"QCD and Standard Model"
Problem Set 4

1. Electroweak Symmetry Breaking in the Standard Model

Consider the Lagrangian for the electroweak interactions in the Standard Model :

$$\begin{aligned}\mathcal{L}=&-\frac{1}{4}W^a_{\mu\nu}W^{\mu\nu\,a}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}+i\sum_j\bar{\psi}_j\not{D}\psi_j+D_\mu H^\dagger D^\mu H+\mu^2 H^\dagger H-\lambda(H^\dagger H)^2\\&-\bar{E}_L\Lambda^{(e)}He_R-\bar{Q}_L\Lambda^{(u)}\tilde{H}u_R-\bar{Q}_L\Lambda^{(d)}Hd_R+\text{h.c.}\end{aligned}\tag{1}$$

where $D_\mu=\partial_\mu-igW^a_\mu\tau^a-ig'\frac{Y}{2}B_\mu$, $\psi_j=\{Q_L,E_L,u_R,d_R,e_R\}$, $\tilde{H}=i\sigma_2H^*$, and the family indices for the various Yukawa couplings Λ are suppressed.

a) Minimize the potential and identify the vacuum. Based on the result, show that the $SU(2)_\text{L}\times U(1)_\text{Y}$ gauge symmetry is spontaneously broken and discuss the pattern of breaking.

b) Write the potential around the minimum, identify the Higgs mass and write the terms in the potential (quadratic, cubic and quartic) as functions of the mass m_h and the *vacuum expectation value (VEV)* v. Work in the unitary gauge and use the convention

$$H(x)=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\v+h(x)\end{pmatrix}\tag{2}$$

c) Identify the physical $W^\pm_\mu$, Z_μ and A_μ gauge bosons in terms of W^i_μ and B_μ. Furthermore, expand the Higgs kinetic term around the vacuum and determine their masses.

d) Expanding the Yukawa sector around the vacuum, determine the masses of the fermions and all interaction terms.

2. Flavor parameters and CKM matrix

Let us now focus on the Yukawa interactions for leptons and quarks

$$\mathcal{L}_Y = -\Lambda^{(e)}_{ij} \bar{E}^i_L H e^j_R - \Lambda^{(d)}_{ij} \bar{Q}^i_L H d^j_R - \Lambda^{(u)}_{ij} \bar{Q}^i_L \tilde{H} u^j_R + \text{h.c.} \,, \tag{3}$$

where i, j are family indices.

a) Rotate the quark fields in order to diagonalize the Yukawa interactions. Show that the transformations on the right-handed quarks are unphysical but the ones on the left-handed quarks cannot be rotated away : in the diagonal (weak) basis they give rise to a mixing matrix (the CKM matrix) in the charged-current sector of the theory.

b) Given an arbitrary number N of quark families, determine the number of independent real parameters (mixing angles) and imaginary parameters (complex phases) of the CKM matrix. Make sure that the complex phases are really independent, *i.e.*, they cannot be reabsorbed into quark field redefinitions. Apply the formula when $N = 2$ and $N = 3$.

c) Make a CP transformation of the Yukawa term and convince yourselves that invariance under it implies that the Yukawa matrices must satisfy $\Lambda = \Lambda^*$. The existence of complex phases therefore points at CP violation. Using the results of point (b), justify why the experimental evidence of CP violation was a strong indication of the existence of a third generation of quarks.

d) Show that if neutrinos are massless, as the Standard Model assumes, there is no mixing matrix in the lepton sector.