Neutrino Course 2021

Lecture VII

4/5/2021

LMU

Spring 2021

SM: W, Z boson physics

massive U(1) = Proca

$$\mathcal{L}_P = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m_A^2}{2}A_\mu A^\mu$$

$$= \frac{1}{2}A_\mu\left[(\Box + m_A^2)g^{\mu\nu} - \partial^\mu\partial^\nu\right]A_\nu$$

$$\Downarrow$$

$$\partial_\mu \,/\, (\Box + m_A^2)A^\mu = \partial^\mu\partial^\nu A_\nu$$

$$\Downarrow$$

$$(\cancel{\Box} + m_A^2)\partial_\mu A^\mu = \cancel{\Box\,\partial_\nu A_\nu}$$

$$\Downarrow$$

$$\boxed{m_A^2\,\partial_\mu A^\mu = 0}$$

• $m_A = 0 \Rightarrow$ nothing $\Rightarrow$ gauge invariance of Maxwell

you choose a gauge — popular

$$\partial^\mu A_\mu = 0$$

• $m_A \neq 0 \Rightarrow \boxed{\partial_\mu A^\mu = 0}$ must

A_μ (4 d.o.f.)

Proca = 3 d.o.f. = spin 1

↓ momentum space

$$A_\mu = e^{ipx}\, \epsilon_\mu(p)$$

$$\Rightarrow \quad \mathcal{L}_p = \epsilon_\mu \left[(-p^2 + m_A^2) g^{\mu\nu} + p^\mu p^\nu \right] \epsilon_\nu$$

propagator = inverse $\Downarrow$

$$\Delta_{\mu\nu} \left[(-p^2 + m_A^2) g^{\nu\alpha} + p^\nu p^\alpha \right] = g_\mu^{\ \alpha}$$

$$\Downarrow$$

$$\Delta_{\mu\nu} = A g_{\mu\nu} + B p_\mu p_\nu$$

$$\Downarrow$$

$$\boxed{\Delta_{\mu\nu}^{(A)} \propto \frac{g_{\mu\nu} - \frac{p_\mu p_\nu}{m_A^2}}{p^2 - m_A^2}}$$

Proca propagator

$\hookrightarrow$ $\boxed{W^+,\ W^-,\ Z \Rightarrow \text{Proca}}$

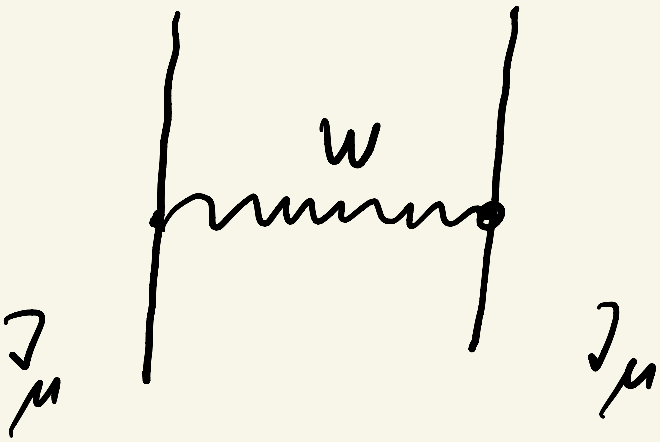
<u>Fermi:</u> $\frac{4\, G_F}{\sqrt{2}}\, J_\mu^{W}\, \overline{J}^\mu_W$

$$J_\mu^{W} = \left(\bar{\nu}_L \gamma_\mu e_L + \bar{u}_L \gamma_\mu d_L \right)$$

$\downarrow$ QED analogy

$$\frac{g}{\sqrt{2}} W_\mu^+ J^\mu_W + h.c.$$

$\Downarrow$



$$\frac{4G_F}{\sqrt{2}} = \left(\frac{g}{\sqrt{2}}\right) J_\mu^W \bar{J}_\nu^W \, \Delta^{\mu\nu}(W)$$

$$= \frac{g^2}{2} \bar{J}_\mu^W J_\nu^W \left[\frac{g^{\mu\nu} - p^\mu p^\nu / m_A^2}{p^2 - m_A^2} \right]$$

$$p \simeq MeV \to GeV \ll M_W$$

$$\Rightarrow \frac{4G_F}{\sqrt{2}} = \frac{g^2}{2M_W^2} \qquad (p=0)$$

$$\Downarrow$$

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}}$$

$M_W = ?$

$g = ?$

↑ interaction

not very useful

1961 Glashow

$$\overset{g}{SU(2)} \times \overset{g'}{U(1)}$$

(A_μ^a) T_a $\quad (B_\mu)$ $Y/2$ $\quad [T_a, Y] = 0$

$$Q_{em} = \overset{(a)}{{}^{\iota}T_3} + \overset{(b)}{{}^{\nu}\frac{Y}{2}}$$

· $b = 0$? $\quad$ ~~$Q_{em} = T_3 = \pm 1/2$~~

· $a = 0$! $\quad Q_{em} = Y/2 \Rightarrow Q_u = Q_d$

$(Y_u = Y_d)$

$[T_a, Y] = 0 \Leftrightarrow$ particles in $SU(2)$

states = same Y

$\Rightarrow$ $\boxed{a \neq 0 \neq b}$

$\Downarrow$

$$\boxed{\begin{aligned} Q_{em} &= T_3 + \frac{Y}{2} \\ Y &= 2\left[Q_{em} - T_3\right] \end{aligned}}$$

$\Downarrow$

$$\boxed{\begin{aligned} A &= \sin\theta\, A_3 + \cos\theta\, B \\ Z &= \cos\theta\, A_3 - \sin\theta\, B \end{aligned}} \qquad \theta \neq 0$$

$$\Downarrow$$

$$A_3 = \sin\theta\, A + \cos\theta\, Z$$

$$B = \cos\theta\, A - \sin\theta\, Z$$

$$i\bar{f}\gamma^\mu D_\mu f = i\bar{f}\gamma^\mu\left(\partial_\mu - ig\, T_3 A_\mu^3 - \right.$$

$$\left. - ig' \frac{Y}{2} B_\mu\right) f$$

$$\rightarrow \bar{f}\left[g T_3 A_\mu^3 + g'\left(Q_{em} - T_3\right) B_\mu\right]\gamma^\mu f$$

$$= \bar{f}\left(T_3\left[g A_\mu^3 - g' B_\mu\right] + g' Q_{em} B_\mu\right)\gamma^\mu f$$

$\left[g A_\mu^3 - g' B_\mu\right]$: not a photon!

$\Downarrow$

Z boson

$$Z = \frac{g A_3 - g' B}{\sqrt{g^2 + g'^2}} \Rightarrow \cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$$

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

$$\Rightarrow \boxed{\tan\theta_W = g'/g}$$

$\Rightarrow$ Neutral current interaction

$$= \bar{f}\gamma^\mu\left[\sqrt{g^2+g'^2}\, T_3 Z_\mu + g\tan\theta_W\, Q_{em}\right.$$

$$\left.(\cos\theta_W A_\mu - \sin\theta_W Z_\mu)\right] f$$

$$= \bar{f}\gamma^\mu\Big[A_\mu Q_{em}\, g\sin\theta_W \; + $$

$$+ \left(\frac{g}{\cos\theta_W} T_3 - \frac{g\sin^2\theta_W}{\cos\theta_W} Q_{em}\right) Z_\mu\Big] f$$

$$\Downarrow$$

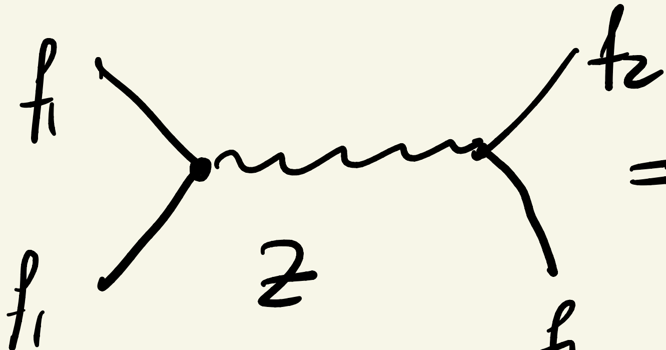
$$= e A_\mu \bar{f} \gamma^\mu Q_{em} f \quad +$$

$$+ \frac{g}{\cos\theta_W} Z_\mu \bar{f} \gamma^\mu \left(T_3 - Q_{em} \sin^2\theta_W \right) f$$

$$\boxed{e = g \sin\theta_W}$$

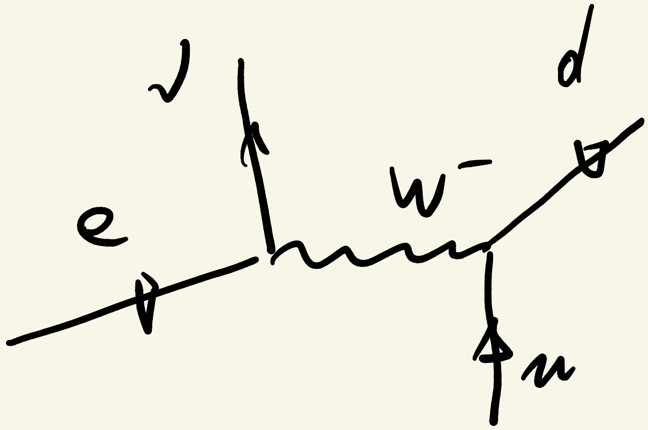
$$= \frac{g}{\cos\theta_W} Z_\mu J^\mu_Z + e A_\mu j^\mu_{em}$$

(T_3, Q_{em}) $\qquad$ (Q_{em})

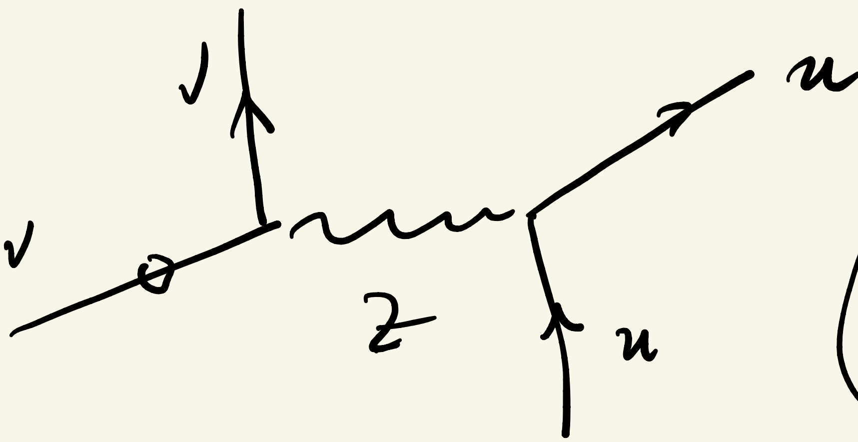


$$= \frac{g^2}{\cos^2\theta_W \, m_Z^2} J_\mu^Z J^\mu_Z$$

1971 't Hooft
(renormalizable SM)



+



'70s

$$J_\mu^Z = \bar{f} \gamma_\mu [T_3 - Q \sin^2\theta_W] f \quad (*)$$

ν: $\bar{\nu}_L \gamma^\mu \frac{1}{2} \nu_L$

$T_3 f_R = 0$

$T_3 f_L = t_3 f_L$

$$\bar{f} \gamma^\mu [T_3 L - Q \sin^2\theta_W] f \quad (**)$$

e_L: $\bar{e}_L \gamma^\mu \left[-\frac{1}{2} + \sin^2\theta_W \right] e_L$

d_L: $\bar{d}_L \gamma^\mu \left[-\frac{1}{2} + \frac{1}{3} \sin^2\theta_W \right] d_L$

$g = \frac{e}{\sin\theta_W}$

measure !

$\Downarrow$

$\theta_W \cong 30^\circ$

$\sin^2\theta_W \approx 0.25$

$\Downarrow$

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8 M_W^2 \sin^2\theta_W}$$

$\alpha = \frac{e^2}{4\pi}$

$\Downarrow$

$$\frac{G_F}{\sqrt{2}} = \frac{\pi \alpha}{2 (M_W \sin\theta_W)^2}$$

$\Downarrow$

$M_W = 80$ GeV

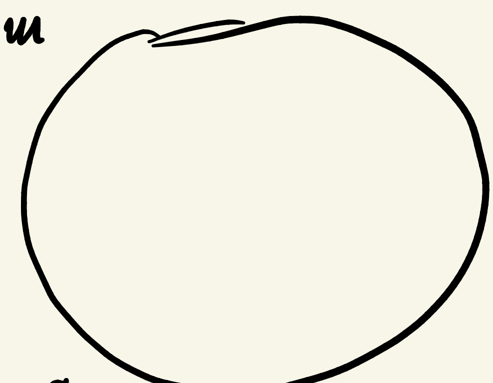
\+ Z current success

$\Downarrow$

$M_Z = 90$ GeV

$\theta_W \simeq 30^\circ$

7km



SPS

$p + \bar{p}$

1983

W, Z

$$d_L + (\bar{u})_R \longrightarrow W^- \quad (M_W \simeq 100 \text{ GeV})$$

at rest

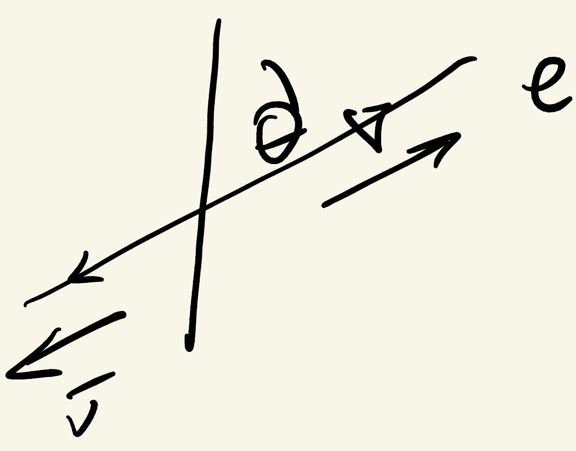
$$m_u \simeq m_d \simeq \text{MeV} \cong 0 \Rightarrow \underline{\text{helicity!}}$$

$$W^- \longrightarrow \left\{ \begin{array}{l} (e)_L + (\bar{\nu}_e)_R \\ (d)_L + (\bar{u})_R \end{array} \right\} + \text{gen.} \quad (m_f = 0)$$

⇑ $S_z = +1$ How?

W at rest

$$\frac{d\Gamma}{d\Omega}(\theta)$$



(3)

• quarks carry color

• $m_t > M_W$

• $\varepsilon_\mu(W) = ?$

$\partial_\mu A^\mu = 0 \Rightarrow \varepsilon_\mu p^\mu = 0$

at rest $\Rightarrow \varepsilon_0 = 0$

$(T_i)_{jk} = -i\,\varepsilon_{ijk}$

$i = 1, 2, 3$

$\Uparrow$

$[T_i, T_j] = i\,\varepsilon_{ijk} T_k$

$A_\mu \bar{f}_L \gamma^\mu f_L \quad (L, R \ldots)$

$$\to A_i \bar{f} \gamma^i f \longrightarrow A_i \, u^+ \sigma^i u$$

sw.

$$u = u_L, \; u_R$$

$$u_{L,R} \longrightarrow e^{i \vec{\sigma}/2 \cdot \vec{\theta}} \, u_{L,R}$$

ROT

$$\Rightarrow \boxed{(T_i)_{jk} = -i \, \varepsilon_{ijk}}$$

$$T_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T_3 \, \epsilon^{(+)} = \epsilon^{(+)} \Rightarrow \varepsilon^{(+)} = \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}$$

$$T_3\, \epsilon^{(-)} = \epsilon^{(-)} \Rightarrow \epsilon^- = \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$$

$$T_3\, \epsilon^{(0)} = 0 \Rightarrow \epsilon^0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\Downarrow$$

$$\epsilon_\mu^{(+)} = (0;\ 1,\ i,\ 0)\ \frac{1}{\sqrt{2}} \quad S_z = +1$$

$$\epsilon_\mu^{(-)} = (0;\ 1,\ -i,\ 0)\ \frac{1}{\sqrt{2}} \rightarrow S_z = -1$$

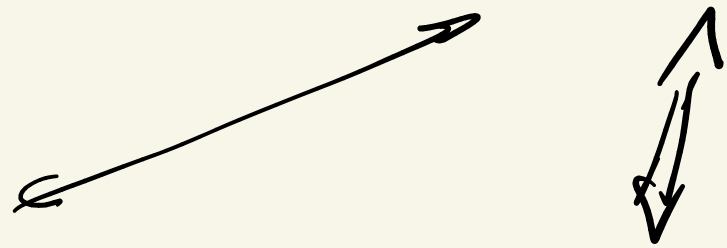
$$\epsilon_\mu^{(0)} = (0;\ 0,\ 0,\ 1) \rightarrow S_z = 0$$

$$\Downarrow$$

$$\sum_{\substack{i=1,2,3 \\ (+,-,0)}} \epsilon_\mu \epsilon_\nu^* = -g_{\mu\nu} + \frac{p_\mu p_\nu}{M_W^2}$$

$p_0 = M_W$

$\vec{p} = 0$



$$\Delta_{\mu\nu}(W) = \frac{g_{\mu\nu} - \frac{p_\mu p_\nu}{M_W^2}}{p^2 - M_W^2}$$

$$= \frac{\sum \epsilon_\mu \epsilon_\nu^*}{p^2 - M_W^2}$$

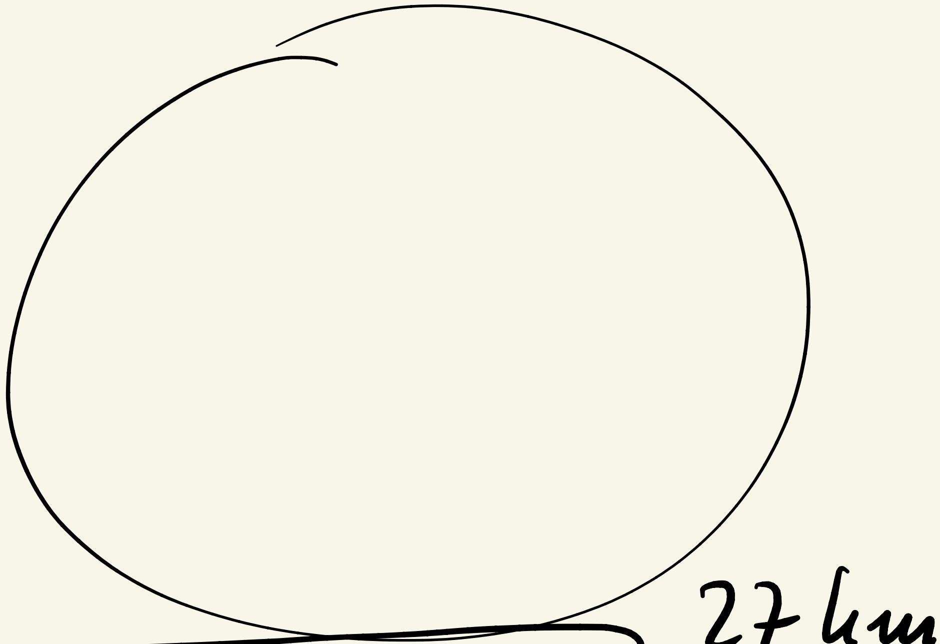
$$\Delta(\phi) = \frac{1}{p^2 - m_\phi^2} \quad \text{scalar} \left.\right\} \begin{array}{l} \to 0 \\ p \to 0 \end{array}$$

<u>Theory</u> $\Rightarrow$ $M_W, M_Z \simeq 100\ \text{GeV}$

(+ low energy exp)

LEP

$e + \bar{e}$



$E_{LEP} = 205\,GeV$

Machines = accelerators

discovery

high precision

$\Downarrow$

(hadron) [(lepton)]

E_q = free

discovery!

SPS = $p + \bar{p}$ goes first

- SM = high precision theory

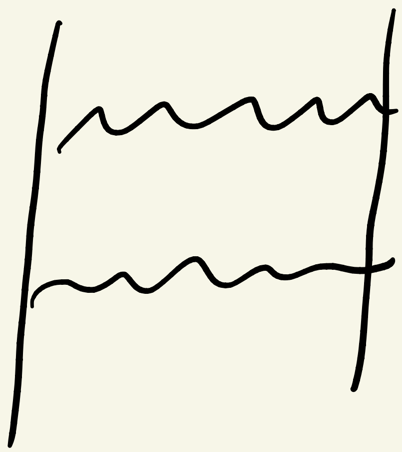
  = loops

$\Rightarrow$ M_W, Γ_W, M_Z, Γ_Z = compute

LEP = W, Z factory that measures ($\leq 1\%$)

$\sin^2\theta_W = 0.23$
LEP

· loop?



$$\int_0^\infty d^4k \; \Delta(W)\,\Delta(W) \cdots$$

$$\Delta_{\mu\nu}(W) = \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}}{k^2 - M_W^2}$$

· $M_W \to 0 \Rightarrow$ singularity

$M_W \to 0 \iff k \to \infty$

$\Delta_{\mu\nu} \xrightarrow[k\to\infty]{} \frac{1}{M_W}$ (does not vanish)

$\Downarrow$

$\int d^4k \; \ldots = \infty$!!!

Proca = wrong!

LEP : $\sin^2\theta_W = 0.23$

Neutral current: $\sin^2\theta_W^{exp} = 0.20$

Grand Unification: SU(5)

weak + em + strong $\Downarrow$ Georgi, Glashow

proton decay

$$\sin^2\theta_W^{SU(5)} = 0.20$$

Proca: $\mathcal{L}_P = A_\mu \blacksquare A_\nu$

U(1)

<u>SU(2)</u> $A_\mu^i \blacksquare A_\nu^i$

$\longrightarrow Z_\mu \blacksquare Z_\nu$

$$\rightarrow \quad W_\mu^{(+)} \otimes W_\nu^{(-)}$$

$$\|$$

$$A_\mu^{(1)} \otimes A_\nu^{(1)} + A_\mu^{(2)} \otimes A_\nu^{(2)}$$

$$W_\mu^{\pm} = \frac{A_\mu^1 \pm i A_\mu^2}{\sqrt{2}}$$

$$W_\mu^{+} W_\nu^{-} = (1)(1) + (2)(2)$$

$$A = s A_3 + c B$$

$$Z = c A_3 - s B$$

$$A_3 \otimes A_3 + B \otimes B$$

$$= A \,\blacksquare\, A + Z \,\blacksquare\, Z$$

$\Downarrow$ $\Downarrow$

$m_A = 0$ $\qquad$ $M_Z \neq 0$