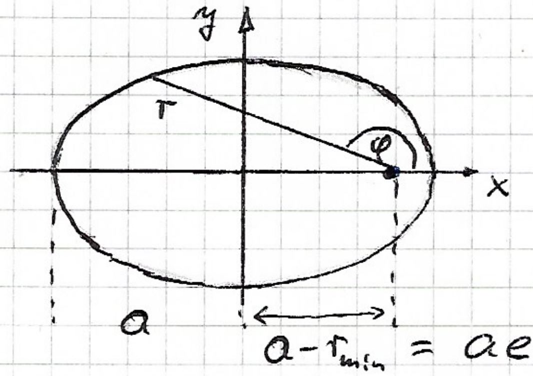
1. Kepler-Ellipse in kartesischen Koordinaten



$$r = \frac{p}{1 + e\cos\varphi}$$

$$0 \leq e < 1$$

- Bestimme r_{min}, r_{max}, a

$$r_{min} = r(0) = \frac{p}{1+e}, \quad r_{max} = r(\pi) = \frac{p}{1-e}$$

$$a := \frac{1}{2}(r_{min} + r_{max}) = \frac{p}{2}\left(\frac{1}{1+e} + \frac{1}{1-e}\right) = \underline{\underline{\frac{p}{1-e^2}}}$$

$$a - r_{min} = \frac{p}{1-e^2} - \frac{p(1-e)}{1-e^2} = ae$$

- $x = ae + r\cos\varphi \qquad\qquad y = r\sin\varphi$

- $y_{max} \equiv b = ?$

$$\frac{dy}{d\varphi} = \frac{d}{d\varphi}\frac{p\sin\varphi}{1+e\cos\varphi} = \frac{(1+e\cos\varphi)p\cos\varphi + p\sin\varphi\, e\sin\varphi}{(1+e\cos\varphi)^2} =$$

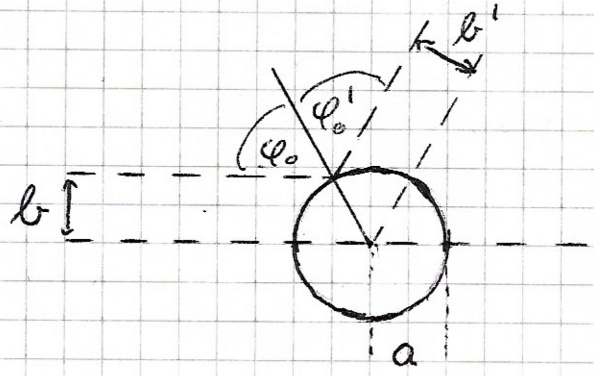
$$= \frac{p\cos\varphi + pe}{(1+e\cos\varphi)^2} \overset{!}{=} 0 \quad \Rightarrow \quad \underline{\cos\varphi = -e} \quad \Rightarrow \quad b = \frac{p\sqrt{1-e^2}}{1-e^2} = \underline{\underline{\frac{p}{\sqrt{1-e^2}}}}$$

- $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \left(e + \frac{(1-e^2)\cos\varphi}{1+e\cos\varphi}\right)^2 + \left(\frac{\sqrt{1-e^2}\,\sin\varphi}{1+e\cos\varphi}\right)^2 =$$

$$= \frac{(e + \cancel{e^2\cos\varphi} + (1-\cancel{e^2})\cos\varphi)^2 + (1-e^2)(1-\cos^2\varphi)}{(1+e\cos\varphi)^2}$$

$$= \frac{\cancel{e^2} + 2e\cos\varphi + \cancel{\cos^2\varphi} + 1 - \cancel{\cos^2\varphi} - \cancel{e^2} + e^2\cos^2\varphi}{(1+e\cos\varphi)^2} \equiv 1 \quad ✓$$

2. Streuung an harter Kugel



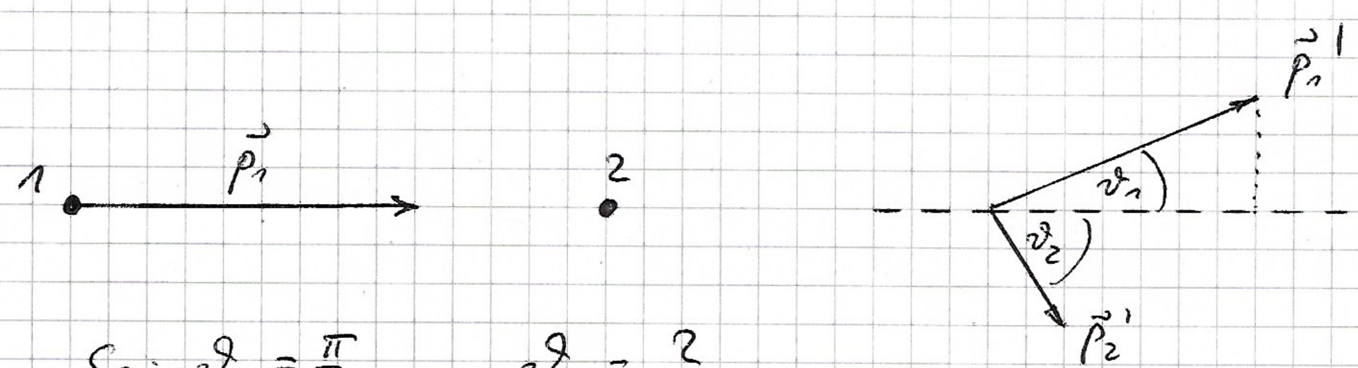
Zeige, dass $\varphi_0' = \varphi_0$

(mit Energie- und Drehimpulserhaltung)

$$E = \frac{m}{2} v^2 , \quad E = E' \Rightarrow v = v'$$

$$\left.\begin{aligned} l &= m v b = m v a \sin\varphi_0 \\ l' &= m v' b' = m v a \sin\varphi_0' \end{aligned}\right\} \; l = l' \Rightarrow \underline{\varphi_0 = \varphi_0'}$$

3. Streuung im Laborsystem



Sei $\vartheta_2 = \frac{\pi}{4}$; $\vartheta_1 = ?$

$$\vartheta_2 = \frac{\pi - \chi}{2} \Rightarrow \chi = \pi - 2\vartheta_2$$

$$\tan\vartheta_1 = \frac{m_2 \sin\chi}{m_1 + m_2 \cos\chi} = \frac{m_2 \sin 2\vartheta_2}{m_1 - m_2 \cos 2\vartheta_2} \overset{\vartheta_2 = \frac{\pi}{4}}{=} \frac{m_2}{m_1}$$

Spezialfall $m_2 = m_1 \Rightarrow \tan\vartheta_1 = 1 \Rightarrow \vartheta_1 = \frac{\pi}{4}$

$\vartheta_1 + \vartheta_2 = \frac{\pi}{2}$ ✓