“QCD and Standard Model”
Problem Set 6

The low-energy description of weak interactions. II. The neutral sector of the Fermi theory

In this exercise we will complete our overview on the effective description of electroweak interactions, with a particular focus on the neutral sector. The absence of which do not conserve flavor—called *Flavour-Changing Neutral Currents* (FCNCs)—is not at all obvious from the bottom-up construction of the EW theory and had to be addressed in a very careful manner. This is exactly what motivated Glashow, Iliopoulos and Maiani to predict the existence of a fourth quark, in addition to the light ones (u, d, s). This mechanism shows why FCNCs are not allowed at tree-level, while they are induced in a very suppressed way in loop diagrams, such as the ones regulating the rare decay $K^0 \to \mu^+\mu^-$.

Moreover, another very important implication is the selection rule $\Delta S = 1$ (S is strangeness) for charged weak interactions.

a) As the $W^{\pm}_{\mu}$ bosons, also the Z_μ and the photon couple effectively to currents. Write down these interactions after Spontaneous Symmetry Breaking in the neutral sector of the electroweak Lagrangian. Introduce the Weinberg angle to rotate W^3_μ and B_μ appropriately.

b) Integrate out Z_μ classically, as you did for $W^{\pm}_{\mu}$ in the previous problem set, in order to obtain the neutral sector of the 4−Fermi theory.

c) Explain why it is not possible to integrate the photon out in the same way. This corresponds to the fact that 4−Fermi theory disregards EM interactions, for they are described by QED.

d) Is the neutral current flavour-diagonal ?

e) In the early stages of the Standard Model, the only known quark content was the light triplet (u, s, d). In this case we only have one left-handed quark doublet

$$Q^1_L = \begin{pmatrix} u_L \\ d_L \cos\theta_C + s_L \sin\theta_C \end{pmatrix} , \tag{1}$$

in mass basis. Show that the neutral current is not flavour diagonal.

Now, complete the second generation of quarks by introducing the charm c, *i.e.*

$$Q^2_L = \begin{pmatrix} c_L \\ d_L V_1 + s_L V_2 \end{pmatrix} . \tag{2}$$

Prove that the coefficients $V_{1,2}$ can be set in such a way that the neutral current is flavour-diagonal at tree-level, as desired. What is the consequence on the Cabibbo matrix, which mixes d_L and s_L ?