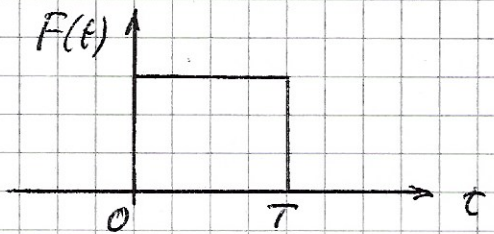
1. Zeige: $1-\cos\varphi = 2\sin^2\frac{\varphi}{2}$

$$2\sin^2\frac{\varphi}{2} = 2\left(\frac{e^{i\varphi/2}-e^{-i\varphi/2}}{2i}\right)^2 = -\frac{1}{2}\left(e^{i\varphi}+e^{-i\varphi}-2\right) = 1-\cos\varphi$$

2.



harmonischer Oszillator,
$x = \dot{x} = 0$ für $t<0$

a) Berechne $x, \dot{x}, E, a$ für $t>T$

$$\xi(t) = e^{i\omega t}\left[\int_0^T \frac{F}{m} e^{-i\omega t'}\,dt'\right] =$$

$$= \frac{F}{m} e^{i\omega t} \frac{e^{-i\omega t'}}{-i\omega}\Big|_0^T = i\frac{F}{m\omega} e^{i\omega t}\left(e^{-i\omega T}-1\right)$$

$\Rightarrow$

$$x = \frac{1}{\omega}\operatorname{Im}\xi = \frac{F}{m\omega^2}\left(\cos\omega(t-T)-\cos\omega t\right) = \frac{2F}{m\omega^2}\sin\frac{\omega T}{2}\sin\left(\omega t-\frac{\omega T}{2}\right.$$

$$\dot{x} = \operatorname{Re}\xi = \frac{F}{m\omega}\left(-\sin\omega(t-T)+\sin\omega t\right)$$

$$E = \frac{m}{2}|\xi|^2 = \frac{m}{2}\frac{F^2}{m^2\omega^2}\left(1-e^{-i\omega T}\right)\left(1-e^{i\omega T}\right) =$$

$$= \frac{F^2}{m\omega^2}\left(1-\cos\omega T\right) = \frac{2F^2}{m\omega^2}\sin^2\frac{\omega T}{2} \overset{!}{=} \frac{m}{2}\omega^2 a^2$$

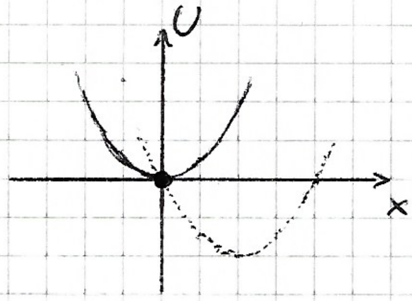
$$\Rightarrow a = \frac{2F}{m\omega^2}\sin\frac{\omega T}{2}$$

Trigonometrische Relation

$$\underline{2\sin\alpha\sin\beta} = -\tfrac{1}{2}\left(e^{i\alpha}-e^{-i\alpha}\right)\left(e^{i\beta}-e^{-i\beta}\right) =$$

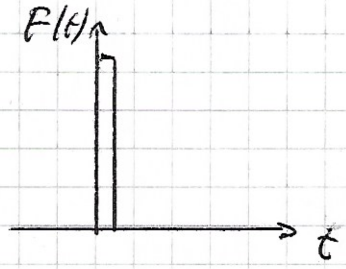
$$= -\tfrac{1}{2}\left(e^{i(\alpha+\beta)}+e^{-i(\alpha+\beta)}-e^{i(\alpha-\beta)}-e^{-i(\alpha-\beta)}\right) = \underline{\cos(\alpha-\beta)-\cos(\alpha+\beta)}$$

b.) Was passiert wenn $T = \frac{2\pi}{\omega}$?



$\Rightarrow$ $a = 0$, $x(t) \equiv 0$ f. $t > T$

3. Für Lsg. von 2a.), bilde Limes $T \to 0$, $F \cdot T \equiv p = \text{const.}$



$$x(t) = \lim_{T\to 0} \frac{p}{m\omega^2} \frac{\cos\omega(t-T) - \cos\omega t}{T} =$$

$$= \lim_{T\to 0} \frac{p}{m\omega^2} \frac{-(-\omega)\sin\omega(t-T)}{1} = \underline{\frac{p}{m\omega}\sin\omega t}$$

$$\dot{x} = \frac{p}{m}\cos\omega t$$

$$a = \frac{2p}{m\omega^2 T}\sin\frac{\omega T}{2} \overset{T\to 0}{=} \frac{2p}{m\omega^2 T}\frac{\omega T}{2} = \frac{p}{m\omega} \quad ✓$$

$$\Delta E = \frac{p^2}{2m}$$

$$E = \frac{m}{2}\omega^2 a^2 = \frac{p^2}{2m} \quad ✓$$