_0 = v \neq 0}$ $\quad Q\phi_0 = \phi_0$

- $T \gg v \quad \Rightarrow \langle \phi_0 \rangle_T = 0$

$\uparrow$ mass scale (~ 100 GeV)

come back

1967 Weinberg

"A model of leptons"

Higgs to the Glashow model $SU(2) \times U(1)$

$\Downarrow \boxed{SM}$

- $G_{SM} = \overset{g}{SU(2)_L} \times \overset{g'}{U_Y(1)}$ $\quad \tan\theta_W = g'/g$

- matter = quarks + leptons

$$q_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L^{\alpha} \qquad u_R^{\alpha},\ d_R^{\alpha} \qquad \alpha = r, y, b$$

$$\ell_L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R$$

- Higgs

$$\Downarrow$$

$$m_f = 0$$

$$(m_f \bar{f}_L f_R + h.c.)$$

$\shortparallel$

doublet

$$\mathcal{L}_y \equiv \bar{f}_L f_R \phi + h.c.$$

$\uparrow$ choose

invariant

$\Downarrow$ SM

$$\mathcal{L}_y = y_d \, \bar{q}_L \, \Phi \, d_R$$

$\uparrow$ doublet

$$\boxed{Y_\Phi = ?}$$

(1)

$SU(2)_L$: $q_L \longrightarrow U q_L$, $\quad q_R \rightarrow q_R$

$$U \equiv U_L \equiv e^{i\vec{\theta}\cdot\vec{\sigma}/2}$$

$a = 1,2,3$: $\quad T_L^a \equiv \frac{\sigma_a}{2} \qquad T_R^a = 0$

$$\boxed{Q_{em} = T_3 + \frac{Y}{2}} \qquad [Y, T_a] = 0$$

$\nRightarrow$

$Q_{em} = T_3 \leftarrow$ wrong $\Leftrightarrow q = \pm 1/2$

$Q_{em} = Y \Rightarrow Q_u = Q_d \Leftarrow$ wrong

$$y = 2(Q_{em} - T_3)$$

$$\Rightarrow \quad y_l = -1, \quad y_{e_R} = -2$$

$$y_q = 1/3, \quad y_{u_R} = \frac{4}{3}, \quad y_{d_R} = -2/3$$

$$(1) \Longrightarrow \boxed{y_\Phi = +1}$$

$$\Downarrow$$

$$\boxed{SU(2)}$$

$$\mathcal{L}_y = y_d \, \bar{q}_L \, \bar{\Phi} \, d_R + y_d \, \bar{q}_L i\sigma_2 \Phi^* u_R$$

$$y: \quad -\tfrac{1}{3} \; (+1) \; -2/3 \qquad -\tfrac{1}{3} \qquad 4/3$$

$$\cancel{E = ma^2} \Rightarrow$$

$$\cancel{E = mb^2}$$

$$E = mc^2$$

$SU(2)$: $\quad \underbrace{D_1,\ D_2}_{\text{dublets}} \quad D_i \to U D_i$

(1) $D_1^+ D_2 + h.c. = \text{inv.}$

$\to D_1^+ U^+ U D_2 = D_1^+ D_2$

(2) $D_1^T i\sigma_2 D_2 = \uparrow_1 \downarrow_2 - \downarrow_1 \uparrow_2$

$D_i = \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix}_i$

$D_1^T U^T i\sigma_2 U D_2 = D_1^T i\sigma_2 U^+ U D_2$

$= D_1^T i\sigma_2 D_2$

$\left(\underbrace{\psi_L^T C \psi_L}_{\text{Majorana}} = u_L^T i\sigma_2 u_L \right) \quad \psi_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix}$

$\Downarrow$ complete Yukawa

$$\boxed{\begin{aligned}\mathcal{L}_y = & \, y_d \overline{q_L} \, \Phi \, d_R + y_u \, \overline{q_L} \, i\sigma_2 \Phi^* u_R \\ & + y_e \, \overline{l_L} \, \Phi \, e_R + h.c.\end{aligned}}$$

$$\left(\text{if } \Phi_0 \neq 0 \overset{?}{\Rightarrow} \text{masses} \right)$$

$$\mathcal{L}_{SM} = i \bar{f} \gamma^\mu D_\mu f \quad (f = q, l \ldots)$$

$$+ \frac{1}{2} (D_\mu \Phi)^+ (D^\mu \Phi) - V(\Phi) - \mathcal{L}_y$$

$$- \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$a = 1, 2, 3$

$$\boxed{V(\Phi) = \frac{\lambda}{4}\left(\Phi^{+}\Phi - v^2\right)^2 \quad (d=4)}$$

Higgs sign

1 SU(2) invariant

SU(2) : $r = 1$ $\qquad$ $r = \#$ of Cartan

$[T_i^c, T_j^c] = 0$

$\Rightarrow T^c = T_3$

but ?

$\Phi^T i\sigma_2 \Phi$ Q1. <u>why not ?</u>

A1. breaks $U_Y(1)$

Q2. $(\Phi^T i\sigma_2 \Phi)(\Phi^{+} i\sigma_2 \Phi^*)$ <u>why not?</u>

Y : $-1 \quad -1 \quad +1 \quad +1 = 0$

A2. $\Phi_1^T i\sigma_2 \Phi_2 = \varphi_u \varphi_d - \varphi_d \varphi_u = 0$

$$\Phi = \begin{pmatrix} \varphi_u \\ \varphi_d \end{pmatrix}$$

Q3. $(\Phi^\dagger \Phi)^2 \leftarrow$ ok

$$\underbrace{(\Phi^\dagger \vec{\sigma} \Phi)}_{\text{vector}} \underbrace{(\Phi^\dagger \vec{\sigma} \Phi)}_{\text{vector}} = \frac{\text{invariant}}{\text{only rot}}$$

A3. $(\Phi^\dagger \vec{\sigma} \Phi)^2 \propto (\Phi^\dagger \Phi)^2$

$$\Downarrow$$

$$V = \frac{\lambda}{4} (\Phi^\dagger \Phi - v^2)^2$$

$$\mathcal{M}_0 = \{ \Phi_0 : V = V_{min} = 0 \}$$

$$= \{ \Phi_0^\dagger \Phi_0 = v^2 \} = S_3$$

$$\Uparrow$$

$$\Phi = \begin{pmatrix} \varphi_u \\ \varphi_d \end{pmatrix} = \begin{pmatrix} R_1 + i R_2 \\ R_3 + i R_4 \end{pmatrix}$$

$\varphi_u, \varphi_d \in \mathbb{C}$

$$\Rightarrow \Phi_0^\dagger \Phi_0 = \sum_{i=1}^{4} R_i^2 = \underbrace{R_1^2 + R_2^2 + R_3^2 + R_4^2}$$

$$\Downarrow$$

$$\frac{1}{2} (D_\mu \Phi)^\dagger (D^\mu \Phi) \rightarrow (D_\mu \Phi_0)^\dagger (D^\mu \Phi_0)$$

vacuum

$$\| \qquad (g' = 0) \qquad \Downarrow \qquad \boxed{SU(2)}$$

$$D_\mu \phi_0 = (-ig\, T_a A_\mu^a)\,\Phi_0$$

$$\Downarrow$$

$$\frac{1}{2}|D_\mu \Phi_0|^2 = \frac{1}{2} g^2\, \Phi_0^+ T_a A_\mu^a\, T_b A^\mu_b\, \Phi_0$$

$$= \frac{1}{2} g^2\, \Phi_0^+ \frac{\sigma_a}{2} \frac{\sigma_b}{2} A_\mu^a A^\mu_b\, \Phi_0$$

$$= \frac{1}{2} \cdot \frac{1}{4} g^2\, \Phi_0^+ \sigma_a \sigma_b\, \Phi_0\, A^\mu_a A_\mu^b$$

$$= \frac{1}{2}\, \frac{1}{4} g^2\, \Phi_0^+ (\delta_{ab} + \underbrace{i\,\varepsilon_{abc}}_{\text{anti sym}} \sigma_c)\, \Phi_0\, \underbrace{A^\mu_a A_\mu^b}_{\text{sym.}}$$

$$= \frac{1}{2} \cdot \frac{1}{4} g^2\, \Phi_0^+ \Phi_0\, A_\mu^a A^\mu_a$$

$$\Downarrow \qquad \boxed{\Phi_0^+ \Phi_0 = v^2}$$

$$= \frac{1}{2} m_A^2 A_\mu^a A^\mu_a \qquad m_A = \frac{g}{2} v$$

$$\boxed{\begin{array}{l} \text{all } 3\ A_\mu^a = \text{massive} \\ \Leftrightarrow T_a \Phi_0 \neq 0 \Rightarrow \text{massive } A_a \\ T_a = \frac{\sigma_a}{2} \Rightarrow \text{all } T_a \Phi_0 \neq 0 \end{array}}$$

$$\boxed{\begin{array}{l} SU(2) \Longrightarrow \mathbb{1} \\ \langle \Phi \rangle = \Phi_0 \end{array}}$$

- Breaking does <u>not</u> depend on the choice of Φ_0

- all of $SU(2)$ broken by Φ_0

- all $m_A^i = m_A = g v \frac{1}{2}$

(equal masses)

why?

$\boxed{\text{Hint: } V(\Phi) \text{ must have more symmetry}}$

$\Downarrow$ task

$\boxed{SU(2) \times U(1)}$

$\Rightarrow$ close $\boxed{\Phi_0^G = \begin{pmatrix} 0 \\ v \end{pmatrix}}$ $\Phi_0^\dagger \Phi_0 = v^2$

(for complicated) $\Phi_0^S = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ $v_1^2 + v_2^2 = v^2$

$$\Phi_0^S = U \, \Phi_0^G \leftrightarrow U = ??$$

$$\Downarrow$$

$$D_\mu \Phi \to D_\mu \Phi_0 =$$

$$= \left(-ig \, T_a A_\mu^a - ig' \frac{Y}{2} B_\mu \right) \Phi_0$$

$$\boxed{\frac{1}{2} |D_\mu \Phi_0|^2 = \left| \left(g \, T_a A_\mu^a + g' \frac{Y}{2} B_\mu \right) \Phi_0 \right|^2}$$

$$T_{1,2} = \frac{\sigma_{1,2}}{2} \leftarrow \text{separately}$$

$$\Downarrow$$

$$\boxed{m^2_{A_1} = m^2_{A_2} = \frac{1}{4} g^2 v^2}$$

$$W^{\pm}_{\mu} = \frac{(A_1 \mp i A_2)_\mu}{\sqrt{2}}$$

$$\Rightarrow \boxed{M_W = \frac{g}{2} v}$$

$\boxed{\text{Glashow} \quad 1961 \qquad SU(2) \times U(1)}$

$$A_3, \ B \longrightarrow (\exists A), (\exists Z) \qquad (Z \perp A)$$

$$\updownarrow$$

$$e \bar{f} \gamma^\mu Q_{em} f A_\mu$$

$$\Uparrow$$

$$A_\mu = \sin\theta_W\, A_{3\mu} + \cos\theta_W\, B_\mu$$

$$Z_\mu = \cos\theta_W\, A_{3\mu} - \sin\theta_W\, B_\mu$$

$$\tan\theta_W = g'/g$$

$$e = g\sin\theta_W$$

we <u>had to</u> assume $\exists A$

$\downarrow$

$Z!$ $\quad M_Z = ???$

$$\frac{1}{2}|D_\mu\Phi_0|^2 = \frac{1}{2}\left|\left(g\frac{\sigma_3}{2}A_\mu^3 + g'\frac{1}{2}B_\mu\right)\Phi_0\right|^2$$

$$\Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{1}{2} \left| \begin{pmatrix} 0 \\ -\frac{g}{2} A_\mu^3 + \frac{g'}{2} B_\mu \end{pmatrix} \right|^2$$

$$= \frac{1}{2} \frac{1}{4} \begin{pmatrix} 0 & (g A_\mu^3 - g' B_\mu) \end{pmatrix} \begin{pmatrix} 0 \\ (g A_\mu^3 - g' B_\mu) \end{pmatrix}$$

$$= \frac{1}{2} \frac{1}{4} \left(\frac{g A_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}} \right)^2 (g^2 + g'^2)$$

$$\boxed{\begin{aligned} Z_\mu &\equiv \frac{g A_3 - g' B}{\sqrt{g^2 + g'^2}} = \cos\theta_W A_3 - \sin\theta_W B \\ A_\mu &\equiv \frac{g' A_3 + g' B}{\sqrt{g^2 + g'^2}} = \sin\theta_W A_3 + \cos\theta_W B \end{aligned}}$$

$\therefore$

$$M_A = 0, \quad M_Z^2 = \frac{1}{4}(g^2+g'^2)v^2$$

$$M_W^2 = \frac{1}{4} g^2 v^2$$

$$\Downarrow$$

$$\boxed{M_W = M_Z \cos\theta_W}$$

$\sin^2\theta_W = 0.25$

$$\boxed{\begin{array}{l} M_W = 80\ \text{GeV} \\ M_Z = 90\ \text{GeV} \end{array}}$$

$$\boxed{\text{Why } M_A = 0\,?}$$

$$\Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix} \Rightarrow T_a \left(\equiv \frac{\sigma_a}{2}\right) \Phi_0 \neq 0$$

$Y\Phi = \Phi$

$Y_{\Phi} = 1$

$y\,\Phi_0 \neq 0$

$\Downarrow$ all are broken!?

all gauge bosons are massive!?

<u>neutral generators</u> $T_3, \frac{Y}{2}$

- $T_3\,\Phi_0 = -\frac{1}{2}\,\Phi_0 \neq 0$
- $\frac{Y}{2}\,\Phi_0 = \frac{1}{2}\,\Phi_0 \neq 0$

$$\boxed{Q_{em}\,\Phi_0 = \left(T_3 + \frac{Y}{2}\right)\Phi_0 = 0}$$

$$\left[\,SU(2)_L \times U_Y(1) \xrightarrow[\langle \Phi_0 \rangle]{} U(1)_{em}\,\right]$$

$\Phi_0^G = \begin{pmatrix} 0 \\ v \end{pmatrix}$ $\qquad$ $\Phi_0^S = \begin{pmatrix} v \\ 0 \end{pmatrix}$

$\Downarrow$

$Q_{em}\,\Phi_0^G = 0$

$\parallel$

Q_{em}^G

$Q_{em}^G\,\Phi_0^S \neq 0$

$T_3\,\Phi_0^S = +\frac{1}{2}\,\Phi_0^S$

$\frac{Y}{2}\,\Phi_0^S = +\frac{1}{2}\,\Phi_0^S$

$$\boxed{\begin{gathered} Q_{em}^S = T_3 - \frac{Y}{2} \Rightarrow \\ Q_{em}^S\,\Phi_0^S = 0 \end{gathered}}$$

$\Phi_0^u = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \Rightarrow \left(a T_3 + b \frac{Y}{2}\right)\Phi_0^u \neq 0$

$v_1^2 + v_2^2 = v^2$

T_a^{o} , $T_a^{u} = U\, T_a^{o}\, U^{+}$

Given

claim: $Q_{em}^{k}\, \Phi_0^{u} = 0$!

Must be

Answer

$$Q_{em}^{u} = c_i T_i + \frac{y}{2} b \qquad c_i = ?$$

<u>here</u>

$$\mathcal{L}_Y = y_d \bar{Q}_L \; \Phi \; d_R$$

$y_\Phi = +1$

$$Q_{em} = T_3 + Y/2 \Leftarrow\!\!\!\!\!/ \;\Longleftrightarrow\; \boxed{\Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix}}$$

$$Q_u = +2/3, \quad Q_d = -1/3$$

$$\Leftrightarrow \quad q_L = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$\text{if} \quad \Phi_0 = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \Longleftrightarrow q = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$\Downarrow$$

$$\mathcal{L}_y = y_d \left(\bar{\psi}_{1L} \, d_R \, v_1 + \bar{\psi}_{2L} \, v_2 \, d_R \right) + h.c.$$

$$= y_d \, \frac{\left(\bar{\psi}_{1L} \, v_1 + \bar{\psi}_{2L} \, v_2 \right)}{\sqrt{v_1^2 + v_2^2}} \, d_R \, \sqrt{v_1^2 + v_2^2} + h.c.$$

$$\Rightarrow \quad d_L = \left(\psi_{1L} \, v_1 + \psi_{2L} \, v_2 \right) \frac{1}{\sqrt{v_1^2 + v_2^2}}$$

$$\Rightarrow \quad u_1 = \frac{\psi_{11} v_2 - \psi_{21} v_1}{\sqrt{v_1^2 + v_2^2}}$$

rotation of states = ensures conservation of charge