Neutrino BBSM course

Lecture I

LMO

Spring 2020

It's neutrino, stupid!

Lecture I

21/4/2020

- $m_\nu = 0 \Leftarrow$ SM (Standard Model)

  $m_\nu \neq 0$ — window to BSM

- theoretical $\left\{ \begin{array}{l} \text{positron} \\ e \longrightarrow \bar{e} \end{array} \right.$

- aloof

  λ = mean free path

  $\lambda_G \simeq 1\,m$ $\quad \lambda_e \ll cm$

$$\lambda_\nu \simeq 10^{20}\,\text{cm} \Big/ E^2(\text{MeV})$$

reactor bom

Natural units

$$\left[\begin{aligned} c &= 3\times 10^{10}\,\text{cm/sec} = 1 \\ \hbar &= 10^{-33}\,\text{J sec} = 1 \end{aligned}\right]$$

$$[m] = [E] = [d^{-1}]$$

$$r_c(p) = \frac{\hbar}{m_p c} \Rightarrow \frac{1}{m_p} \simeq 10^{-14}\,\text{cm}$$

$$m_p \simeq \text{GeV} \simeq 10^{-24}\,\text{gr}$$

$$r_c(G) \simeq \frac{1}{m_\odot} \simeq \frac{1}{100\,kg} \simeq \frac{1}{10^{29}\,GeV}$$

$$\simeq 10^{-43}\,cm$$

$$\boxed{GeV\,cm \simeq 4 \times 10^{14}\,cm}$$

$$V_{em} \simeq \frac{e^2}{4\pi}\frac{1}{r} = \alpha_{em}\frac{1}{r} \qquad \alpha_{em} \simeq \frac{1}{100}$$

$$V_{gr} \simeq G_N \frac{m_1 m_2}{r} \longrightarrow G_N = \frac{1}{M_p^2}$$

$$M_p = 10^{19}\,GeV$$

$$\boxed{\text{unit of mass} = m_{proton}} \qquad m_p \simeq GeV$$

$$V_{gr} \simeq G_N \frac{m_p^2}{r} \simeq 10^{-38}/r \simeq 10^{-36}\, V_{em}$$

proton – protons

irrelevant!

LHC $E \simeq 10^3\,\text{GeV}$

Why does gravity matter?

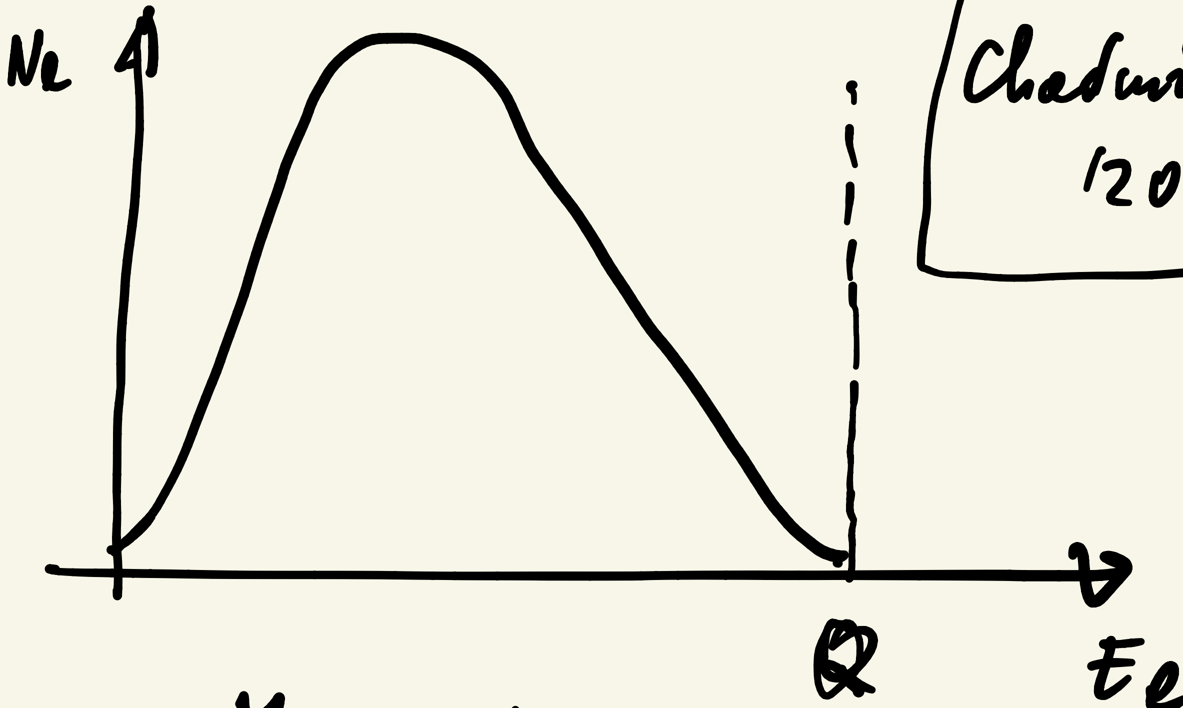
Why does matter gravitate?

$M_\odot \simeq 10^{57}\,\text{GeV}$

β decay

$Co \to Ni + e$

Chadwick
'20s



$$M_i = M_f + E_e = M_f + m_e + T_e$$

$$\boxed{Q \equiv M_i - M_f - m_e}$$

kinetic energy

$$Q = T_e$$

Tubingen

December 4, '30

Pauli

$\boxed{\text{"Not even wrong"}}$

Fermi '34 — $\boxed{\text{Newton}}$

$\boxed{\text{"Natural philosophy"}}$

effective theory of em

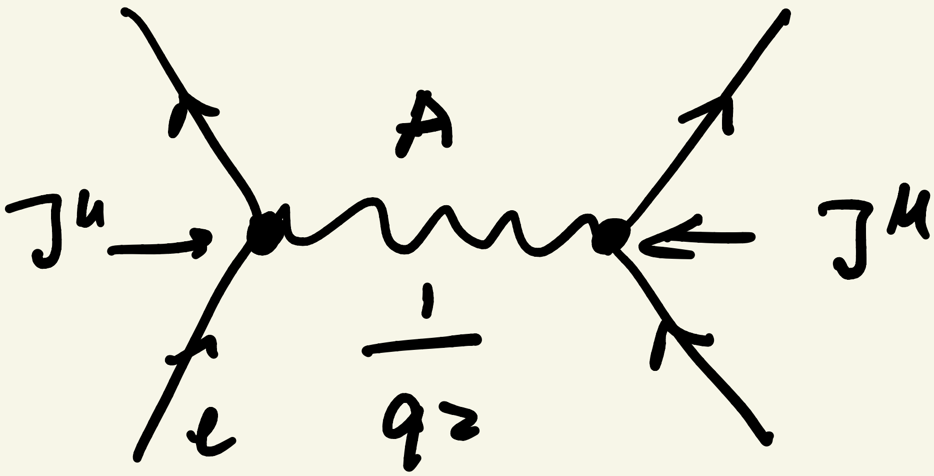
$$V_{em}(r) \simeq \frac{\alpha}{r}$$

$$\int d^3r \, \frac{1}{r} e^{i\vec{q}\cdot\vec{r}} \propto \frac{1}{\vec{q}^2} \quad \text{NR}$$

Rel $\quad \dfrac{1}{q^2} \qquad q^2 = q_0^2 - \vec{q}^2$

$$\left[\frac{1}{q^2} \right] = \frac{1}{M^2}$$

REL em $\longrightarrow$ $J_\mu J^\mu \frac{1}{q^2} = \mathcal{H}_{eff}^{em}$



$$\boxed{q \simeq \text{MeV}}$$

by analogy

$$\mathcal{H}_{eff}^{W} \simeq G_F \, J^W \, \bar{J}_W$$

$$G_F \simeq \frac{1}{\Lambda_F^2} \simeq 10^{-5}\,\text{GeV}^{-2}$$

$\Lambda_F \sim 100\,\text{GeV}$

$$\mathcal{H}_{em} \simeq \frac{1}{q^2} \qquad \mathcal{H}_W \simeq G_F$$

$$\sigma_{em} \simeq \frac{1}{q^2} \qquad \sigma_W \simeq G_F^2\, q^2$$

$$\boxed{\begin{array}{l} G_F \simeq 10^{-5}\ \mathrm{GeV}^{-2} \\ q \simeq \mathrm{MeV} \end{array}}$$

$$\boxed{\sigma_W \simeq 10^{-22}\, \sigma_{em}} \qquad \underline{q \simeq E = \mathrm{MeV}}$$

$$\lambda_W \simeq \lambda_\nu \simeq \frac{1}{\sigma \cdot n \cdot v} \simeq 10^{20}\ \mathrm{cm}$$

$$\boxed{J_\mu = \bar{e}\gamma_\mu e \quad (e = \psi_e)}$$

spinor

$$\boxed{n \to p + e + \bar{\nu}_e}$$

$m_p \simeq \text{GeV}$

$m_n \simeq \text{GeV} \gtrsim m_p$

$\boxed{\text{digression}}$

$$\boxed{\underbrace{n,\ p}_{\text{Baryons}}}$$

$$\underbrace{\pi,\ K}_{\text{Mesons}}$$

hadron = thick

$$\underbrace{e,\ \nu_e}_{\text{leptons}}$$

lepton = thin

- $\boxed{\Delta B = 0, \quad \Delta L = 0}$

$$\boxed{m_e \simeq m_n \simeq m_p \simeq 10^3 m_e}$$

$$J_W = \bar{p}\, O_B\, n + \bar{\nu}\, O_L\, e$$

$$O = \gamma_\mu,\ 1,\ \gamma_5,\ \gamma_\mu\gamma_5$$

em

W^+

$$\boxed{\begin{aligned} &Q_n = 0;\ Q_p = 1 \\ &Q_e = -1,\ Q_\nu = 0 \end{aligned}}$$

Fermi $\longleftrightarrow$ Newton

$E \approx$ MeV $\longleftrightarrow$ $M_W \simeq 100$ GeV

makes no sense

- neutrino
- $\theta_W = ?$ θ_B, θ_L

$$\boxed{1956 - '57}$$

Cowan, Reines '56

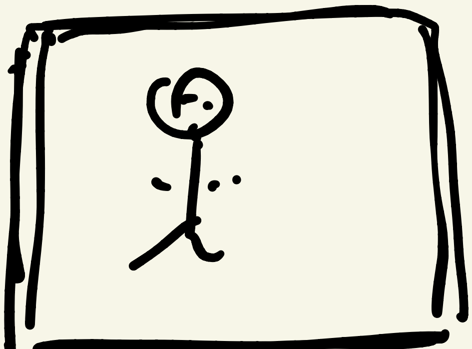
(reactor neutrino)

Pontecorvo '40s

$$\Phi \simeq 10^{13} / \text{cm}^2\,\text{sec}$$

$$n \to p + e + \bar{\nu}_e$$

$$\boxed{\bar{\nu}_e + p \to n + \bar{e}}$$



water

"good old days"

$$\#\text{ of events} = \sigma_w \cdot n \cdot \Phi \cdot V$$

cross section

density

$$\sigma_w \simeq 10^{-44}\,\text{cm}^2 \iff G_F \simeq 10^{-5}\,\text{GeV}^{-2}$$

$$[\, E \simeq \text{MeV} \,]$$

$$n \simeq 10^{24}/\text{cm}^3 \qquad V \simeq 10^{5}\,\text{cm}^3$$

$$\simeq 10^{-44} \cdot 10^{24} \cdot 10^{13} \cdot 10^{5}/\text{sec} \simeq 10^{-2}/\text{sec}$$

$$\simeq 100/\text{hour}$$

· 1956 — PARITY VIOLATION

T.D. Lee, C.N. Yang

1957 Marshak, Sudarshan

$$\boxed{O_B = O_L = \gamma_\mu \frac{1+\gamma_5}{2}}$$

Spinors

$\nu, e,$

u u d

p

u d d

n

$$\boxed{\begin{aligned} e_d &= -\tfrac{1}{3} \\ e_u &= \tfrac{2}{3} \end{aligned}}$$

$$\boxed{d \to u + e + \bar{\nu}_e}$$

spin ½ = spinors

$\psi \to \Lambda \psi$ Lorentz

$$\Lambda = \exp(i\,\theta_{\mu\nu}\Sigma^{\mu\nu})$$

$$\Sigma_{\mu\nu} = \frac{1}{4i}[\gamma_\mu, \gamma_\nu]$$

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} \;(\eta_{\mu\nu})$$

$$g_{\mu\nu} = \eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$$

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix} = \begin{pmatrix} 0 & \sigma^\mu_+ \\ \sigma^\mu_- & 0 \end{pmatrix}$$

$$\sigma^\mu = \sigma^\mu_+ = (1; \vec\sigma)$$

$$\bar\sigma^\mu \equiv \sigma^\mu_- = (1; -\vec\sigma)$$

$$\{\gamma_5, \gamma_\mu\} = 0 \qquad \gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3$$

$$\gamma_5^2 = 1$$

$$\Rightarrow [\gamma_5, \Sigma_{\mu\nu}] = 0 \qquad L(R) = \frac{1 \pm \gamma_5}{2}$$

$$\psi \longrightarrow \psi_L \equiv L\psi \Rightarrow \text{Lorentz inv.}$$

$$\boxed{L^2 = L, \quad R^2 = R, \quad LR = 0}$$

$$\gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \qquad L = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

$$R = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}$$

$$\boxed{\psi_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix}, \quad \psi_R = \begin{pmatrix} 0 \\ u_R \end{pmatrix}}$$

$$\boxed{\text{Fund.} = u_L, u_R}$$

- Parity = LR symmetry

P $\qquad \psi \xrightarrow{P} \gamma_0 \psi$

$$\boxed{\begin{gathered} A_i \to -A_i, \quad A_0 \to A_0 \\ \vec{x} \to -\vec{x}, \quad t \to t \end{gathered}}$$

$$\Leftrightarrow \quad A_\mu \bar{\psi} \gamma^\mu \psi \qquad \boxed{\bar{\psi} \equiv \psi^\dagger \gamma^0}$$

- <u>Charge conjugation</u>

$$\psi \to \psi^c \equiv C \bar{\psi}^T = C \gamma_0 \psi^*$$

$$\boxed{\psi^c \to \Lambda \psi^c} \qquad \boxed{\text{spinor}}$$

$$\psi \to \Lambda \psi \qquad \Lambda = \exp \cdots$$

$$\boxed{u_{L,R} \to e^{i\vec{\sigma}/2 \cdot (\vec{\theta} \pm i\vec{\varphi})} u_{L,R}}$$

$\boxed{\text{ROT}}$ $\boxed{\text{BOOST}}$

$$\varphi_i = \Theta_{0i}\,, \quad \theta_i \equiv \varepsilon_{ijk} A_{jk} \frac{1}{2}$$

Boost

Euler angles

z direction $\boxed{v = \tanh \varphi_3}$

DIRAC $\boxed{\psi = \psi_L + \psi_R = \begin{pmatrix} u_L \\ u_R \end{pmatrix}}$

$$\boxed{\mathcal{L}_D = i \bar{\psi} \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi}$$

$$m \bar{\psi} \psi = m \left(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \right)$$

$$\bar{\psi} \gamma^\mu \partial_\mu \psi = \bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + L \leftrightarrow R$$

$$\boxed{i \gamma^\mu \partial_\mu \psi = m \psi}$$

$\boxed{u_L, u_R}$ $\boxed{m=0}$ $\boxed{E \gg m}$

electron at LHC $E_e \simeq 10^3$ GeV

$m_e = 0$

momentum space $\psi(x) = e^{-ipx}\tilde{\psi}(p)$

$\Rightarrow p_\mu \gamma^\mu \tilde{\psi}(p) = 0$

$$\begin{pmatrix} 0 & E - \vec{\sigma}\cdot\vec{p} \\ E + \vec{\sigma}\cdot\vec{p} & 0 \end{pmatrix} \begin{pmatrix} u_L \\ u_R \end{pmatrix}_{(p)} = 0$$

$\Rightarrow E\, u_{L,R} = \mp \vec{\sigma}\cdot\vec{p}\, u_{L,R}$

$E = |\vec{p}|$

$\vec{S} = \vec{\sigma}/2$ $\boxed{h \equiv \vec{S}\cdot\hat{p} \quad \text{helicity}}$

$\Downarrow$

$$\boxed{h\, u_{L,R} = \mp \frac{1}{2} u_{L,R}}$$

defines L and R

$$L = \frac{1+\gamma_5}{2} \quad \left(\begin{array}{l} \gamma_5^2 = 1 \\ \gamma_5 \text{ — up to a sign} \end{array} \right)$$

$$\psi^c = C \bar{\psi}^T$$

$$\psi^c \to \Lambda \psi^c$$

$$C = i \gamma^2 \gamma^0$$

$$\boxed{\begin{array}{c} C \gamma^\mu C^T = -\gamma_\mu^T \\ C^T C = C^+ C = 1 \\ C^T = -C \end{array}}$$

$$\bar{\psi}^T = \gamma_0 \psi^*$$

$$\bar{\psi} = \psi^+ \gamma^0$$

$$\Rightarrow \boxed{\psi^c = i \gamma_2 \psi^*}$$

$$\psi_L \xrightarrow[C]{} \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix} \begin{pmatrix} u_L \\ 0 \end{pmatrix}^* = \begin{pmatrix} 0 \\ -i\sigma_2 u_L^* \end{pmatrix}$$

$\boxed{C: \quad L \longleftrightarrow R}$

$\nu_L \rightarrow (\nu^c)_R = (\bar{\nu})_R$

$\boxed{e_L \,,\; (e^c)_R}$

Summary

$\underbrace{u_L \,,\; u_R}$ $\quad\quad u_L \xleftrightarrow{P} u_R$

$\Downarrow$

spinors $\Rightarrow$ same under ROT

$\Rightarrow$ opposite under BOOST