“QCD and Standard Model”
Problem Set 9

1. The (massive) neutrino

As we saw, neutrinos are taken to be massless in the (canonical or minimal) Standard Model. If this were actually true, the number of electron neutrinos reaching the Earth from the Sun should be twice of what it is observed. The *solar neutrino problem* can be explained if neutrinos experience flavor oscillations, a phenomenon that requires them to have non-zero masses. In other words, observations suggest that we should go beyond the Standard Model and modify it accordingly in order to accommodate neutrino masses.

The purpose of this exercise is to go through this topic to give some clarifications.

a) Discuss whether neutrinos can be Dirac or Majorana particles.

b) Use these results to generalize the Yukawa part of the electroweak Lagrangian density

$$\mathcal{L}_Y = -\Lambda^{(e)}_{ij}\bar{E}^i_L H e^j_R - \Lambda^{(d)}_{ij}\bar{Q}^i_L H d^j_R - \Lambda^{(u)}_{ij}\bar{Q}^i_L \tilde{H} u^j_R + \text{h.c.}, \tag{1}$$

by writing renormalizable mass terms for the neutrinos.

c) If neutrinos are massive, leptons can oscillate in flavour as the quarks do. To see it, we repeat the same steps which brought us to define CKM matrix (see Exercise 4). Apply unitary transformations on the leptons in order to rotate them from the flavour to the mass basis. Then, show that this implies flavour mixing in the charged current sector of the electroweak theory.
What you just found is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix.

d) The intertwining between non-zero masses and flavour oscillations can be enlightened by a simple quantum mechanical argument. Expand the neutrino mass eigenstates in plane waves and compute the flavour mixing phase. Discuss the dependence of this quantity on the masses.

e) The most well-known mechanism offering an explanation of why neutrinos are so incredibly light as compared to the quarks and the other leptons, is the so-called “seesaw mechanism.” Show that the Dirac and Majorana mass terms for neutrinos can be recast in the following form

$$\mathcal{L}^{\text{mass}}[N] = -m N^T_L C \nu_L - \frac{M}{2} N^T_L C N_L + h.c. \,, \tag{2}$$

if one introduces the left handed spinor

$$N_L = \nu^c_R = C\bar{\nu}^T_R. \tag{3}$$

For the sake of simplicity, work with one family. Write eq (2) in matrix form and find the eigenvalues of the corresponding mass matrix. Estimate the value of the smallest eigenvalue, assuming that $m \lll M$. Can you explain why this is a reasonable assumption ?

2. Kaon oscillations and CP violation

In this exercise you will see how Kaon decays led to the discovery of CP violation in nature. In the following, let's first assume that CP is conserved for the weak interactions.

a) Consider the neutral particle

$$|K^0\rangle = |d\bar{s}\rangle \ ,$$

called Kaon, and its anti-particle

$$|\bar{K}^0\rangle = |\bar{d}s\rangle \ ,$$

where d and s are the down and strange quarks, respectively. Argue that, although the Kaon can be produced by strong interactions, it can only decay via the weak interaction.

b) Using the fact that Kaons are pseudoscalars, show how they transform under CP.

c) Use this to write two linear combinations of $|K^0\rangle$ and $|\bar{K}^0\rangle$, which are eigenstates of CP, with eigenvalues +1 and -1.

d) We know from observations that K^0 can decay into two pions, $\pi^+\pi^-$ or $\pi^0\pi^0$. Using CP conservation, determine which of the above combinations participates in this decay.

e) We also know from observations that K^0 can decay into three pions $\pi^0\pi^0\pi^0$. Which combination participates in this decay ?

Since the decay into 3 pions is much less probable than the decay into 2 pions, the above linear combinations are sometimes called K_S^0 and K_L^0, where one of them is "short-lived" and the other is "long-lived". However, in 1964 it was discovered that the "long-lived" state could indeed decay into 2 pions, around 0.2% of the time. This was the first indication that CP is violated in weak interactions. So the "real" short-lived and long-lived particles are

$$|K_S^0\rangle = N(p\,|K^0\rangle + q\,|\bar{K}^0\rangle) \ ,$$
$$|K_L^0\rangle = N(p\,|K^0\rangle - q\,|\bar{K}^0\rangle) \ ,$$

where $N = (|p|^2 + |q|^2)^{-1/2}$ is a normalization factor. Due to CP violation the (complex) numbers p and q are not equal.

f) Finally, we want to better understand the phenomenon of $K - \bar{K}$ *oscillations*. Draw the lowest-order Feynman diagrams responsible for the process $K^0 \leftrightarrow \bar{K}^0$.

g) The dynamics of this process can be effectively captured by the following equation

$$i\frac{d}{dt}\begin{pmatrix}|K^0\rangle \\ |\bar{K}^0\rangle\end{pmatrix} = \begin{pmatrix} m - i/2\Gamma & -p^2 \\ -q^2 & m - i/2\Gamma\end{pmatrix}\begin{pmatrix}|K^0\rangle \\ |\bar{K}^0\rangle\end{pmatrix} \ ,$$

where the 2×2-matrix may be understood as an "effective" Hamiltonian with Γ the decay rate, modeling the decay into the pions. Find the eigenstates and the eigenvalues of this Hamiltonian. Explain why $|K_S^0\rangle$ and $|K_L^0\rangle$ have different masses and different life-times.