BBSM Neutrino Course

Lecture IX

May 22, 2020

LMU

Spring 2020

It's neutrino, stupid!

Weak interactions $\Leftrightarrow$ messenger $\begin{pmatrix} W^{\pm} \\ Z \end{pmatrix}$

neutral

- $M_W \neq 0 \neq M_Z$
- $\underline{SU(2)_L}$ symmetry

QED $\longleftrightarrow$ $U(1)_{em}$

QFD $\longleftrightarrow$ $SU(2)_L$

flavour

$\underbrace{\nu \longleftrightarrow e}_{\text{lepton flavour}}$

$$\boxed{\text{Massive gauge fields}}$$

$$\boxed{U(1)}$$

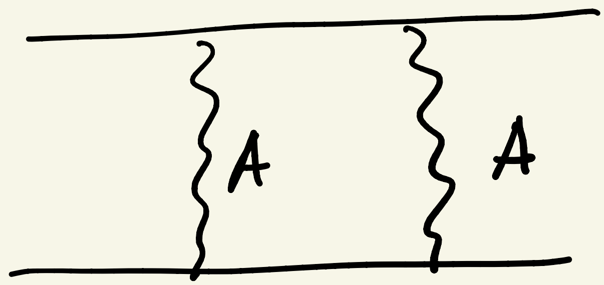
(i) $\mathcal{L}_P = -\frac{1}{4} F^2 + \frac{1}{2} m_A^2 A^2$

(a) $\sum_{i=1}^{3} \epsilon_\mu^{(i)} \epsilon_\nu^{*(i)} = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m_A^2}$

$$\boxed{m_A \to 0 \quad \text{divergent}}$$

(b) $\Delta_{\mu\nu} = -i \frac{\sum \epsilon_\mu \epsilon_\nu^*}{k^2 - m_A^2}$

$\xrightarrow[k \to \infty]{} \frac{1}{m_A^2}$ $\boxed{\text{divergence}}$



$$\int d^4k \; I(k) = \text{finite}$$

$$\underset{k\to\infty}{\hookrightarrow 0}$$

$$\text{(ii)} \quad \mathcal{L}_D = -\frac{1}{4}F^2 + \frac{1}{2}m_A^2 \tilde{A} + \boxed{g^2 \tilde{A}^4}$$

$$\tilde{A} = \underset{\substack{\uparrow \\ \text{Maxwell}}}{A} + \frac{1}{m}\partial G$$

$$A \to A + \partial\alpha, \qquad G \to G - m\alpha$$

$$\text{(iii)} \quad \mathcal{L}_3^G = -\frac{1}{4}F^2 + \frac{1}{2}m_A^2 \underbrace{\left(A + \frac{1}{m}\partial G\right)^2}_{\Downarrow}$$

$$+ \mathcal{L}_{gf}$$

$$\Downarrow \qquad - m_A (\partial^\mu A_\mu) G$$

$$\Downarrow$$

$$\mathcal{L}_{gf} = \frac{1}{2\xi} (\partial A + \xi m_A G)^2$$

$$\Downarrow$$

$$D(G) = \frac{i}{k^2 - m_A^2 \xi} \Leftarrow \underline{\text{not}} \text{ physical}$$

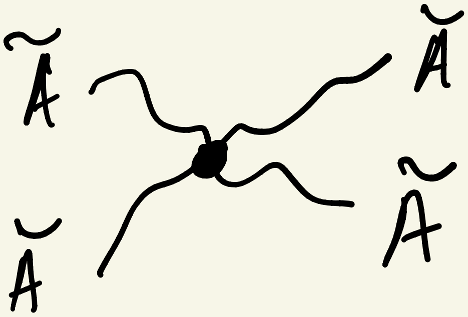
$$\Delta_{\mu\nu}(A) = -i \, \frac{g_{\mu\nu} + (\xi - 1) \frac{k_\mu k_\nu}{k^2 - \xi m_A^2}}{k^2 - m_A^2}$$

$$\boxed{\text{Proca}: \lim \xi \to \infty \Leftarrow \text{after}}$$

$$\boxed{\text{Physics does } \underline{\underline{\text{not}}} \text{ depend on } \xi}$$

$$\bullet \quad \sum \epsilon_\mu \epsilon_\nu^* = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m_A^2}$$

<u>U(1)</u>



$$j_\mu A^\mu$$

$$= 0$$

$$\partial^\mu j_\mu = 0$$

$$\Downarrow$$

$$k^\mu j_\mu = 0$$

$$\epsilon_\mu^{(L)} = \left(\frac{p}{m} ; 0, 0, \frac{E}{m} \right) \qquad p = p_3$$

longitudinal

$$E \simeq p + \frac{m^2}{2p} + \cdots$$

$$p \gg m$$

$$\boxed{\epsilon_\mu^{(L)} \simeq p_\mu / m_A}$$

$$\boxed{SU(2) = \text{non Abelian}}$$

$$[T_a, T_b] = i\,\epsilon_{abc}\, T_c$$

$$D_\mu = \partial_\mu - ig\, T_a A_\mu^a$$

$$\boxed{\text{charged}}$$

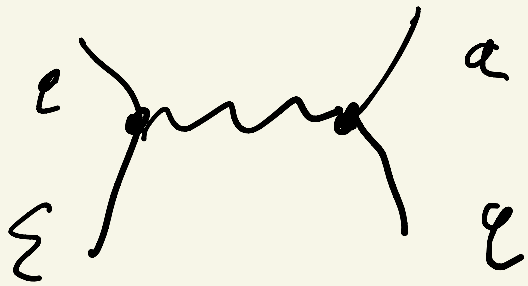
$$j_\mu^a = \bar{\psi}\gamma^\mu T_a \psi \Leftrightarrow j_\mu^{em} = \bar{\psi}\gamma^\mu Q \psi$$

$$\partial_\mu j^{\mu a} = 0 \not\Rightarrow$$

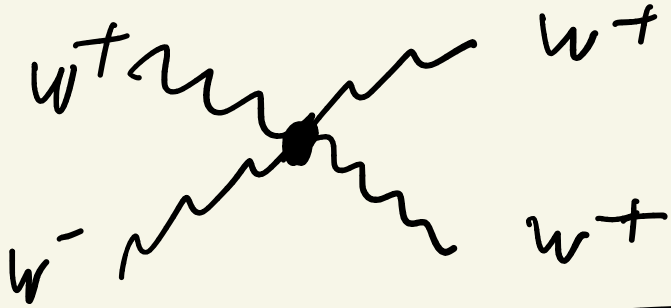
$$\boxed{D^\mu j_\mu^a = 0}$$

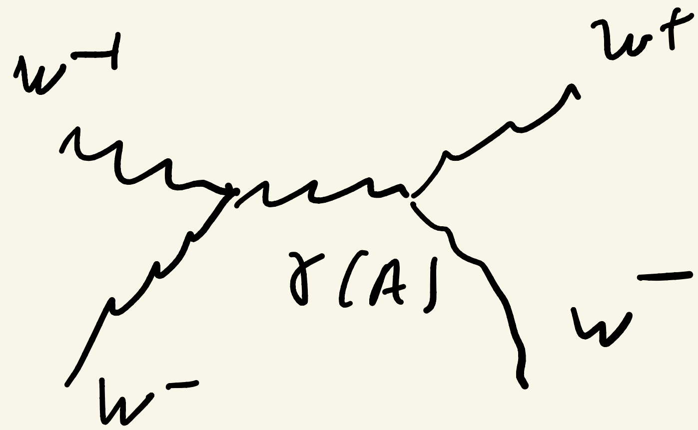
$$\boxed{j_\mu^a = \bar{\psi}\gamma^\mu T^a \psi + g\epsilon^{abc} F_{\mu\nu}^b A^{\nu c}}$$

QED: $\cancel{F_{\mu\nu} A^\nu}$

$W^\pm \longrightarrow$ 

↳ $g_{\mu\nu}$ matters





$$\boxed{\epsilon_\mu^L \sim p^\mu/m} \qquad \boxed{p^2 = q^2 = 1}$$

$$\cancel{\frac{q^4}{m_A^4}} + \frac{q^2}{m_A^2} + O(1)$$

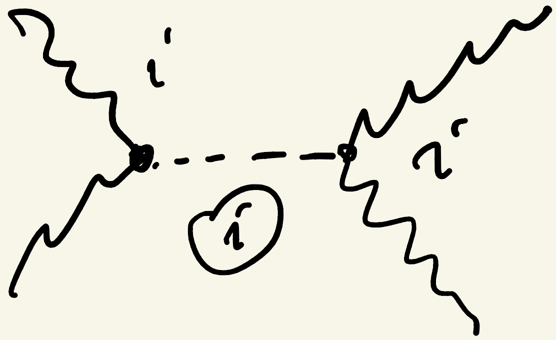
good

(i)

Scalar

$\mu h A_\mu A^\mu$

↳ dim. mass



↳ $\dfrac{\mu^2}{q^2 - \cancel{m_h^2}}$ $\qquad \dfrac{\underbrace{\epsilon_L \epsilon_L \epsilon_L \epsilon_L}\, q^4}{m_A^4}$

$q^2 \gg m_A^2, m_h^2 \ldots$

(**) $\simeq \dfrac{\mu^2}{m_A^4} q^2 (-1)$

$\boxed{\mu = g m_A \Rightarrow \text{cancel!}}$

(*) against (**)

$\boxed{h \longleftrightarrow \text{couples to mass}}$

$\boxed{h \bar{e} e \dfrac{m_e}{M_W}}$

h - dim 1

$\bar{e}_L = F4\ (3)$

$$\boxed{h \bar{b} b \frac{m_t}{M_W}}$$

Massive gauge fields

- gauge inv. $\quad m_A \left(A + \frac{1}{m} \partial G \right)^2$

$$\Downarrow$$

A, G — good high E propagators !

$\xrightarrow[k \to \infty]{} \frac{1}{k^2}$

- non - unitary ! $\quad \sigma \sim \frac{q^2}{m_A^2}$

$\epsilon_\mu \sim \mathcal{P}^\mu / m_A$

- solution : add a h

$$g\, m_A\, h\, A_\mu A^\mu$$

healthy Theory

- $E \to \infty$: μ / E = finite

small

$$\frac{\mu^2}{q^2} \frac{q^4}{m_A^2} + \text{---} \text{———}$$

$$g\, m_A \equiv \mu$$

cancel

- $g\, h\, m_A\, A_\mu A^\mu$ — true at all T

$\boxed{\phi \in R}$

$$V = \pm \frac{\mu^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4 \quad (\phi \to -\phi)$$

$+\circledast$ symmetry

$$\Downarrow$$

$$\frac{\partial V}{\partial \phi}\Big|_{\phi_0} = 0 \Rightarrow \phi_0^2 = \mu^2/\lambda$$

$$\boxed{\phi = \phi_0 + h}$$

dim. of mass

$$\boxed{\text{masses} \propto \phi_0}$$

$$\boxed{T \gg \mu \quad (\phi_0)}$$

Kirzhnits '72
Linde - -
Weinberg '74
Dolan, Jackiw '74

$k = 1$ (Boltzmann)

$\sigma(T) = 1$ (mass)

$$\boxed{V_T = V_{(0)} + a_T T^2 \phi^2 + c T^4}$$

$a = \lambda$

$(a > 0)$

$\cancel{m \sigma\, T^3}$

$$V = \cancel{-\frac{\mu^2}{2}\phi^2} + aT^2\phi^2 + \lambda\phi^4 + + T^4$$

$$\boxed{\frac{\partial V}{\partial \phi} = 0 \Rightarrow \phi = 0}$$

$\cancel{hAA}$

$m_A \to 0$

O_4

$$a_T = \lambda + g^2 + y^2 > 0$$

$\boxed{\phi_1\ \phi_2}$

$$V = \frac{\lambda_1}{4}\phi_1^4 + \frac{\lambda_2}{4}\phi_2^4 + \frac{\lambda_3}{2}\phi_1^2\phi_2^2$$

$\lambda_1, \lambda_2 > 0$, $\boxed{\lambda_3 < 0}$

$\lambda_1 \lambda_2 - \lambda_3^2 > 0$

$\phi_1 \to -\phi_1$

$\phi_2 \to -\phi_2$

$a_T^1 \phi_1^2 T^2 + a_T^2 \phi_2^2 T^2$

$a_T^1 = \lambda_1 + \lambda_3 < 0$

$a_T^2 = \lambda_2 + \lambda_3$

Rochelle salt

$T \to$ no melting

thermal bath $\boxed{E_{av} \sim T}$

D: $\phi_i \to -\phi_i \Longleftrightarrow$ domain walls

G (non Abelian) $\Longleftrightarrow$ magnetic monopoles

$\boxed{\text{dw, monopole}}$ problems

Dvali, G.S. '94-'95

Melfo '95

NO

$\boxed{\text{symm. broken at high } T}$

$\Downarrow$

$\boxed{\text{Neutrino mass}}$

door to new !

$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$ W-boson $\| \; \nu_R \; e_R$ ← add

$\boxed{m_\nu \neq 0}$ $\boxed{\text{oscillation}}$

• $\nu_L^T C \nu_L$ Majorana

breaks $SU(2)_L$

• electron $\boxed{\bar{e}e = \bar{e}_L e_R + \bar{e}_R e_L}$

Higgs

$m_e(\bar{e}_L e_R + h.c.)$ electron mass

$m_\nu(\bar{\nu}_L \nu_R + h.c.)$ neutrino mass

$Q\, \nu_R = 0$ (neutral)

$T_3\, \nu_R = 0$

sterile

phantom

$m_\nu \leq 1\,eV$

$m_W = 10^{11}\,eV$

Higgs

$h\bar{e}e \quad \frac{m_e}{M_W}$ $\boxed{h \to \bar{e}e}$

$h\bar{\nu}\nu \quad \frac{m_\nu}{M_W}$ $\boxed{h \to \bar{\nu}\nu}$

$\lesssim 10^{-11}$

$$\boxed{B(h \to \nu\bar{\nu}) \lessapprox 10^{-22}}$$

$\cdot\ h b \bar{b} \ \frac{m_b}{M_W}, \quad h WW \ M_W$

10^{-1} $\longleftrightarrow$ $\boxed{m_h \simeq 125\ \text{GeV}}$

$$\boxed{B(h \to \nu\bar{\nu}) \leq 10^{-20}}$$

$$\boxed{\nu_L \longleftrightarrow \nu_R} \qquad Q = 0, \quad T_3 = 0$$

$$(\bar{\nu}_L \nu_R + h.c.) + \boxed{\underbrace{\nu_R^T C \nu_R}}$$

Lorentz, QED, $SU(2)_L$ — I'W

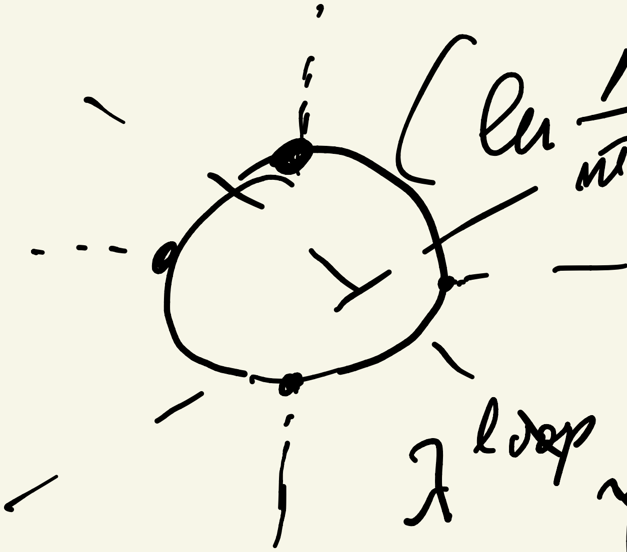
- $\underbrace{\nu_L^T C \nu_L}_{\cancel{SU(2)_L}}$

- $\underbrace{e_L^T C e_L}_{\cancel{\text{charge}}}$

no QED

consistent perturbative QFT

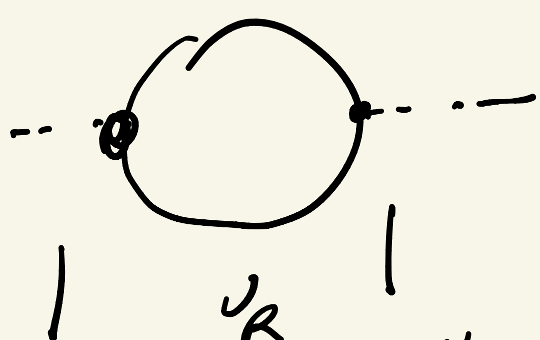
$\underline{\lambda h^4}$



$\lambda^{\text{loop}} \sim \left(\frac{m_f}{M_W}\right)^4$

$\updownarrow$

$\underbrace{(10^{-11})^{44}}_{10^{-44}}$



$\Lambda \simeq M_{Pl}$

$\ln \Lambda/m_h \simeq 100$

$\Downarrow$

$$\boxed{m_D(\bar{\nu}_L \nu_R + h.c.)} + \boxed{M_R \nu_R^T C \nu_R}$$

$M_R \gtrsim M_W \quad (\gg M_W)$

$\boxed{\bullet\ M_R \to 0 \Rightarrow \text{Lepton Number}}$

$$\underline{\cdot\,(\bar{e}_L e_R + \bar{e}_R e_L)}$$

$$\text{"}\ m_{e_L} = m_{e_R}\ \text{"}$$

$\cancel{e_L^T C e_L}$

$\cancel{e_R^T C e_R}$

charge inv.

$\boxed{m_e = m_D^e}$ $\Leftrightarrow$ $SU(2)_L \times U(1)$ symmetry

$\boxed{m_q = m_D^q}$

<u>NO</u> Majorana for charged fermions

$m_D^e (\bar{l}_L l_R + h.c.) + \cancel{e_R^T C e_R}$

$U(1)_Q$

$$\boxed{\Delta_R^{++} e_R^T C e_R}$$

$\Uparrow$

exp.

$$\boxed{\begin{gathered} m_R^e \, e_R^T C e_R \\ \Downarrow \\ \frac{m^e_R}{m^e_D} \leq 10^{-20} \end{gathered}}$$

$\langle \Delta_R^{++} \rangle = v_\Delta$

$\frac{v_\Delta}{M_W} \leq 10^{-20}$

Higgs $\boxed{\delta^{++} \therefore m_{\delta^{++}} \simeq v_\Delta}$

$$\boxed{m_\Delta \simeq e\, v_\Delta \simeq 10^{-14}\,eV}$$

$$v_\Delta \leq 10^{-14}\,eV$$

$$\Rightarrow \boxed{m_{\delta^{++}} \leq 10^{-14}\,eV}$$

$$SU(2)_L \times U(1)$$

$$\Downarrow$$

$$M_R\, \nu_R^T C \nu_R$$