BBSM Neutrino Course

Lecture XXI

LMU

Spring 2020

Axial anomaly and B-L

<u>Gorey</u> $\mathcal{L}_Y^G = \bar{\psi}_L Y \phi \psi_R + \bar{\psi}_R Y^* \phi^* \psi_L$

$\phi \in \mathbb{C}$ general

$\psi_L \to e^{i\alpha} \psi_L, \quad \phi \to e^{i\alpha} \phi, \quad \psi_R \to \psi_R$

<u>Max</u> $\boxed{\mathcal{L}_Y^G = \bar{\psi} \psi h \quad , \quad h \in \mathbb{R}}$

$+ i \bar{\psi} \gamma_5 \psi G$

$$G_{LR} = SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$\boxed{B-L = \text{accidental global symm. of SM - anomaly free}}$$

$\boxed{SM}$ $W \bar{f}' f$ $(\bar{u} d, \bar{\nu} e)$

h, Z, A $\bar{f} f$

$B \longrightarrow B$ (baryon = B)

$L \longrightarrow L$ (lepton = L)

$$\boxed{W, Z, A, h : \text{zero } B, L}$$

$\Downarrow$

$$\boxed{B \text{ and } L = \text{conserved}}$$

$$\boxed{\text{Conservation laws}}$$

$$\psi \to e^{i\alpha}\psi \Rightarrow \partial^\mu j_\mu = \bar{\psi}\gamma^\mu\psi$$

B: $j^\mu_B = \bar{\psi}\gamma^\mu B\psi$ B lepton $=0$

L: $j^\mu_L = \bar{\psi}\gamma^\mu L\psi$ L baryon $=0$

<u>axial U(1)</u>: $\psi \to e^{i\beta\gamma_5}\psi$ (?)

$$\partial^\mu j^5_\mu = m_\psi \bar{\psi}\gamma_5\psi$$

$$j_\mu^5 = \bar{\psi} \gamma_\mu \gamma_5 \psi$$

$$m_\psi \bar{\psi} \psi = m_\psi (\bar{\psi}_L \psi_R + h.c.)$$

breaks chiral sym.

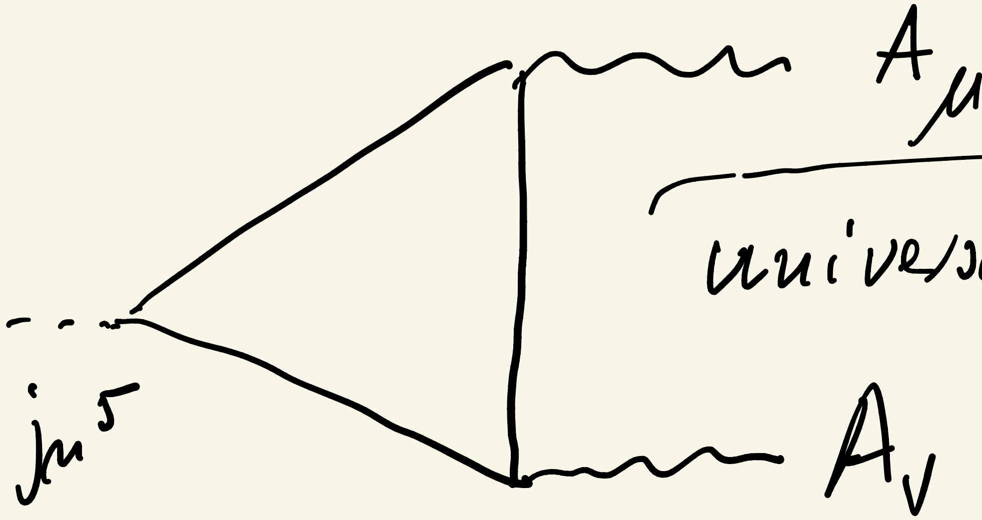
$$\boxed{m_\psi = 0 \Rightarrow \partial^\mu j_\mu^5 = 0}$$

chiral: $\psi_L \to e^{i\alpha} \psi_L, \; \psi_R \to \psi_R$ (2)

equiv.

Anomaly [(γ_5) = axial]

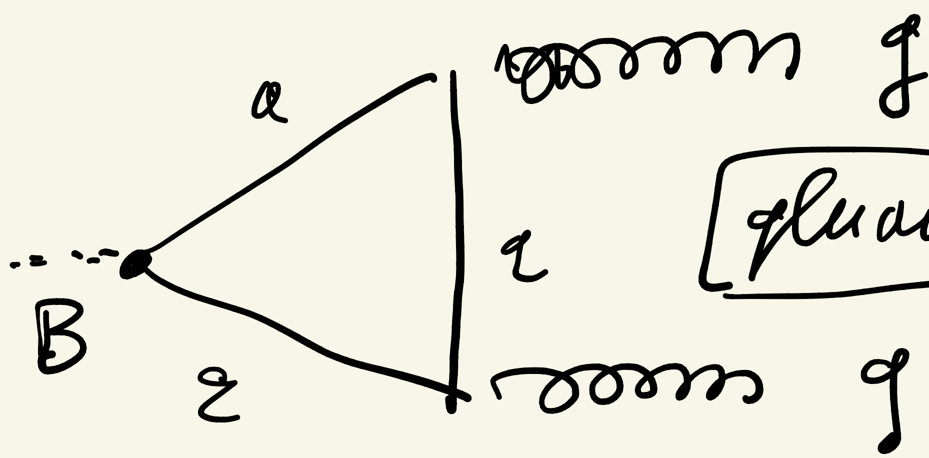
$$\boxed{\partial^\mu j_\mu^5 = \frac{g^2}{16\pi^2} \epsilon_{\mu\nu\alpha\beta} F_a^{\mu\nu} F_a^{\alpha\beta}}$$



universal = same for all f with same quantum #

$\boxed{\text{Baryon number} \equiv B}$

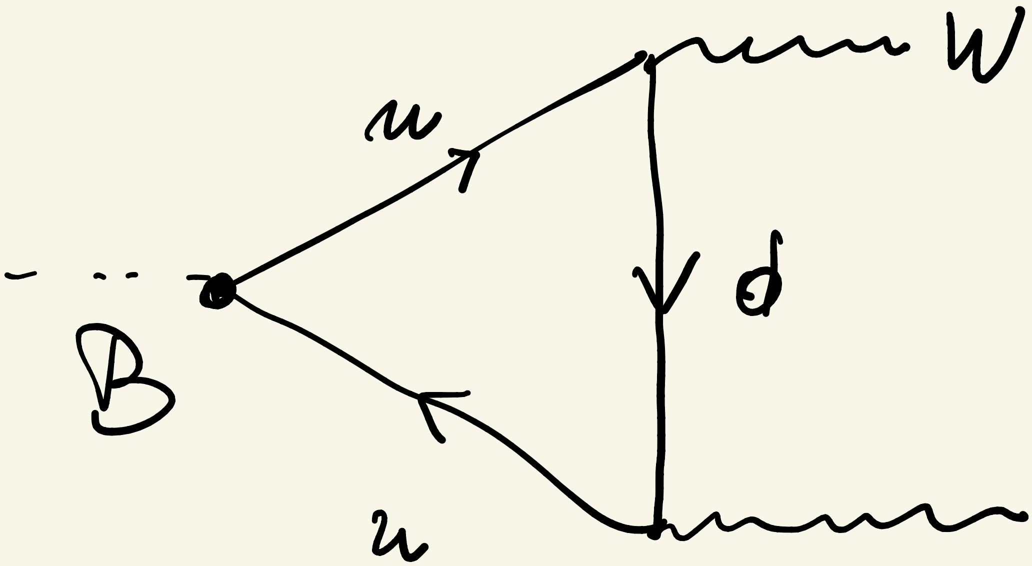
$$j_\mu^B = \bar\psi \gamma_\mu B \psi$$



gluons

QCD $\neq$ <u>not</u> chiral

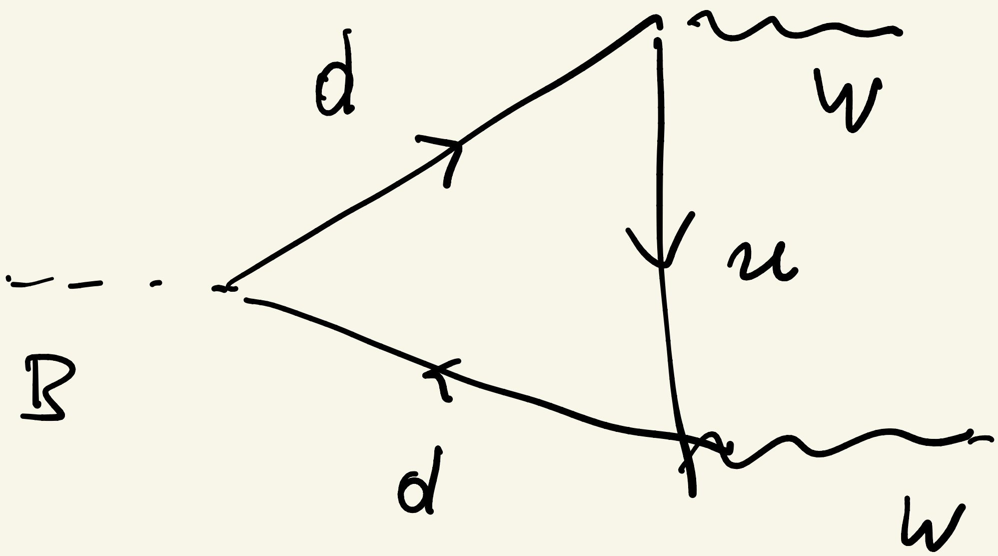
$q_L \leftrightarrow q_R$ couple to glue equally



colour

$$C_A^u(B) \underset{B}{=} \frac{1}{3} \boxed{C_A} \times 3$$

universal



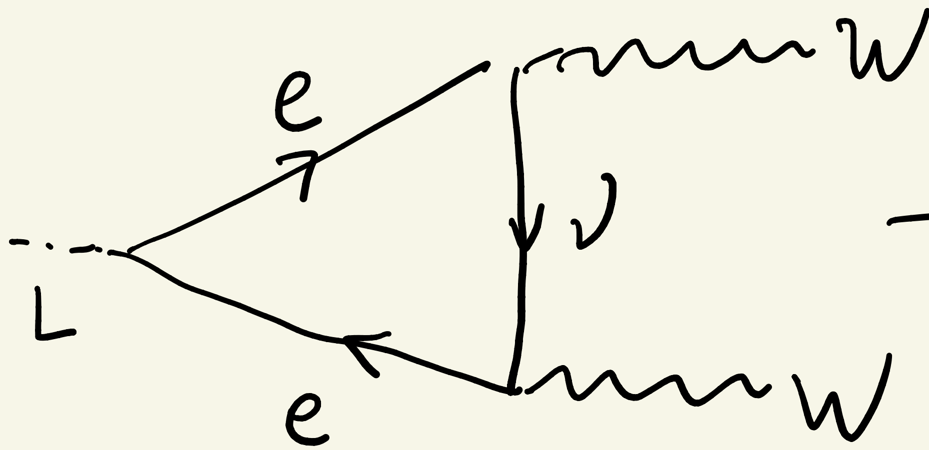
$$\boxed{C_A^d(B) \underset{B}{=} \frac{1}{3} C_A} \times 3$$

colour

$\Downarrow$

$$\boxed{C_A(B) = \frac{2}{3}\, C_A \cdot \underbrace{3}_{\text{color}} = 2\, C_A}$$

$$\boxed{\text{Lepton number} \equiv L}$$



$+\; e \leftrightarrow \nu$

$$C_A^{e}(L) = \underset{\underset{L}{=}}{1} \cdot C_A$$

$$C_A^{\nu}(L) = \underset{\parallel}{1} \cdot C_A$$

$$C_A(L) = 2\,C_A$$

$$\Downarrow$$

Both B and L are broken!

$$\Downarrow$$

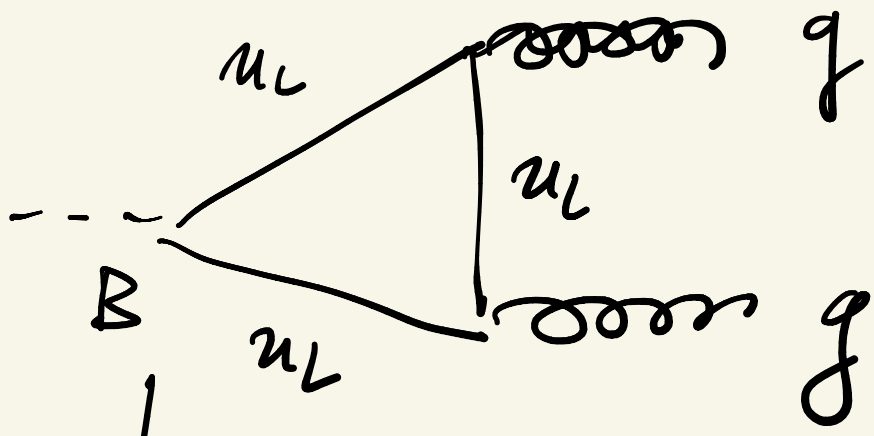
$$T_{proton} \simeq M_W\, e^{-\frac{4\pi}{\alpha}}$$

$$\Downarrow$$

$$\tau_p \simeq 10^{120}\ \mathrm{yr}$$

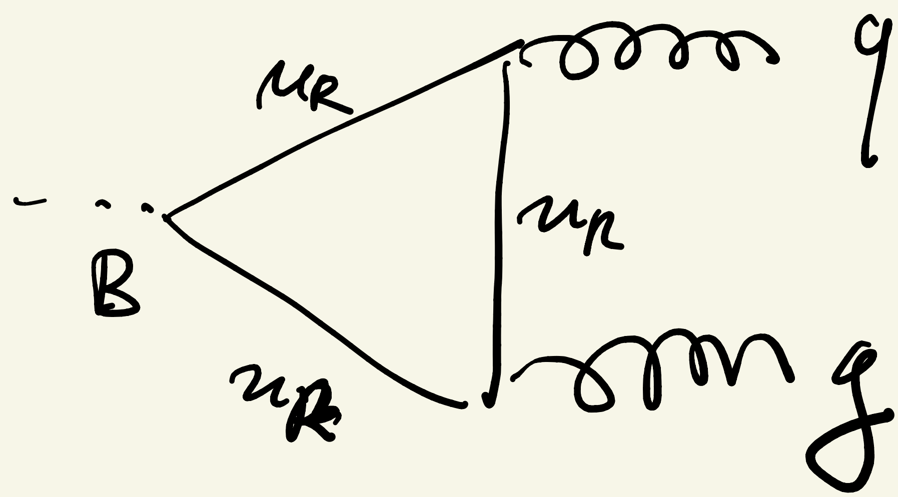
gluon

$$u_L = \frac{1-\gamma_5}{2} u$$



$\downarrow$

$$\frac{1}{3} C_A (+1)$$



$$u_R = \frac{1+\gamma_5}{2} u$$

$$\frac{1}{3} C_A (-1)$$

$\Rightarrow$ <u>no</u> QCD anomaly:

$u_L \leftrightarrow u_R$ cancels

$c_A(B) = 2\, c_A$

$c_A(L) = 2\, c_A$

$\Downarrow$

$$\boxed{c_A(B-L) = 0}$$

B-L = anomaly free
"accidental" symmetry

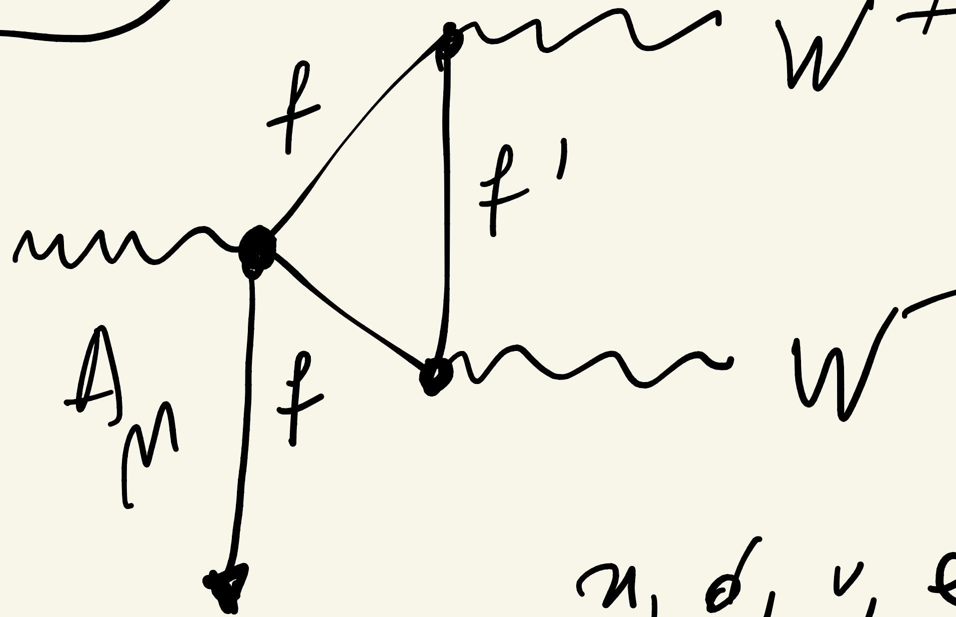
QED = renormalizable theory

$$\boxed{\partial^\mu j_\mu^{em} = 0}$$ crucial

$\Downarrow$

$$\boxed{\text{gauge inv} \Rightarrow \text{anomaly free}}$$

SM



q_f

$$\underbrace{u,\ d,\ \nu,\ e}_{\text{same coupling}}$$

$$\boxed{c_A(A) = \underbrace{\left(\sum q_f\right)}_{=\,0} c_A}$$

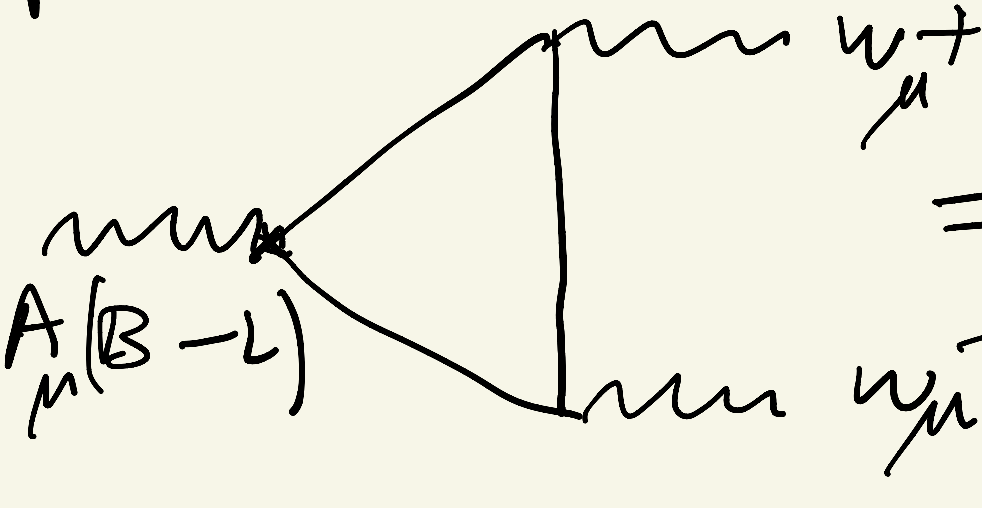
$B-L$ = global symmetry in SM

LR model

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2}$$

gauge

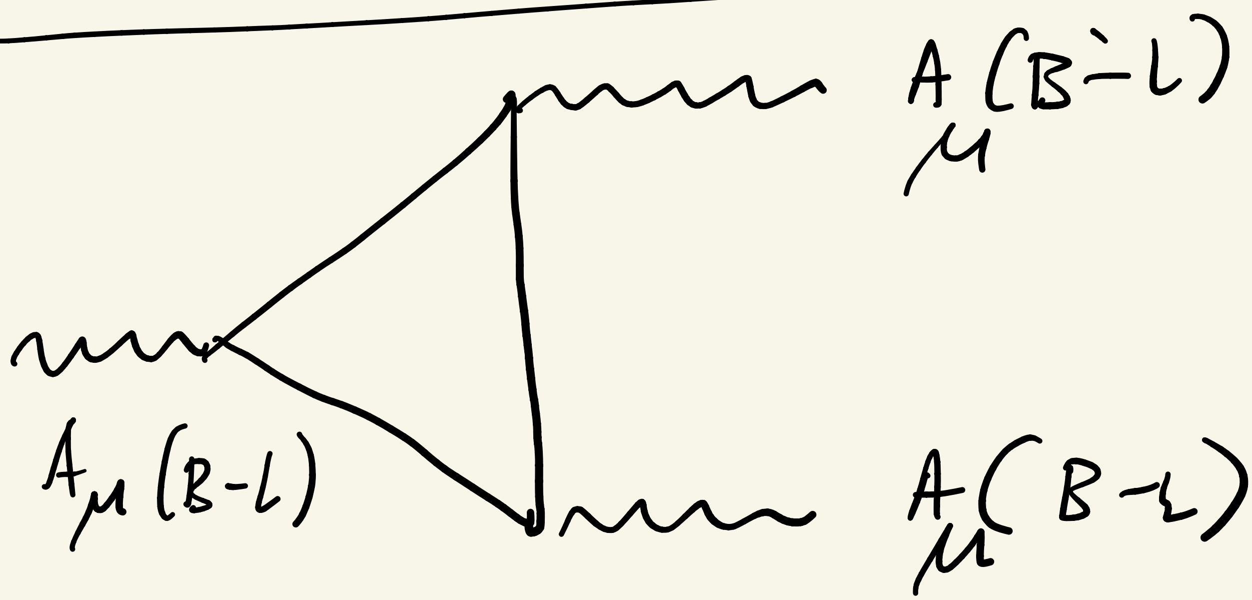
can I gauge $B-L$ in SM?



$$= 0 \quad (1)$$

$$\left. u_L,\ d_L,\ e_L,\ \nu_L \right\} \quad CA$$

$$\frac{1}{3}\cdot 3 + \frac{1}{3}\cdot 3 + (-1) + (-1) = 0$$



$$u_L + u_R \longrightarrow 0$$

$$d_L + d_R \longrightarrow 0$$

$$e_L + e_R \longrightarrow 0$$

$$\boxed{\nu_L \to \ \neq 0}$$

Anomaly!!

$$\Downarrow$$

$$\boxed{\exists\, \nu_R \Longleftrightarrow B-L = \text{anomaly free, } \underline{\underline{\text{gauge}}}}$$

$$\boxed{B-L \longleftrightarrow \exists\, \nu_R}$$

LR

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \leftrightarrow \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$\Downarrow$$

B-L gauged

B-L gauged $\Rightarrow \exists\, \nu_R$

$$\underbrace{SU(2)_L \times SU(2)_R \times U(1)_{B-L}}$$

$$\Big\downarrow \; M_R = M_{LR}$$

B-L is broken!

$$U(1)_Y$$

$\boxed{M_R}$

$\Delta_L \longleftrightarrow \Delta_R$

SU(2) triplets , $B-L=2$

$$\Delta_L \to U_L \Delta_L U_L^\dagger \,, \quad \Delta_R \to U_R \Delta_R U_R^\dagger$$

basic building blocks

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad l_R = \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$\mathcal{L}_y = l_L^T \underset{\text{SU(2)}}{i\sigma_2} \Delta_L \underset{\text{Lorentz}}{C} l_L Y_{\Delta_L} + L \leftrightarrow R$$

$$\to l_L^T \cancel{U_L^T i\sigma_2 U_L} \Delta_L \cancel{U_L^\dagger U_L} l_L$$

$\boxed{\text{SM}}$ $\mathcal{L}_Y = \overline{\ell_L}\, \Phi Y_e e_R$

$\hookrightarrow$ $\boxed{\underline{\text{no}} \text{ int. with quarks}}$

$$\ell_L^{\alpha\,T}\, i\sigma_2\, \Delta_L\, C\, \overline{q}^{\beta}_L$$

B-L $\quad \frac{1}{3} + 2 + \frac{1}{3} \neq 0$

$\underline{\text{Color}}$ $\quad \boxed{3 \times 3 = 6 + 3^*}$

$$\boxed{3 \times \bar{3} = 8 + 1}$$

$$\Downarrow$$

$$\mathcal{L}_Y^{\Delta} = \ell_L^T Y_{\Delta_L} i\sigma_2 C \Delta_L \ell_L + L \leftrightarrow R$$

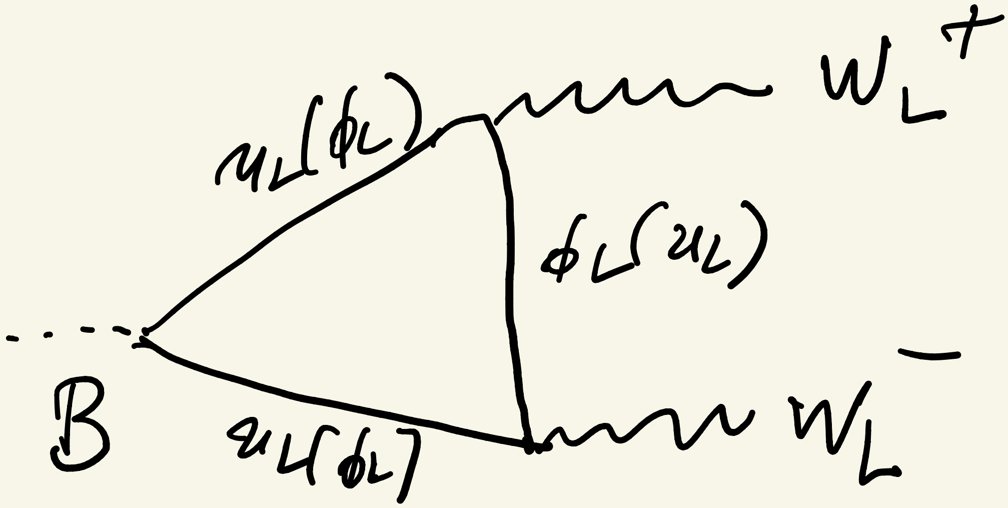
$$\boxed{\begin{array}{l} P: \quad \Delta_L \longleftrightarrow \Delta_R \\ \quad \Rightarrow \quad Y_{\Delta_L} = Y_{\Delta_R} = Y_\Delta \end{array}}$$

$$\boxed{\begin{array}{l} \cdot\ \langle \Delta_L \rangle = 0 \,, \quad \langle \Delta_R \rangle \neq 0 \\ \quad \cancel{P} \text{ spontaneous} \end{array}}$$

<u>oto</u>: $\boxed{B+L = \text{anomaly?}}$

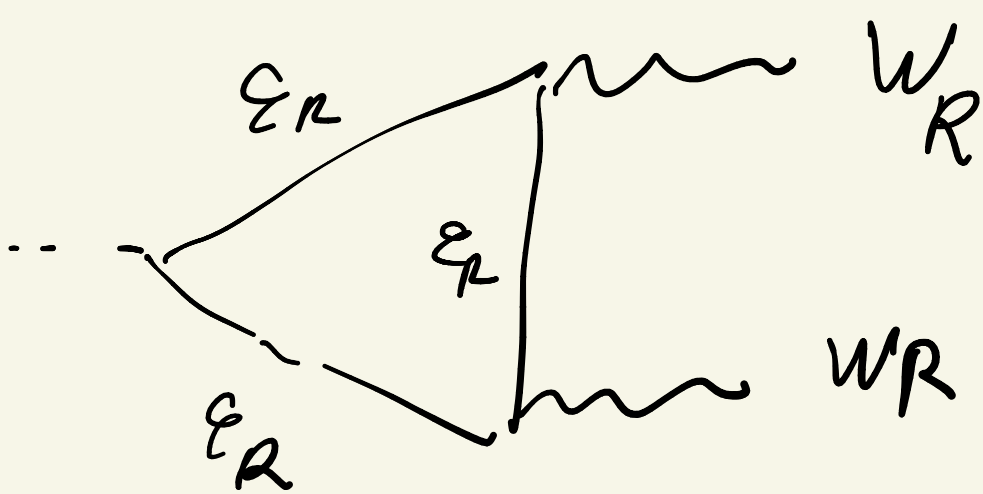
$\boxed{B-L = \text{anomaly free}}$ LR

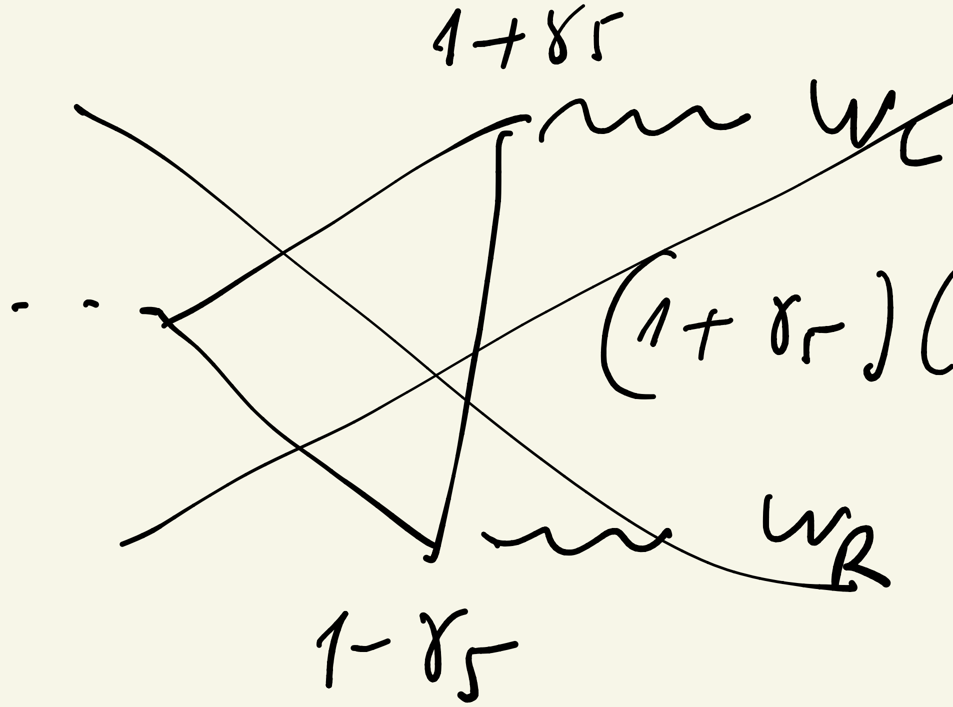
$\boxed{LR : \text{what about } B?}$



$\Downarrow$

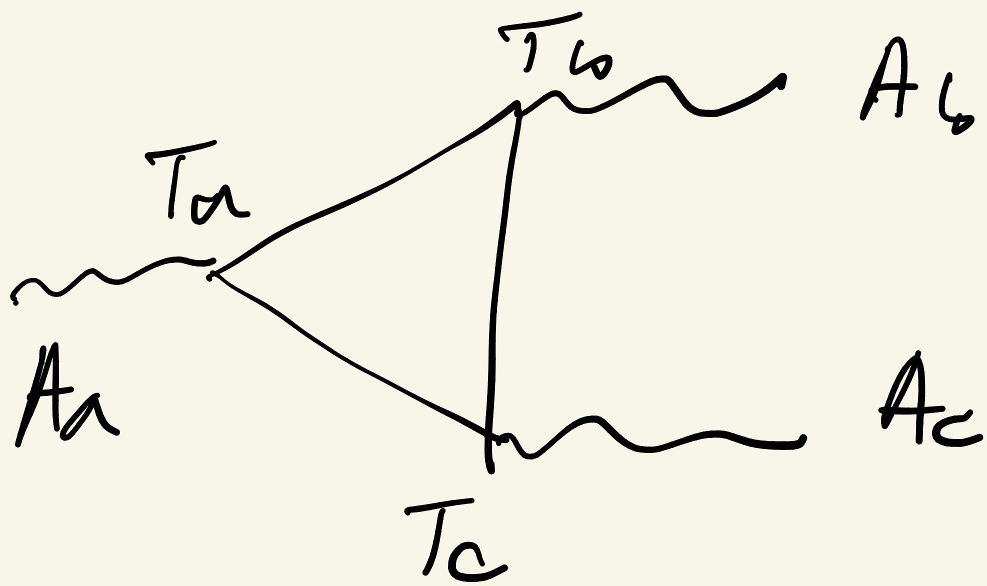
$$\left.\begin{array}{lll} u: & \frac{1}{3}\cdot 3 & c_A \\ d & \frac{1}{3}\cdot 3 & c_A \end{array}\right\} = 2\,c_A$$





$$(1+\gamma_5)(1-\gamma_5)=0$$

C_A^{abc} -

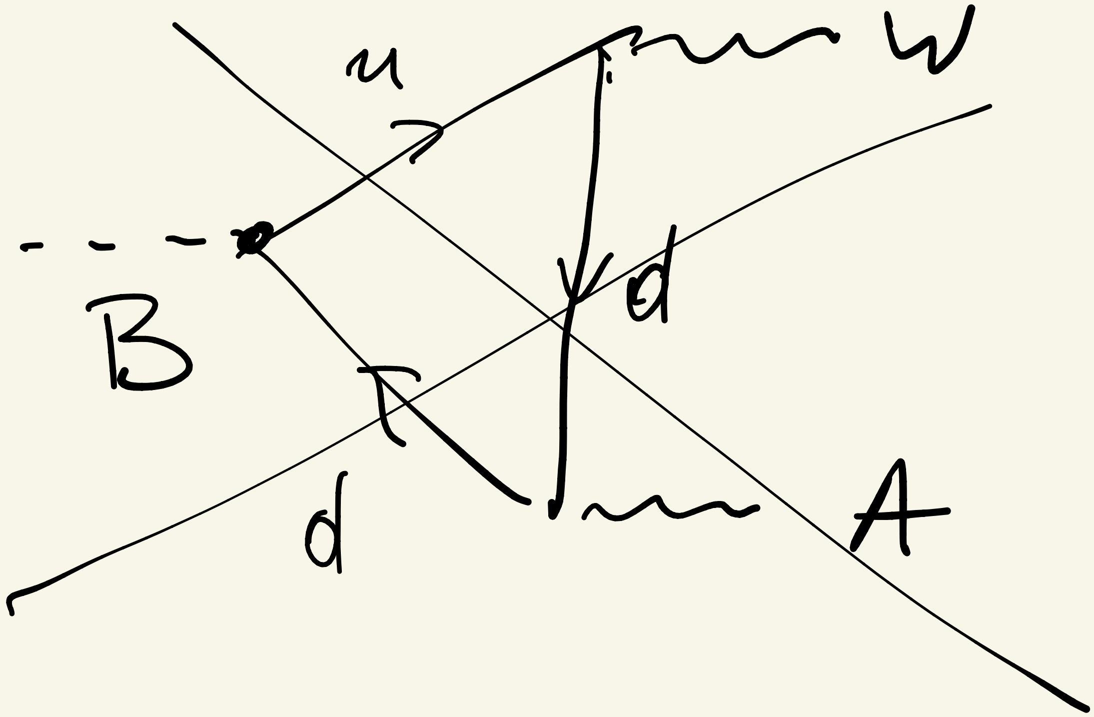


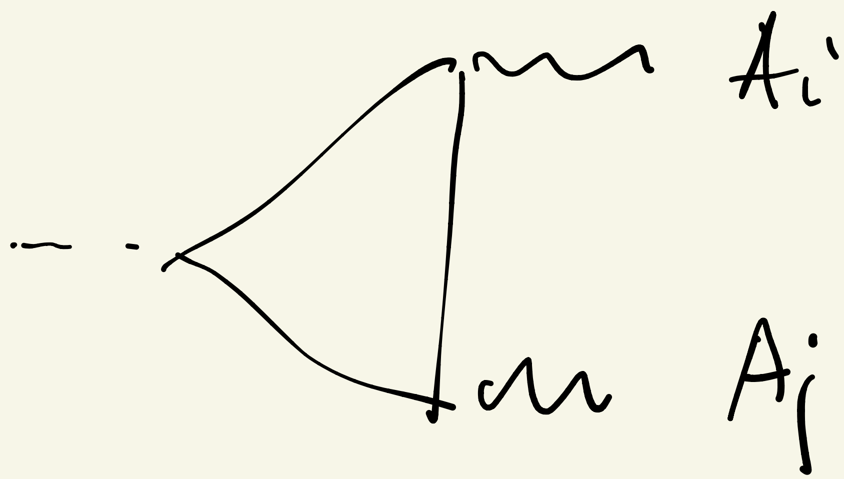
$$= \mathrm{Tr}\, T_a T_b T_c + \mathrm{Tr}\, T_a T_c T_b$$

$$\Downarrow \quad + \ldots$$

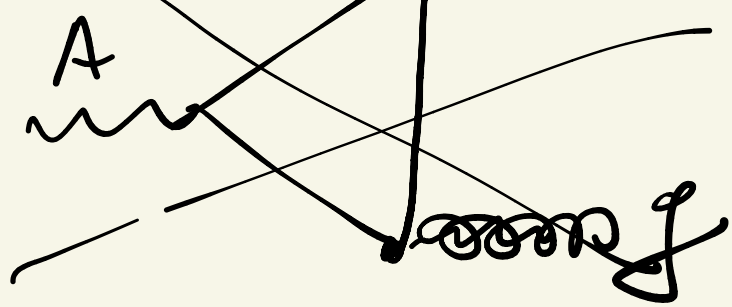
$$\boxed{C_A^{abc} = \mathrm{Tr}\{T_a, T_b\} T_c}$$

show





$SM = SU(2) \times U(1) \times SU(3)$



$SU(2) \times U_Y(1)$

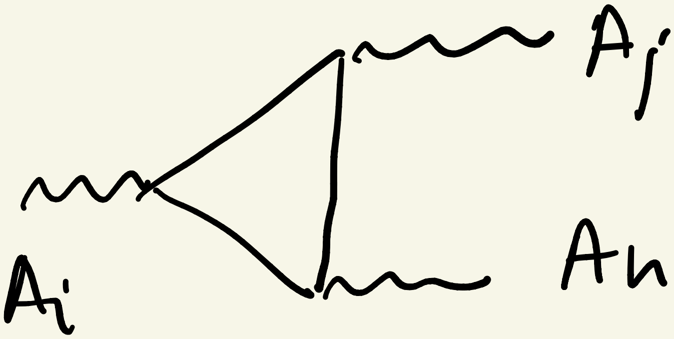
$$Q = T_3 + Y/2$$

$$\mathrm{Tr}\, Q = 0 \quad \begin{cases} u: 2/3 \cdot 3 \\ d: -1/3 \cdot 3 \\ e: -1 \end{cases} \qquad \nu : 0$$

$$= 2/3 \cdot 3 + (-1/3) \cdot 3 - 1$$

$$= 0$$

$$\boxed{\mathrm{Tr}\, T_3 = 0}$$



$$\boxed{SU(2)}$$

$$\boxed{i, j, h = 1, 2, 3}$$

$$\boxed{C_A^{ijk} = \mathrm{Tr}\,\{T_i, T_j\}\,T_k}$$

SU(2)

anomaly free

$$\boxed{T_i = \sigma_i/2}$$

$$\{T_i, T_j\} = \frac{1}{2}\delta_{ij}$$

$$\boxed{C_A^{ijk} = \mathrm{Tr}\,\frac{1}{2}\delta_{ij}\,\frac{\sigma_k}{2} = 0}$$

$$\boxed{SU(2) \leftrightarrow SO(3)}$$

$$\boxed{SU(2)\times SU(2) = \text{anomaly free}}$$

$$\|$$

$$\boxed{SO(4)} = \text{anomaly free}$$

$\boxed{SU(4) = SO(6)}$

# of gen = same = 15
rank = 3 = same

SO(4) = anomaly free

$\boxed{LR}$ only B-L = anomaly free
$\exists\ \nu_R$

$\Downarrow$ Δ int.

$$\mathcal{L}_Y = l_R^T\, i\sigma_2 C\, \Delta_R Y_\Delta\, l_R + R \to L$$
$$+ h.c.$$

$\langle \Delta_R \rangle \neq 0 \qquad \langle \Delta_L \rangle \neq 0$

$$\Delta = \begin{bmatrix} \delta^+ & \delta^{++} \\ \delta^0 & -\delta^+ \end{bmatrix}$$

$$\langle \Delta_R \rangle = v_R \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$\Downarrow$ $\qquad\qquad\qquad\qquad \overset{C}{\downarrow}$

$$v_R \left(\nu_R^T\ e_R^T\right) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_R Y_\Delta$$
$$+ h.c.$$

$$= v_R Y_\Delta \left(\nu_R^T e_R^T\right) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$
$$\underset{C}{\uparrow} \quad + h.c.$$

$$= \boxed{Y_\Delta v_R\ \nu_R^T C \nu_R} + h.c.$$

$$\boxed{M_N = y_\Delta v_R \gg M_W}$$

"natural"

$\ell_L \longleftrightarrow \ell_R$

D T ??

$$\boxed{T(\Delta) \Rightarrow M_N \simeq M_R \simeq M_{W_R}}$$

$D(\chi_L, \chi_R) \Rightarrow$ <u>no</u> int. with f

$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d \end{pmatrix}_R$

$\underline{\mathrm{I}}$: $\quad m_\nu \propto \frac{1}{M_N} \quad (\Delta)$

(x)

$\underline{\mathrm{D}}$: $\quad m_\nu = m_D \simeq m_e$

$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_R$ Brenco, G.S.

$\boxed{\underline{M} \, L \, R \, S \, \underline{M}}$

Minimal LR Symmetric Model

$$\boxed{\begin{matrix} \Delta_L, & \Delta_R \\ & \Phi \end{matrix}}$$

SM

$$\Lambda_{QCD}^3 = \underbrace{\langle \bar{u}_L u_R \rangle}_{\text{Doublet}} \neq 0 \quad (QCD)$$

$\boxed{\text{Too small!}}$