Neutrino BBSM course

Lecture I

LMU

Spring 2020

It's neutrino, stupid!

Lecture 2

Spinors and chirality

$$\psi = \begin{pmatrix} u_L \\ u_R \end{pmatrix} \qquad h\, u_L = -\frac{1}{2}\, u_L$$

$$\qquad\qquad\qquad u_R \quad + \quad u_R$$

$$L = \frac{1+\gamma_5}{2} \iff \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$\boxed{V - A} \iff \cancel{\gamma_\mu (1-\gamma_5)} \quad \boxed{\gamma_5^2 = 1}$$

· u_L, u_R (2 comp.)

$$u_{L,R} \longrightarrow V_{Rot}\, u_{L,R}$$

$$V^\dagger V = V V^\dagger = 1 \qquad SU(2)$$

$$\det V = 1$$

$$a = 1, 2, 3$$

$$V = e^{i\Theta_a T_a} \qquad T_a = T_a^\dagger$$

$$\mathrm{Tr}\, T_a = 0$$

$$T_a = \frac{\sigma_a}{2}$$

$$u_{L,R} \to B_{L,R}\, u_{L,R}$$

$$B_{L,R} = e^{\pm \varphi_a T_a}$$

· spinor inv. $\quad u^\dagger u \,, \; u_1^\dagger u_2$

$$u_L^\dagger u_R \to u_L^\dagger B_L^\dagger B_R u_R$$

$$= \text{inv.}$$

$$\mathcal{L} = i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - m\bar{\Psi}\Psi$$

$$\Psi = \begin{pmatrix} u_L \\ u_R \end{pmatrix} = \Psi_L + \Psi_R$$

$$\Psi_{L,R} = \frac{1 \pm \gamma_5}{2}\Psi \qquad \boxed{\gamma_5^2 = 1}$$

$$\{\gamma_5, \gamma_\mu\} = 0$$

$$\Psi \to \lambda\Psi \qquad \boxed{\Psi^c = C\bar{\Psi}^T}$$

$$\Psi^c \to \lambda\Psi^c$$

- $\boxed{u_1 + u_2}$, $u + n$ (D) $\Big\}$ $S = 0$

$s = -\frac{1}{2}$ $\quad$ $s = +1/2$

- $$u = \begin{pmatrix} \varphi_u \\ \varphi_d \end{pmatrix} = \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} = \begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix}$$

D = Dirac

$\underline{S=0}$ $\qquad \underbrace{|\uparrow\downarrow - \downarrow\uparrow\rangle}$ $\qquad S=0$

$$u^T \epsilon u \qquad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\underline{= u^T i \sigma_2 u} \quad (M)$$

Majorana

$$\boxed{\tilde{u} = i\sigma_2 u^*}$$

$$\tilde{u}^+ = u^T(-i\sigma_2) \quad \Rightarrow \quad \underbrace{\tilde{u}^+ u = -u^T i \sigma_2 u}$$

INV

spinor $\Longleftrightarrow$ $\boxed{\tilde{u} \rightarrow V\tilde{u}}$

$$\psi^c = C\bar{\Psi}^T = i\gamma_2\gamma_0\gamma_0\psi^* = i\gamma_2\psi^*$$

$$\boxed{\psi^c = \begin{pmatrix} i\sigma_2 u_R^* \\ -i\sigma_2 u_L^* \end{pmatrix}}$$

$\underline{P}$: $\underbrace{u_L \longleftrightarrow u_R}_{\text{parity}} \iff \psi' = \gamma^0 \psi$

C : $u_L \longrightarrow i\sigma_2 u_R^*$

$$\mathcal{L}_{QED} = i\bar{\psi}\gamma^\mu D_\mu \psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$D_\mu = \partial_\mu - ie\underbrace{Q_\psi}_{\text{charge of } \psi} A_\mu$$

e: $Q_e = -1$

u: $Q_u = 2/3$

$$D_\mu \psi^c = \partial_\mu + ieQ_\psi A_\mu$$

$$\boxed{Q_{\psi^c} = -Q_\psi}$$

$$\psi \to \psi^c$$
$$p \to \text{anti-}p$$

$$\boxed{e_L \xrightarrow[C]{} (e^c)_R}$$

LH particle $\leftrightarrow$ RH anti-particle

$\cancel{\psi_L \leftrightarrow \psi_R}$ $\quad \boxed{u_L \leftrightarrow u_R}$

$$\psi \to \gamma^0 \psi$$

$$\begin{pmatrix} u_L \\ u_R \end{pmatrix} \to \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u_L \\ u_R \end{pmatrix} = \begin{pmatrix} u_R \\ u_L \end{pmatrix}$$

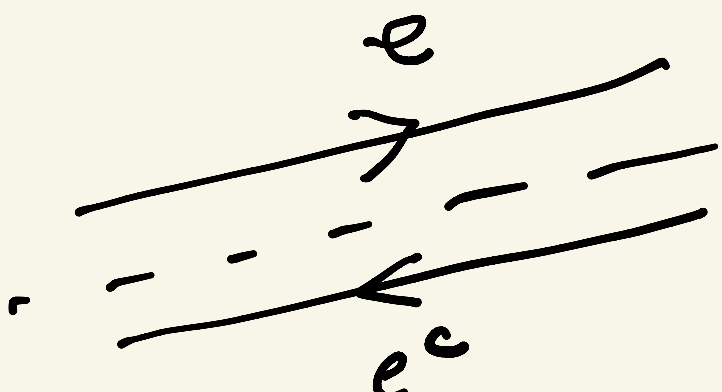
$$\psi_L \equiv L\psi \xrightarrow{P} L\gamma^0\psi$$

$$= \gamma^0 R\psi = \gamma_0\psi_R$$

$$\boxed{\boxed{\psi_L \to \gamma^0\psi_R} \; \boxed{\Leftrightarrow u_L \leftrightarrow u_R}}$$

$\boxed{\beta \text{ decay}}$ $\quad n \to p + e + \bar{\nu} \, (= \nu^c)$

$\boxed{(\bar{\nu})_R \Leftrightarrow \nu_L}$



Feynman rules

$[d \to u + e + \bar{u}]$

$$Co \to Ni + e + \bar{\nu}_e$$

$\boxed{\cancel{P}}$

'34 Fermi

$$\mathcal{H}^{w}_{eff} = G_F\, J_w \bar{J}_w$$

$$\mathcal{H}^{em}_{eff} = \frac{e^2}{q^2} J^{em}_{\mu} J^{\mu}_{em}$$

$$J^{em}_{\mu} = \bar{\psi} \gamma_\mu \psi$$

$$\boxed{\begin{array}{l} J_w = \bar{u}\, O_q\, d + \bar{\nu}\, O_e\, e \\ O_{q,e} = ? \end{array}}$$

P

Co $(s=5)$
N_i^* $(s=4)$ $\Big\}$ $\Delta S_z = 1$

Wu et al

Lee, Yang '56

$\Uparrow \qquad \cancel{\uparrow e_R \Uparrow} \qquad \downarrow e_L \Uparrow$

$\Delta S_z = 1 \qquad \cancel{\downarrow (\bar\nu)_L \Uparrow} \qquad \downarrow (\bar\nu)_R \Uparrow$

$$l_z = 0 \Rightarrow J_z = \cancel{l_z} + S_z$$

$$\left[\begin{array}{l} m_e = 0 \text{ (rel. electron)} \\ m_\nu = 0 \end{array}\right]$$

<u>Dirac eq.</u>

$$\boxed{\begin{array}{ll} h u_L = -\frac{1}{2} u_L & h = \vec{S}\cdot\hat{p} \\ h u_R = +\frac{1}{2} u_R & \end{array}}$$

<u>weak int</u> $\boxed{e_L, \ \nu_L}$

Marshak, Sudarshan '57
Gell-Mann, Feynman '58

$\boxed{u_L, d_L \text{ — also selected}}$

$$O_q = O_\ell = \gamma_\mu \frac{1+\gamma_5}{2}$$

$\boxed{h\, u_L = -\frac{1}{2} u_L}$

$\Downarrow$

$\boxed{\text{V–A theory}}$

$\boxed{\text{"V–A was the key"}}$

$\boxed{\text{history + convention}}$

<u>QED</u>: $\frac{1}{q^2} J_\mu J^\mu \leftrightarrow \boxed{e A_\mu J^\mu}$

<u>weak</u> $\frac{4 G_F}{\sqrt{2}} J_\mu^w \bar{J}^{\mu w}$

$$\boxed{J_\mu^W = \left(\bar{u}_L \gamma_\mu d_L + \bar{\nu}_L \gamma_\mu e_L\right)}$$

vector index (μ) + only L

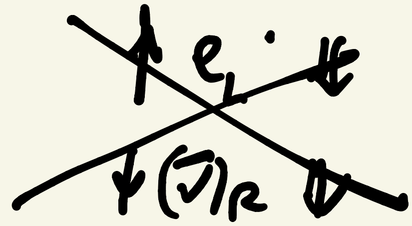
$$\boxed{\frac{g}{\sqrt{2}} J_\mu^W W^{\mu +} + h.c.}$$

Fermi resp, but <u>useless</u>

heavy

$$\boxed{M_W = 80\ GeV} \quad \boxed{E \sim MeV}$$

$$\left(\cdots \frac{g^2}{M_W^2} = G_F\right)$$

$W \Uparrow \; S_z = 1$ 

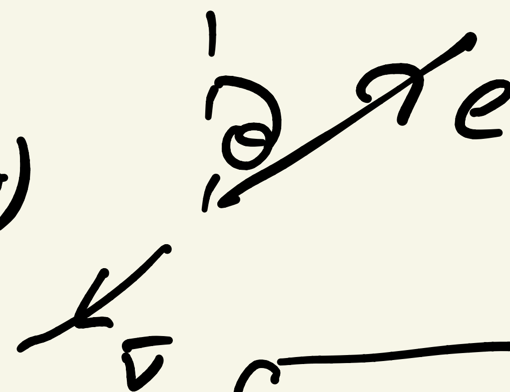
'83 Darulet, Della Negra, Rubbia

hadron collider $p-\bar{p}$ (SPS)

(~ 7 km)

$$W \to e + \bar{\nu}$$

$$\frac{d\Gamma}{d\Omega} = f(\theta)$$



Homework

Neutrino mass

$n \to p + e + \bar{\nu}_e$ ← measure mass

$\Updownarrow$

$N_i \to N_f$ -"-

nuclei $\quad E_e + E_\nu = M_i - M_f$

$$E_e = m_e + T_e \quad \leftarrow \text{kinetic } E$$

$$T_e + E_\nu = Q \qquad \boxed{Q \equiv M_i - M_f - m_e}$$

$$d\Gamma \propto \cdots \underline{E_\nu}\,\underline{p_\nu}$$

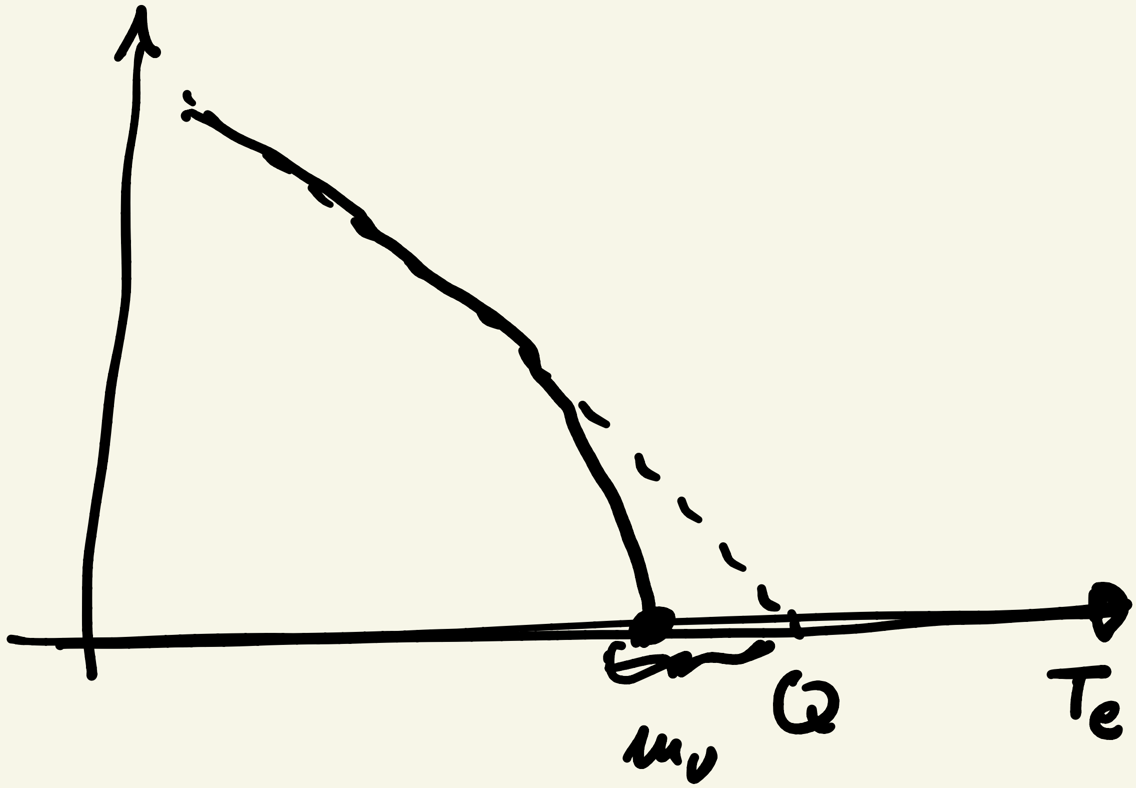
$$p_\nu = \sqrt{E_\nu^2 - m_\nu^2} \qquad E_\nu = Q - T$$

$$d\Gamma \propto \underbrace{(Q-T)\sqrt{(Q-T)^2 - m_\nu^2}}_{K^2}$$

Curie function

$$K = \left[(Q-T)\sqrt{(Q-T)^2 - m_\nu^2}\right]^{1/2}$$

$$= Q - T, \quad m_\nu = 0$$



- Q as small as possible
- $\tau_{1/2} \simeq 10\,yr$

$$^{3}_{1}H \longrightarrow {}^{3}_{2}He + e + \bar{\nu}_e$$

$$\boxed{Q \simeq 18.6\ keV}$$

$\boxed{T \gtrsim Q - \sim eV}$ $\boxed{\text{limits} \sim eV}$

$\boxed{\text{Troitzk Mainz} \quad 2.4\,eV \gtrsim m_\nu}$

KATRIN $\boxed{m_\nu \leq 1.1\,\mathrm{eV}}$

KArlsruhe TRItium Neutrino

- cosmology limit
- $0\nu 2\beta$ neutrino-less double beta

$\sim$ eV

$\longleftrightarrow$ LHC scale

$\boxed{\cdot\ q, l \text{ mix}}$ $\boxed{1 \text{ generation}}$

$\boxed{u, d, e, \nu}$ $\boxed{\gamma}$

$\boxed{\Delta m^2 \neq 0 \quad \text{oscillation}}$