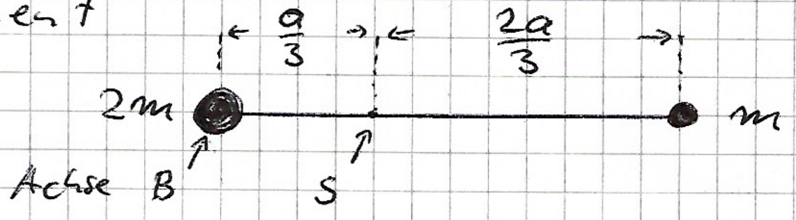
1. Trägheitsmoment

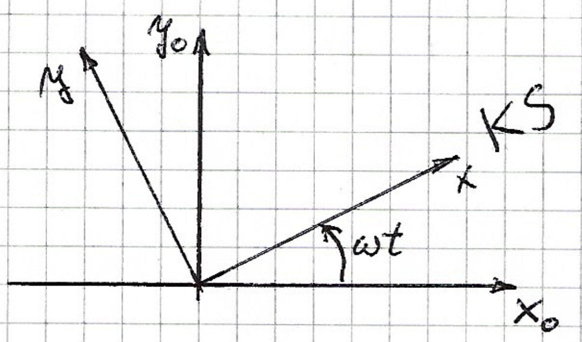
J_S, J_B ?



$$J_S = 2m\left(\frac{a}{3}\right)^2 + m\left(\frac{2a}{3}\right)^2 = \underline{\frac{2}{3} m a^2}$$

$$J_B \overset{\text{Steiner}}{=} J_S + M\left(\frac{a}{3}\right)^2 = \frac{2}{3} m a^2 + \frac{1}{3} m a^2 = \underline{m a^2} \checkmark$$

2. Rotierendes Bezugssystem ($\omega = \text{const.}$)

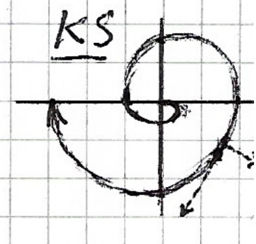


freie Bewegung <u>in IS</u>:

$$x_0(t) = ut$$
$$y_0(t) = 0$$

a.) Bestimme die Bewegung des Punkts in <u>KS</u>-Koord. <u>x, y</u>

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \cos\omega t & \sin\omega t \\ -\sin\omega t & \cos\omega t \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} ut\cos\omega t \\ -ut\sin\omega t \end{pmatrix}$$



b.) Berechne für $\vec{r} \equiv (x, y, 0)^T$: $\vec{v} = \dot{\vec{r}}$, $\dot{\vec{v}} = \ddot{\vec{r}}$

$$\vec{r} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \qquad \vec{v} = \dot{\vec{r}} = \begin{pmatrix} u\cos\omega t - u\omega t\sin\omega t \\ -u\sin\omega t - u\omega t\cos\omega t \\ 0 \end{pmatrix}$$

$$\dot{\vec{v}} = \begin{pmatrix} -2u\omega\sin\omega t - u\omega^2 t\cos\omega t \\ -2u\omega\cos\omega t + u\omega^2 t\sin\omega t \\ 0 \end{pmatrix}$$

P10.2

c.) Verifiziere $\underline{\dot{\vec{v}} = 2\,\vec{v}\times\vec{\omega} + (\vec{\omega}\times\vec{r})\times\vec{\omega}}$

$$\vec{\omega} = \begin{pmatrix}0\\0\\\omega\end{pmatrix} \qquad \dot{\vec{\omega}} = 0$$

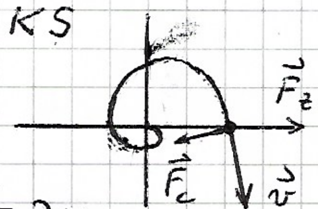
$$\vec{v}\times\vec{\omega} = \begin{pmatrix}\dot{x}\\\dot{y}\\0\end{pmatrix}\times\begin{pmatrix}0\\0\\\omega\end{pmatrix} = \begin{pmatrix}\omega\dot{y}\\-\omega\dot{x}\\0\end{pmatrix} = \begin{pmatrix}-u\omega\sin\omega t - u\omega^2 t\cos\omega t\\-u\omega\cos\omega t + u\omega^2 t\sin\omega t\\0\end{pmatrix}$$

$$(\vec{\omega}\times\vec{r})\times\vec{\omega} = \omega^2\,\vec{r} = \begin{pmatrix}u\omega^2 t\cos\omega t\\-u\omega^2 t\sin\omega t\\0\end{pmatrix}$$

$$(\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}\cdot\vec{c})\,\vec{b} - (\vec{b}\cdot\vec{c})\,\vec{a}$$

$$\Rightarrow 2\,\vec{v}\times\vec{\omega} + \omega^2\vec{r} = \begin{pmatrix}-2u\omega\sin\omega t - u\omega^2 t\cos\omega t\\-2u\omega\cos\omega t + u\omega^2 t\sin\omega t\\0\end{pmatrix} \equiv \dot{\vec{v}} \quad \checkmark$$

d.)



$\underline{\omega t = 2\pi}$

$$\vec{r} = \frac{2\pi u}{\omega}\begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \vec{v} = \begin{pmatrix}u\\-2\pi u\\0\end{pmatrix}, \quad 2\,\vec{v}\times\vec{\omega} = \begin{pmatrix}-4\pi u\omega\\-2u\omega\\0\end{pmatrix}$$

$$\dot{\vec{v}} = \begin{pmatrix}-2\pi u\omega\\-2u\omega\\0\end{pmatrix}$$

e.) Energieerhaltung in KS? $\quad E = \frac{m}{2}\vec{v}^{\,2} - \frac{m}{2}(\vec{\omega}\times\vec{r})^2$

$$\vec{v}^{\,2} = (uc - u\omega t s)^2 + (us + u\omega t c)^2 = \underline{u^2 + u^2\omega^2 t^2}$$

$$\vec{\omega}\times\vec{r} = \begin{pmatrix}0\\0\\\omega\end{pmatrix}\times\begin{pmatrix}x\\y\\0\end{pmatrix} = \begin{pmatrix}-\omega y\\\omega x\\0\end{pmatrix} = \begin{pmatrix}u\omega t\sin\omega t\\u\omega t\cos\omega t\\0\end{pmatrix}$$

$$\Rightarrow (\vec{\omega}\times\vec{r})^2 = \underline{u^2\omega^2 t^2} \qquad \Rightarrow \qquad \underline{E = \frac{m}{2}u^2 = \text{const.}}$$