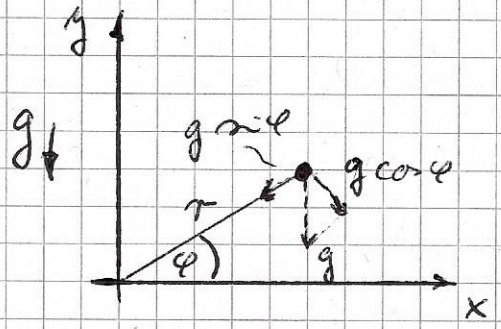
1. Koordinatentrafo: Newton vs. Lagrange



$$m\ddot{x} = 0$$

$$m\ddot{y} = -mg$$

$$L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) - mgy$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial x} \Leftrightarrow m\ddot{x} = 0$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} = \frac{\partial L}{\partial y} \Leftrightarrow m\ddot{y} = -mg$$

Polarkoord.: $x = r\cos\varphi \quad y = r\sin\varphi$

Newton: $\dot{x} = \dot{r}\cos\varphi - r\sin\varphi\,\dot{\varphi}$, $\dot{y} = \dot{r}\sin\varphi + r\cos\varphi\,\dot{\varphi}$

$$\ddot{x} = \ddot{r}\cos\varphi - \dot{r}\sin\varphi\,\dot{\varphi} - \dot{r}\sin\varphi\,\dot{\varphi} - r\cos\varphi\,\dot{\varphi}^2 - r\sin\varphi\,\ddot{\varphi}$$

$$\ddot{y} = \ddot{r}\sin\varphi + \dot{r}\cos\varphi\,\dot{\varphi} + \dot{r}\cos\varphi\,\dot{\varphi} - r\sin\varphi\,\dot{\varphi}^2 + r\cos\varphi\,\ddot{\varphi}$$

$$\left.\begin{aligned} \ddot{r}\cos\varphi - 2\dot{r}\sin\varphi\,\dot{\varphi} - r\dot{\varphi}^2\cos\varphi - r\ddot{\varphi}\sin\varphi &= 0 \\ \ddot{r}\sin\varphi + 2\dot{r}\cos\varphi\,\dot{\varphi} - r\dot{\varphi}^2\sin\varphi + r\ddot{\varphi}\cos\varphi &= -g \end{aligned}\right\} \Leftrightarrow \begin{aligned} \ddot{r} - r\dot{\varphi}^2 &= -g\sin\varphi \\ 2\dot{r}\dot{\varphi} + r\ddot{\varphi} &= -g\cos\varphi \end{aligned}$$

nicht: $\ddot{r} = -g\sin\varphi$, $r\ddot{\varphi} = -g\cos\varphi$

Lagrange: $\dot{x}^2 + \dot{y}^2 = \dot{r}^2\cos^2\varphi - 2r\dot{r}\sin\varphi\cos\varphi\,\dot{\varphi} + r^2\sin^2\varphi\,\dot{\varphi}^2 +$

$$+ \dot{r}^2\sin^2\varphi + 2r\dot{r}\sin\varphi\cos\varphi\,\dot{\varphi} + r^2\cos^2\varphi\,\dot{\varphi}^2$$

$$= \dot{r}^2 + r^2\dot{\varphi}^2 \qquad U = mgr\sin\varphi$$

$$L = \frac{m}{2}(\dot{r}^2 + r^2\dot{\varphi}^2) - mgr\sin\varphi$$

$$L = \frac{m}{2}(\dot r^2 + r^2\dot\varphi^2) - mgr\sin\varphi$$

ELG:

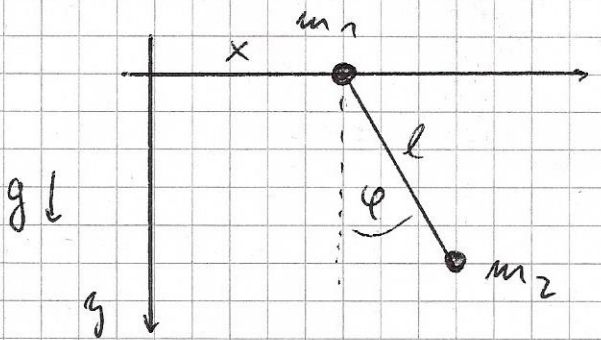
$$\frac{d}{dt}\frac{\partial L}{\partial \dot r} = \frac{\partial L}{\partial r} \Leftrightarrow \ddot r = r\dot\varphi^2 - g\sin\varphi \quad \checkmark$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\varphi} = \frac{\partial L}{\partial \varphi} \Leftrightarrow \frac{d}{dt}(r^2\dot\varphi) = -gr\cos\varphi$$

$$\Leftrightarrow 2r\dot r\dot\varphi + r^2\ddot\varphi = -gr\cos\varphi \Leftrightarrow 2\dot r\dot\varphi + r\ddot\varphi = -g\cos\varphi \quad \checkmark$$

‖ ELG forminvariant unter Trafo $(x,z)\to(r,\varphi)$
‖ im Ggs. zu Newton

2.



Bewegungsgln.: (1) $(m_1+m_2)\ddot x + m_2 l\ddot\varphi\cos\varphi = p_x = \text{const.}$

(2) $l\ddot\varphi = -\ddot x\cos\varphi - g\sin\varphi$

Grenzfall $m_1 \gg m_2$, $\frac{m_2}{m_1}\to 0$

$$(1) \Leftrightarrow \left(1+\frac{m_2}{m_1}\right)\ddot x + \frac{m_2}{m_1} l\ddot\varphi\cos\varphi = \frac{p_x}{m_1}$$

$$\to \dot x = \text{const.} \Rightarrow \ddot x = 0 \overset{(2)}{\Rightarrow} l\ddot\varphi = -g\sin\varphi$$

(einfaches Pendel)