1. Zeige explizit die Invarianz der ELG unter einer Eichtransformation von $L \to L'$

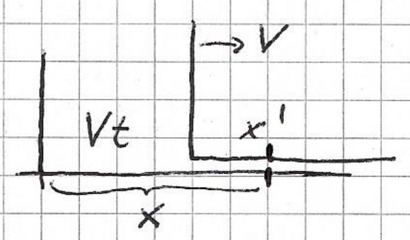
$$L'(q,\dot q,t) = L(q,\dot q,t) + \frac{d}{dt} f(q,t)$$

$$\frac{d}{dt}\frac{\partial L'}{\partial \dot q} - \frac{\partial L'}{\partial q} = \underbrace{\frac{d}{dt}\frac{\partial L}{\partial \dot q} - \frac{\partial L}{\partial q}}_{=0} + \frac{d}{dt}\frac{\partial f}{\partial q} - \frac{\partial^2 f}{\partial q^2}\dot q - \frac{\partial^2 f}{\partial q\, \partial t} = 0$$

wegen

$$\frac{d}{dt} f(q,t) = \frac{\partial f}{\partial q}\dot q + \frac{\partial f}{\partial t}, \quad \frac{d}{dt}\frac{\partial f}{\partial q} = \frac{\partial^2 f}{\partial q^2}\dot q + \frac{\partial^2 f}{\partial q\, \partial t}$$

2. $L = \frac{m}{2}\dot x^2$ eigentliche Galilei-Trafo $\to$?



$x' = x - Vt \qquad \dot x' = \dot x - V$

$$L = \frac{m}{2}(\dot x' + V)^2 = \frac{m}{2}\dot x'^2 + mV\dot x' + \frac{m}{2}V^2$$

$$= \frac{m}{2}\dot x'^2 + \frac{d}{dt} f(x',t)$$

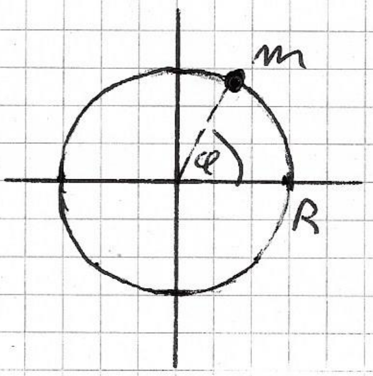
mit $f(x',t) = mVx' + \frac{m}{2}V^2 t$

3. $f(x_1,x_2) = x_1^2 + x_1 x_2 + x_2^2$, verifiziere Euler-Theorem

$$\sum x_i \frac{\partial f}{\partial x_i} = x_1(2x_1 + x_2) + x_2(x_1 + 2x_2) = 2x_1^2 + 2x_1x_2 + 2x_2^2 = 2f \checkmark$$

P3.2

4. Freier Massenpunkt auf kreisförmiger Schiene



Bestimme L,
Euler-Lagrange-Gleichungen

a.) unter Eliminierung der Zwangsbedingung

b.) mit Lagrange-Multiplikator und Bestimmung der Zwangskraft

zu a.) $L = \frac{m}{2}(\dot{r}^2 + r^2\dot{\varphi}^2)$, Zwangsbed. $r = R = \text{const.}$

$\Rightarrow \dot{r} = 0$, $\quad L(\dot{\varphi}) = \frac{m}{2}R^2\dot{\varphi}^2$

ELG: $\frac{d}{dt}\frac{\partial L}{\partial \dot{\varphi}} = \frac{d}{dt}(mR^2\dot{\varphi}) = 0$, $\quad \underline{mR^2\dot{\varphi} = \text{const.}}$

Drehimpuls

zu b.) $L = \frac{m}{2}(\dot{r}^2 + r^2\dot{\varphi}^2) - \lambda(r - R)$

ELG: $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i}$ für $q_i = r, \varphi, \lambda$

$m\ddot{r} = mr\dot{\varphi}^2 - \lambda \quad \overset{(*)}{\Rightarrow} \quad \ddot{r} = 0, \quad \underline{\lambda = mR\dot{\varphi}^2}$

$\frac{d}{dt}(mr^2\dot{\varphi}) = 0 \quad \overset{(*)}{\Rightarrow} \quad \underline{mR^2\dot{\varphi} = \text{const.}}$

$r - R = 0 \quad (*)$

Zwangskraft
= Zentrifugalkraft