1. Zweikörperproblem

Transformiere $L = \frac{m_1}{2}\dot{\vec{r}}_1^{\,2} + \frac{m_2}{2}\dot{\vec{r}}_2^{\,2} - U(|\vec{r}_1 - \vec{r}_2|)$

auf Schwerpunkts- und Relativkoordinaten ($\vec{R}$ und $\vec{r}$)

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} \qquad \vec{r} = \vec{r}_1 - \vec{r}_2 \qquad |\vec{r}| = r$$

$$\Leftrightarrow \quad \vec{r}_1 = \vec{R} + \frac{m_2}{m_1+m_2}\vec{r} \qquad \vec{r}_2 = \vec{R} - \frac{m_1}{m_1+m_2}\vec{r}$$

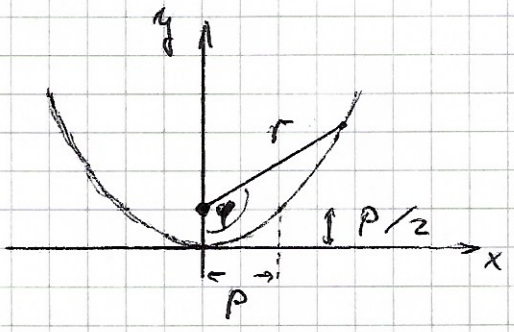
$$L = T - U(r)$$

$$T = \frac{m_1}{2}\left(\dot{\vec{R}} + \frac{m_2}{m_1+m_2}\dot{\vec{r}}\right)^2 + \frac{m_2}{2}\left(\dot{\vec{R}} - \frac{m_1}{m_1+m_2}\dot{\vec{r}}\right)^2 =$$

$$= \frac{m_1+m_2}{2}\dot{\vec{R}}^{\,2} + \frac{1}{2}\frac{m_1m_2^2 + m_2m_1^2}{(m_1+m_2)^2}\dot{\vec{r}}^{\,2} \equiv \frac{M}{2}\dot{\vec{R}}^{\,2} + \frac{m}{2}\dot{\vec{r}}^{\,2}$$

2. Kepler-Problem, Parabelbahn

$$r = \frac{p}{1+\cos\varphi}$$



Bahnkurve in Koord. (x, y) ?

$$x = r\sin\varphi \,, \qquad y = \frac{p}{2} - r\cos\varphi$$

$$2py = p^2 - 2p^2\frac{\cos\varphi}{1+\cos\varphi} = p^2\frac{1-\cos\varphi}{1+\cos\varphi} =$$

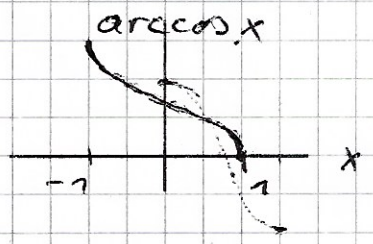
$$= p^2\frac{\sin^2\varphi}{(1+\cos\varphi)^2} = x^2 \qquad \Leftrightarrow \qquad \underline{y = \frac{x^2}{2p}}$$

3. Zeige

$$-\int \frac{du}{\sqrt{c_1 + c_2 u + c_3 u^2}} = \frac{1}{\sqrt{-c_3}} \arccos\left[-\frac{c_2 + 2c_3 u}{\sqrt{c_2^2 - 4c_1 c_3}} \right]$$

$$\frac{d \arccos x}{dx} = -\frac{1}{\sqrt{1-x^2}} \quad (*) \qquad x \equiv \cos u$$

$$1 = \frac{d}{du} \arccos(\cos u) = \frac{d \arccos x}{dx} \underbrace{(-\sin u)}_{-\sqrt{1-x^2}} \quad \Rightarrow (*)$$



$$\frac{1}{\sqrt{-c_3}} \frac{d}{du} \arccos[\dots] = \frac{1}{\sqrt{-c_3}} \frac{-1}{\sqrt{1-[\dots]^2}} \cdot \frac{(-2c_3)}{\sqrt{c_2^2 - 4c_1 c_3}} =$$

$$= \frac{-2\sqrt{-c_3}}{\sqrt{c_2^2 - 4c_1 c_3 - (c_2 + 2c_3 u)^2}} = \frac{-2\sqrt{-c_3}}{\sqrt{c_2^2 - 4c_1 c_3 - c_2^2 - 4c_2 c_3 u - 4c_3^2 u^2}} =$$

$$= \frac{-1}{\sqrt{c_1 + c_2 u + c_3 u^2}} \qquad \checkmark$$