BBSM Neutrino Course

Lecture VII

LMU
Spring 2020

It's neutrino, stupid!

Lecture 7

Massive gauge field

$U(1)$

Proca theory → gauge theory

$\cancel{\mathcal{L}}$ explicitly: $G_F \to \frac{g}{\sqrt{2}} J_\mu^W W_\mu^+$ + h.c.

$$J_\mu^W = \bar{u}_L \gamma_\mu d_L + \bar{\nu}_L \gamma_\mu d_L$$

$\cancel{P}$ maximal (NO f_R ???)

$M_W = 80\,\mathrm{GeV}$

$$\mathcal{L}_{Proca} = \frac{1}{2} A_\mu \left[(\Box + m^2) g^{\mu\nu} - \partial^\mu \partial^\nu \right] A_\nu$$

$\Downarrow$

$$\boxed{\begin{aligned}(\Box + m^2) A_\mu &= 0 \\ \partial^\mu A_\mu &= 0 .\end{aligned}}$$ (1) (2)

(1) $E^2 = \vec{p}^{\,2} + m^2$ $\quad A_\mu = e^{-ipx} \epsilon_\mu(p)$

(2) $p^\mu \epsilon_\mu = 0 \Rightarrow$ $\boxed{\text{at rest} \quad \epsilon_0 = 0}$

$$\boxed{\epsilon_\mu = (0; \epsilon_1, \epsilon_2, \epsilon_3)}$$

$$\frac{1}{2} \epsilon_\mu \left[(-p^2 + m^2) g^{\mu\nu} + p^\mu p^\nu \right] \epsilon_\nu$$

in p-space

propogator = $\Uparrow$

= inverse of 2-form

$$\left[\left(-p^2+m_A^2\right)g_{\mu\alpha}+p_\mu p_\alpha\right]\Delta^{\alpha\nu}\equiv\delta_\mu^\nu$$

$$\Delta^{\alpha\nu}=A\,g^{\alpha\nu}+B\,\frac{p_\alpha p_\nu}{p^2}$$

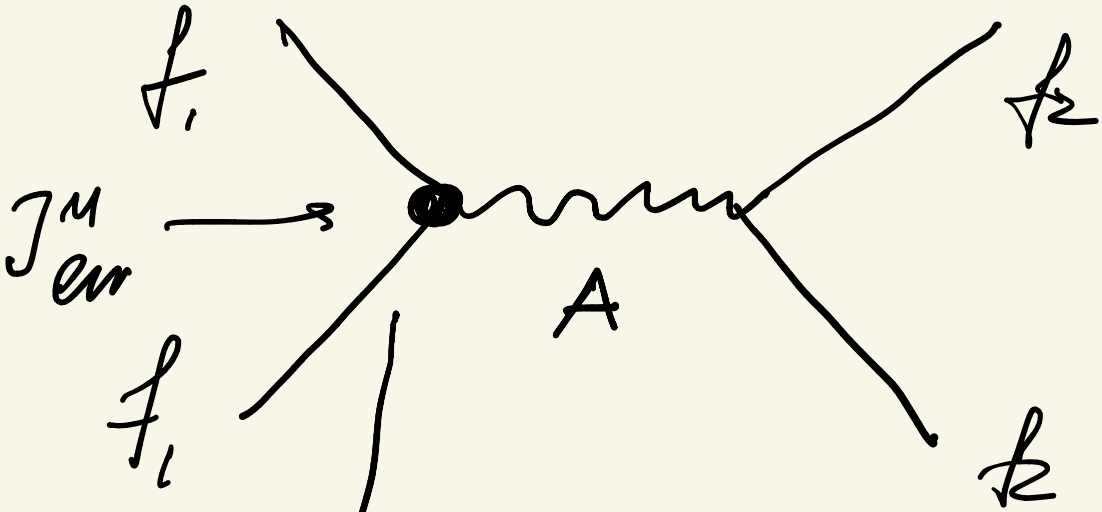
$$\Downarrow$$

$$\boxed{\Delta_{\mu\nu}=-i\,\frac{g_{\mu\nu}-\frac{p_\mu p_\nu}{m_A{}^2}}{p^2-m_A{}^2}}\qquad(3)$$

$$\boxed{p=m_A}$$

$$\Delta_{\mu\nu}\xrightarrow[p\to\infty]{}\frac{p_\mu p_\nu}{p^2}\,\frac{1}{m_A{}^2}\sim\frac{1}{m_A{}^2}$$

$$\boxed{m_A\to0\Rightarrow\Delta_{\mu\nu}\to\infty}$$



$\boxed{\text{QED}}$

$p_\mu J^\mu_{em} = 0$ in p-space

$\Leftrightarrow \partial^\mu J_\mu^{em} = 0$

$\boxed{\text{Imagine}}$

$J^\mu_{em} \longrightarrow J^\mu_L = J^\mu_W$

- $\bar{\psi}_L \gamma^\mu \psi_L A_\mu \quad (m_A \neq 0)$

$\partial_\mu \bar{\psi} \gamma^\mu \psi = 0$ (Dirac eq.)

$$\boxed{\partial^\mu \bar{\psi}\gamma^\mu\gamma_5\psi \;\propto\; m\bar{\psi}\gamma_5\psi}$$

$$i\gamma^\mu\partial_\mu\psi = m\psi \qquad \bar{\psi} \equiv \psi^\dagger\gamma^0$$

$$-i\partial_\mu\bar{\psi}\gamma^\mu = m\bar{\psi}$$

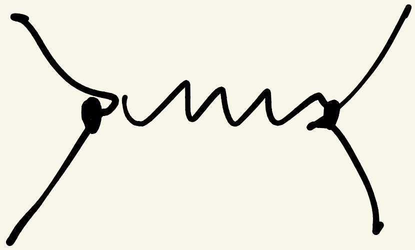
$$\partial_\mu \bar{\psi}\gamma^\mu\psi = \bar{\psi}m\psi - \bar{\psi}m\psi = 0$$

$$\partial_\mu \bar{\psi}\gamma^\mu\gamma_5\psi = \bar{\psi}m\gamma_5\psi + \bar{\psi}m\gamma_5\psi$$

$$\Downarrow$$

$$p^\mu \bar{\psi}_L\gamma^\mu\psi_L \propto m_f \bar{\psi}\gamma_5\psi$$

$\Rightarrow$ A_μ couples to non-conserved current

•  $\longrightarrow \left(\dfrac{m_f}{m_A}\right)^?$

$$\boxed{E \to \infty \iff m_A \to 0}$$

Polarizations of Proca

ϵ at rest $\qquad \epsilon_0 = 0$

• $\vec{V} \longrightarrow \vec{V} + \vec{V} \times \vec{\theta} \qquad$ (ROT)

• $\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} \longrightarrow O \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} \qquad O = e^{i\theta^a L_a}$

$a = 1, 2, 3$

$O \in R$

$O^T O = 1$, $\det O = 1 \implies \boxed{\begin{array}{l} L^* = -L \\ L^T = -L \end{array} \quad \mathrm{Tr}\, L = 0}$

$$\boxed{\begin{aligned}(L_i)_{jk} &= -i\,\epsilon_{ijk}\\ [L_i, L_j] &= i\,\epsilon_{ijk} L_k\end{aligned}}$$

- $A = v_a T_a \qquad T_a = \sigma_a/2 \ , \quad a = 1,2,3$

$v_a \in \mathbb{R}$

$$\boxed{A \to U A U^\dagger}$$

$\boxed{\text{Adjoint}}$

- $A^\dagger = A, \ \mathrm{Tr}\, A = 0$

$U = e^{i\theta_a \sigma_a/2} \qquad SU(2)$

$\boxed{SU(N)}$ — $\boxed{\text{Adjoint}}$ N^2-1

$a, b \ldots$

$i, j \ldots$

$\boxed{\text{digression}}$

$$(L_a)_{jk} = -i f_{ajk}$$

$$[L_a, L_b] = i f_{abc} L_c$$

$U = e^{i\theta_a T_a}$ $\boxed{T_a = \text{fundamental}}$

$\sigma/2$ $\boxed{\text{everywhere}}$

$\boxed{\text{Spin} \Leftrightarrow SO(3)}$

$$S_3 \equiv L_3 = \begin{pmatrix} 0 & -i & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$L_3\, \epsilon_T(+) = +\epsilon_T(+) \qquad \epsilon_T(+) = \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}$$

$$L_3\, \epsilon_T(-) = -\epsilon_T(-) \qquad \epsilon_T(-) = \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$$

$$L_3\, \epsilon_L(0) = 0 \qquad \epsilon_L(0) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\epsilon_T^{(+)} = (0;\ 1, i, 0)\frac{1}{\sqrt{2}}$$

$$\epsilon_T^{(-)} = (0;\ 1, -i, 0)\frac{1}{\sqrt{2}}$$

$$\epsilon_L(0) = (0;\ 0, 0, 1)$$

$$\boxed{\sum_{i=1}^{3} \epsilon_\mu^{(i)} \epsilon_\nu^{(i)*} = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m_A^2} \qquad k_0 = m_A,\ \vec{k} = 0}$$

$' = T(+), T(-), L(0)$

$$(\Box + m_A^2) A_\mu = 0 \quad (\partial^\mu A_\mu = 0)$$

Digression

$$\boxed{D(\phi) = \frac{i}{k^2 - m^2}}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2$$

$$\frac{1}{2}(-p^2 + m^2)$$

$$\Delta_{\mu\nu}(\text{Proca}) = i \frac{\sum_i \epsilon_\mu^{(i)} \epsilon_\nu^{(i)*}}{k^2 - m_A^2}$$

$$= i \frac{-g_{\mu\nu} + k_\mu k_\nu / m_A^2}{k^2 - m_A^2}$$

$\boxed{z\text{ - boost}}$

S_3

$$\epsilon_3' = \frac{\epsilon_3 + v\epsilon_0}{\sqrt{1-v^2}} \qquad \epsilon_2' = \epsilon_2$$

$$\epsilon_1' = \epsilon_1$$

$$\epsilon_0' = \frac{\epsilon_0 + v\epsilon_3}{\sqrt{1-v^2}}$$

$$\boxed{\epsilon_T' = \epsilon_T \quad (+,-)} \quad \boxed{\text{Transverse}}$$

$$\boxed{\begin{aligned}\epsilon_L' &= \left(\frac{v}{\sqrt{1-v^2}};\ 0,\ 0,\ \frac{1}{\sqrt{1-v^2}}\right)\\ &= \left(\frac{|\vec{p}|}{m};\ 0,\ 0,\ E/m\right)\end{aligned}}$$

dir. of motion

$$\boxed{\sum_{i=1}^{3} \epsilon_\mu^{(i)} \epsilon_\nu^{*(i)} = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m_A^2}}$$

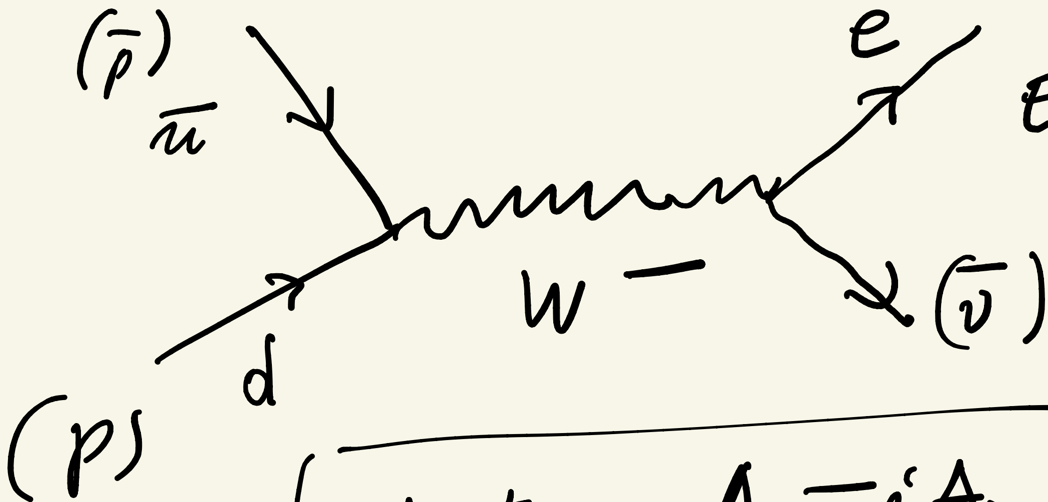
✓

$\boxed{\text{W born at rest}}$ $p + \bar{p}$ '83

$\boxed{E \simeq 100\,\text{GeV}}$ ○ 7 km

$$\frac{g}{\sqrt{2}} W_\mu^+ \left(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L \right)$$

$m_e = 0.5\,\text{MeV}$



$E_e \simeq \frac{M_W}{2}$

$E_{\bar{\nu}}$ //

$$\boxed{W^\pm \equiv \frac{A_1 \mp i A_2}{\sqrt{2}} \qquad A_1, A_2 = \text{Proca}}$$

$$\mathcal{L}_W = \frac{1}{2} \sum_{i=1}^{2} A_\mu^i \left[(\Box + m^2) g^{\mu\nu} \ldots \right] A_\nu^i$$

$$= W_\mu^+ \left[(\Box + m_W^2) g^{\mu\nu} - \partial^\mu \partial^\nu \right] W_\nu^-$$

$\boxed{\Uparrow \; S_z^W = +1}$

$\boxed{E \gtrsim 100 \text{ GeV}}$

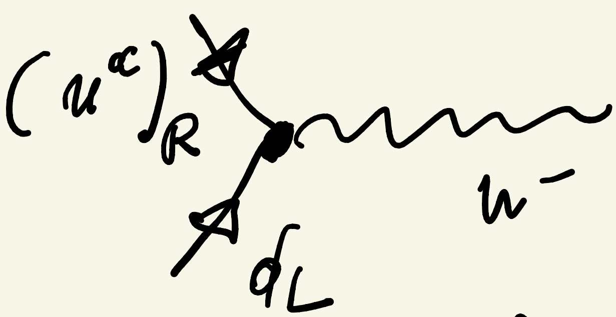
$$\boxed{\begin{array}{l} m_{p,n} \simeq 10^3 m_e \\ m_{u,d} \simeq \text{few MeV} \end{array}}$$

$$\boxed{\Rightarrow m_e = m_q = 0}$$

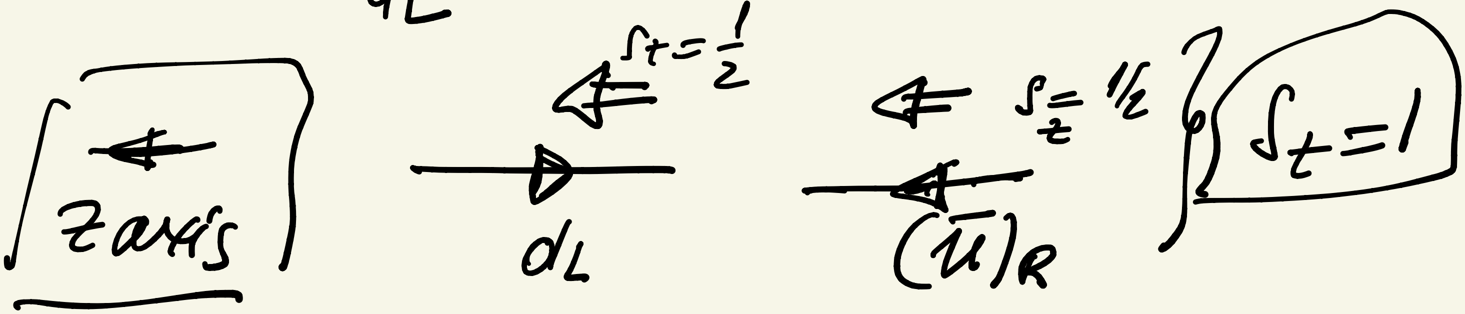
$$\boxed{m_f = 0} \Rightarrow \boxed{h = \text{chirality}}$$

$$\left[\begin{array}{l} h\, u_L = -\frac{1}{2} u_L \\ h\, u_R = +\frac{1}{2} u_R \end{array}\right] \qquad h \equiv \vec{S}\cdot\hat{p} \text{ helicity}$$

$$\boxed{\bar{u}_L \gamma^\mu d_L}$$



$$u^c \equiv C\bar{u}^T$$



$\boxed{S_z^W = +1}$

experiment = high E
p-p (p-$\bar p$)

$\boxed{W^- \longrightarrow e_L^- + (\bar\nu)_R}$ (quarks)

$\epsilon_\mu = (0; 1, i, 0)\frac{1}{\sqrt{2}}$

- $M_W = E_e + E_{\bar\nu}$
- $0 = \vec{p}_e + \vec{q}_\nu$

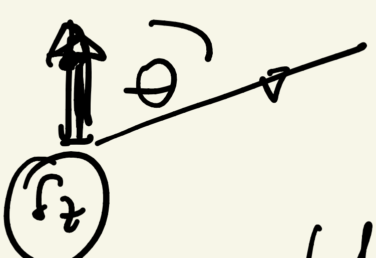
$\boxed{E_e = E_{\bar\nu} = |\vec{p}|}$

$\vec{p} = -\vec{q}$

$p_\mu = \frac{M_W}{2}\left(1; \cos\theta, \sin\theta\cos\varphi\ \ \sin\theta\sin\varphi\right)$

$q_\mu = \frac{M_W}{2}\left(\ ; -\qquad -\ ''\quad -\qquad\right)$

opposite



$|M|^2 = f(\theta, \varphi)$

$\phi = ?$

fix

$$= f(\theta)$$

$$\boxed{\begin{array}{l} k^0 = M_W \\ \vec{k} = 0 \end{array}}$$

$$\frac{d\Gamma}{d\Omega} = \frac{1}{4\pi^2}\frac{1}{2M_H}\int\frac{p^2 dp}{2E_p}\int\frac{d^3q}{2E_q}|M|^2$$

$$\delta^{(4)}(p+q-k)$$

$\underline{\text{Int.}}$ $$\frac{g}{\sqrt{2}}\epsilon_\mu \bar{u}(p)\gamma^\mu L v(q) \equiv \mathcal{M}$$

$$\sum_{s,s'}|\mathcal{M}|^2 = \frac{g^2}{2}\sum_{s,s'}\bar{u}(p)\not{\epsilon} L v(q) \times \bar{v}(q)\not{\epsilon}^* L u(p)$$

$$\sum v\bar{v} = \not{q} \quad (m_\nu = 0)$$

$$\sum u\bar{u} = \not{p} \quad (m_e = 0)$$

$$= \frac{g^2}{2}\mathrm{Tr}\,\not{\epsilon}\not{q}\not{\epsilon}^* L\not{p}$$

$$= \frac{g^2}{2}\mathrm{Tr}\,\not{\epsilon}\not{q}\not{\epsilon}^*\not{p}\frac{1-\gamma_5}{2}$$

$$\left[\begin{array}{l} \mathrm{Tr}\, \gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta = 4\left(g^{\mu\nu} g^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha}\right) \\ \\ \mathrm{Tr}\, \gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta \gamma_5 = -4i\, \epsilon^{\mu\nu\alpha\beta} \end{array}\right]$$

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu_+ \\ \sigma^\mu_- & 0 \end{pmatrix} \qquad \sigma^\mu_\pm = (1, \pm\vec{\sigma})$$

$$\gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \qquad \boxed{\begin{array}{l} L = \dfrac{1+\gamma_5}{2} \\ \\ R = \dfrac{1-\gamma_5}{2} \end{array}}$$

$$\boxed{|M|^2 = \dots\; ?}$$

$$\begin{array}{llll} \Uparrow & \cancel{\uparrow e_L \;\Downarrow} & & \downarrow e_L \;\Uparrow \\ S_z^W = +1 & \cancel{\downarrow (\bar{\nu})_R \;\Downarrow} & & \uparrow (\bar{\nu})_R \;\Uparrow \end{array}$$

$$\Downarrow$$

$$\frac{d\Gamma}{d\Omega}(W^- \to e\bar{\nu}) = \otimes$$

$$\frac{d\Gamma}{d\Omega}(W^- \to \bar{u} d) = 3\,\otimes$$

$$\boxed{\Gamma(W \to SM)}$$

$$u,\ d \longrightarrow \begin{pmatrix} u^r, & u^y, & u^b \\ d^r, & d^y, & d^b \end{pmatrix}$$

$\Longrightarrow$ $SU(3)_c$ quantum number

$$\boxed{\Gamma(W \to \dots)_{W\ \text{total}} \simeq \alpha_W M_W}$$

$$\boxed{\alpha_W = \frac{g^2}{4\pi} \simeq 10^{-2}}$$

$$\boxed{\begin{gathered}[\Gamma] = \text{mass} \\ \Gamma = 1/\tau \quad \hbar = c = 1\end{gathered}}$$

$\simeq$ GeV

$$\boxed{\Gamma_W \simeq 2\,\text{GeV}?}$$

$$\boxed{\text{weak int} \neq \text{weak}}$$

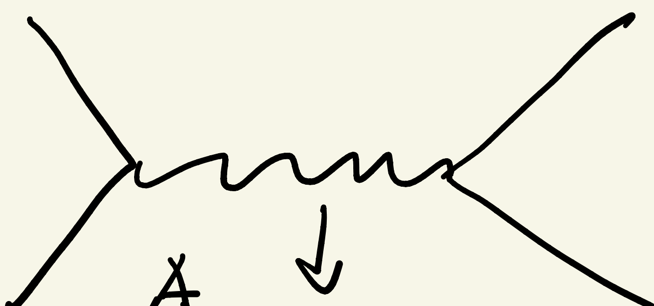
$$e A_\mu \underbrace{\bar\psi \gamma^\mu Q \psi}_{j^\mu_{em}} + \frac{g}{\sqrt{2}} W^+_\mu J^\mu_W$$

$e \simeq 0.3$

$\alpha_{em} = \dfrac{e^2}{4\pi} \simeq 1/100$

$\alpha_W = \dfrac{g^2}{4\pi} \simeq 1/30$

$$\boxed{\text{weak int} = \text{stronger than em!!??}}$$



$\dfrac{1}{q^2}$

photon

em

weak

$$\frac{1}{q^2 - M_W^2}\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right]$$

weak $\quad q \simeq \text{MeV}$ $\boxed{\text{history}}$

$M_W \simeq 100\ \text{GeV}$

$$\frac{1}{q^2} \simeq 10^{-5} \frac{1}{M_W^2} \Rightarrow \boxed{\frac{\text{weak}}{\text{em}} \simeq 10^{-10}}$$

λ_e (mean free path) $\simeq$ cm

λ_ν ( ) $\simeq 10^{20}$ cm

$\boxed{q \simeq \text{TeV}}$ $\quad \boxed{\frac{\text{weak}}{\text{em}} \gtrsim 1}$ $\quad \boxed{g^2 \gtrsim e^2}$

$$\frac{1}{q^2} \longleftrightarrow \frac{1}{q^2 - \cancel{M_W^2}}$$

(u, d, ν_e, e) $\quad (c, s, \nu_\mu, \mu)$ $\quad (\cancel{t, b}, \nu_\tau, \tau)$

$m_u \simeq 0$ $\quad m_s \simeq m_\mu \simeq 100\ \text{MeV}$; $m_c \simeq \text{GeV} = 0$ $\quad m_t \simeq 2 M_W$; $m_\tau = 0$

$W^- \to l\,\bar{\nu}$ $\qquad$ $W^- \to \bar{u}\,d$

$\cancel{\to \bar{t}\,b}$

$W \to$ '1983

top $\to$ '1996 $\to$ Tevatron