BB S_M Neutrino Course

Lecture XIX

LMU

Spring 2020

It's LR, stupid!

actual symmetry of my model

SO(3) $\qquad\qquad \phi_1 \leftrightarrow \phi_2$

$$\phi_1, \phi_2 = 2 \text{ vectors}$$

$$V = \mu^2 (\phi_1^2 + \phi_2^2) + \cdots$$

$$\underbrace{\phi_2 \to -\phi_2}_{D} + \cancel{\phi_1^2\, \phi_1 \cdot \phi_2}$$

$$(\phi_1^2 + \phi_2^2)^2 + (\vec{\phi}_1 \cdot \vec{\phi}_2)^2$$

$$\boxed{SO(3) \times SO(3)}$$

$$SO(6)$$

$V(\phi_i) \leftarrow$ global group G

$\Downarrow$ by accident

$$\boxed{G' > G}$$

$$\boxed{LR}$$

$$\boxed{\begin{aligned} D_\mu = \partial_\mu - i g_L T_a^L A_\mu^L - i g_R T_a^R A_\mu^R \\ - i \bar{g} \frac{B-L}{2} C_\mu \end{aligned}}$$

$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \leftarrow \bar{g}$

$g_L = g_R \equiv g$

LR symmetry

$$A_{\mu L} \longleftrightarrow A_{\mu R}$$

$$f_L \longleftrightarrow f_R$$

$$g_L = g_R$$

Our course

Group $\longrightarrow$ Algebra

$$SO(4) \longleftrightarrow SU(2) \times SU(2)$$

$$SO(3) \longleftrightarrow SU(2)$$

$$G_{LR} = SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$G_{SM} = SU(2)_L \times U(1)_Y$$

<u>SM:</u> $G_{SM} \longrightarrow U(1)_{em}$

$\langle \phi \rangle$

$\searrow$ $SU(2)$ doublet

<u>LR</u>: $G_{LR} \longrightarrow G_{SM}$

$$SU(2)_R \times U(1)_{B-L} \longrightarrow U(1)_Y$$

$\langle \Delta_R \rangle$

$\hookrightarrow$ $SU(2)_R$ triplet (adjoint)

$$L \longleftrightarrow R$$

$$\Delta_L \longleftrightarrow \Delta_R$$

$\boxed{\text{triplets}, \quad B-L=2}$

$$\boxed{\Delta_{L,R} \to U_{L,R}\,\Delta_{L,R}\,U_{L,R}^{+}}$$

2×2 matrices

$\boxed{HW}$ SM $\boxed{\text{triplet}}$

doublet

$$M_W = M_Z \cos\theta_W \qquad \boxed{M_W = 2 M_Z \cos\theta_W}$$

$$\tan\theta_W = g'/g$$

$$\Updownarrow$$

$$\boxed{e = \sin\theta_W \, g}$$

unbroken

SU(2)

$$\Downarrow$$

$$\boxed{M_Z = 2\,\frac{\sqrt{g^2+g'^2}}{g}\,M_W}$$

TRIPLET

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$$

analogy

$$\Downarrow$$

$$\boxed{M_{Z_R} = 2\,\frac{\sqrt{g^2+\bar{g}^2}}{g}\,M_{W_R}}$$

Δ_R = TRIPLET

SM

$$\tan\theta_W = \frac{\sin\theta_W}{\cos\theta_W} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} \equiv \frac{g'}{g}$$

$$\sin^2\theta_W \simeq 1/4$$

$$\theta_W \simeq 30^\circ$$

$$\boxed{g = \sqrt{3}\, g'} \quad \boxed{\text{SM}}$$

- $e = \sin\theta_W\, g = \frac{g'}{\sqrt{g^2+g'^2}}\, g$

$$\updownarrow$$

- $g' = \frac{\bar{g}\, g}{\sqrt{g^2+\bar{g}^2}}$

$$g'/g = \frac{\bar{g}}{\sqrt{g^2+\bar{g}^2}} = \frac{1}{\sqrt{3}}$$

$$\Downarrow$$

$$g^2 + \bar{g}^2 = 3\bar{g}^2$$

generic – any Higgs $\boxed{g^2 = 2\bar{g}^2}$ $\star\star\star$

$$\boxed{\alpha_{a}^{-1}(0) = 137} \qquad \boxed{\alpha_{em}^{-1}(M_W) = 128}$$

$$M_{Z_R} = 2\frac{\sqrt{g^2+\bar{g}^2}}{g} M_{W_R}$$

$$= 2\frac{\sqrt{3}}{2} M_{W_R} = \sqrt{3}\, M_{W_R}$$

$$\boxed{M_{Z_R} \simeq 1.7\, M_{W_R}}$$

$$M_{W_R} \gtrsim 4.5\ \text{TeV}$$

$$M_{Z_R} \gtrsim 8\ \text{TeV}$$

Higgs dependent $\leftrightarrow$ triplet

SM

$$e = g \sin\theta_W$$

Higgs independent

$\exists A$

$\Rightarrow$ Z boson must exist

end

$$\boxed{J^\mu_Z = \bar{f}\gamma^\mu (T_3 - Q\sin^2\theta_W) f}$$

Higgs independent

However; gauge boson masses $\longleftrightarrow$ Higgs

doublet

D: $M_W = M_Z \cos\theta_W$

T: $M_W = \frac{1}{\sqrt{2}} M_Z \cos\theta_W$

triplet

LR: Symmetry Breaking

I. $G_{LR} \xrightarrow[\Delta_R]{} G_{SM}$

$SU(2) \times U_Y(1)$

$\Delta \to U \Delta U^{+}$

$Y \Delta = 2 \Delta$

$$Q = \hat{T}_3 + \frac{Y}{2}$$

$$\boxed{Q \Delta = \hat{T}_3 \Delta + \Delta}$$

$$\Delta \to (1 + i\theta T)\,\Delta\,(1 - i\theta T)$$

$$= \Delta + i\theta\,[T, \Delta]$$

$$\Rightarrow \boxed{\hat{T} = [T, \Delta]} \to \text{fandomentel}$$

$$\Delta = \begin{bmatrix} \delta_1' & \delta_2' \\ \delta_3' & -\delta_1' \end{bmatrix} \qquad \boxed{T, \Delta = 0}$$

$$\hat{T}_3\,\Delta = [T_3, \Delta] = \begin{bmatrix} 0 \cdot \delta_1' & +\delta_2' \\ -\delta_3' & -0 \cdot \delta_1' \end{bmatrix}$$

$$Q\,\Delta = \begin{bmatrix} 1 \cdot \delta_1' & 2 \cdot \delta_2' \\ 0\delta_3' & -(1 \cdot \delta_1') \end{bmatrix}$$

$$\Downarrow$$

$$\boxed{\delta_1' = \delta^{+}, \quad \delta_2' = \delta^{++} \\ \delta_3' = \delta_0}$$

$$\boxed{\Delta \equiv T_a \delta_a}$$

OTO

GORAW

$$\delta_1' = \delta_3$$

$$\delta_2' = \delta_1 + i\delta_2$$

$$\delta_3' = \delta_1 - i\delta_2$$

$$\mathrm{Tr}\,\Delta = 0$$

$\Downarrow$ bottom line

$$\Delta = \begin{bmatrix} \delta^{+} & \delta^{++} \\ \delta^{0} & -\delta^{+} \end{bmatrix}$$

charge indentification

$V(\Delta_L, \Delta_R) \to$ minimize

$$\Downarrow$$

$$\langle \Delta \rangle = \begin{bmatrix} 0 & 0 \\ v & 0 \end{bmatrix}$$

? ? ? ?

General ?

$\boxed{\text{SM}}$

$$\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$

$$\Downarrow$$

$$\Phi_0^G = \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \Phi_0^0 = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$U \Phi_0 =$ most general doublet

$$\Updownarrow$$

$$\boxed{\Phi_0^G = U \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}}$$ always

$$\Downarrow$$

Q is <u>not</u> broken

generically

LR real adjoint $\boxed{T_a \equiv \frac{\sigma_a}{2}}$

$$\Delta_{herm} = T_a \, \delta_{real}$$

$$\Delta_{herm} \longrightarrow U \Delta_{herm} U^\dagger$$

$$= \begin{bmatrix} v & 0 \\ 0 & -v \end{bmatrix}$$

Δ_{LR} $(B-L=2)$

$\Updownarrow$

$\Delta_1^{herm}, \Delta_2^{herm}$ ← hermitian

$$\Delta = \Delta_1^{herm} + i \, \Delta_2^{herm}$$

$$\langle \Delta_1^{hem} \rangle \longrightarrow \begin{bmatrix} v & 0 \\ 0 & -v \end{bmatrix}$$

$$\langle \Delta_2^{hem} \rangle \not\longrightarrow \text{diagonal in general}$$

$\boxed{\text{However}}$

$$\langle \Delta \rangle \longrightarrow U \langle \Delta \rangle U^{\dagger} = \begin{bmatrix} 0 & 0 \\ v & 0 \end{bmatrix}$$

Find U

$\boxed{\text{Potential}}$

$$V(\Delta) = -\mu_\Delta^2 \left(\mathrm{Tr}\, \Delta_L^\dagger \Delta_L + \mathrm{Tr}\, \Delta_R^\dagger \Delta_R \right)$$

$$+ \lambda_1 \left(\mathrm{Tr}\, \Delta_L^\dagger \Delta_L + \mathrm{Tr}\, \Delta_R^\dagger \Delta_R \right)^2$$

$$+ \lambda_2 \left(\mathrm{Tr}\, \Delta_L^\dagger \Delta_L \Delta_L^\dagger \Delta_L + \mathrm{Tr}\, \Delta_R^\dagger \Delta_R \Delta_R^\dagger \Delta_R \right)$$

- - - - - - - -

$$+ \lambda' \, \mathrm{Tr}\, \Delta_L^\dagger \Delta_L \, \mathrm{Tr}\, \Delta_R^\dagger \Delta_R$$

$$\underline{\lambda' > 0} \Rightarrow \boxed{\begin{array}{c} \langle \Delta_L \rangle = 0 \\ \langle \Delta_R \rangle \neq 0 \end{array}}$$

$$\langle \Delta_R \rangle = \begin{bmatrix} 0 & 0 \\ v_R & 0 \end{bmatrix}$$

$$\frac{Y'}{2} = \underbrace{T_{3R} + \frac{B-L}{2}}_{\text{broken}}$$

$$T_{3R} \langle \Delta_R \rangle \neq 0$$

$$(B-L) \langle \Delta_R \rangle \neq 0$$

$$\therefore \boxed{Y' \langle \Delta_R \rangle = 0}$$

$$T_{3L} \langle \Delta_R \rangle = 0$$

$$\Downarrow$$

$$\boxed{Q_{em} \langle \Delta_R \rangle = 0}$$

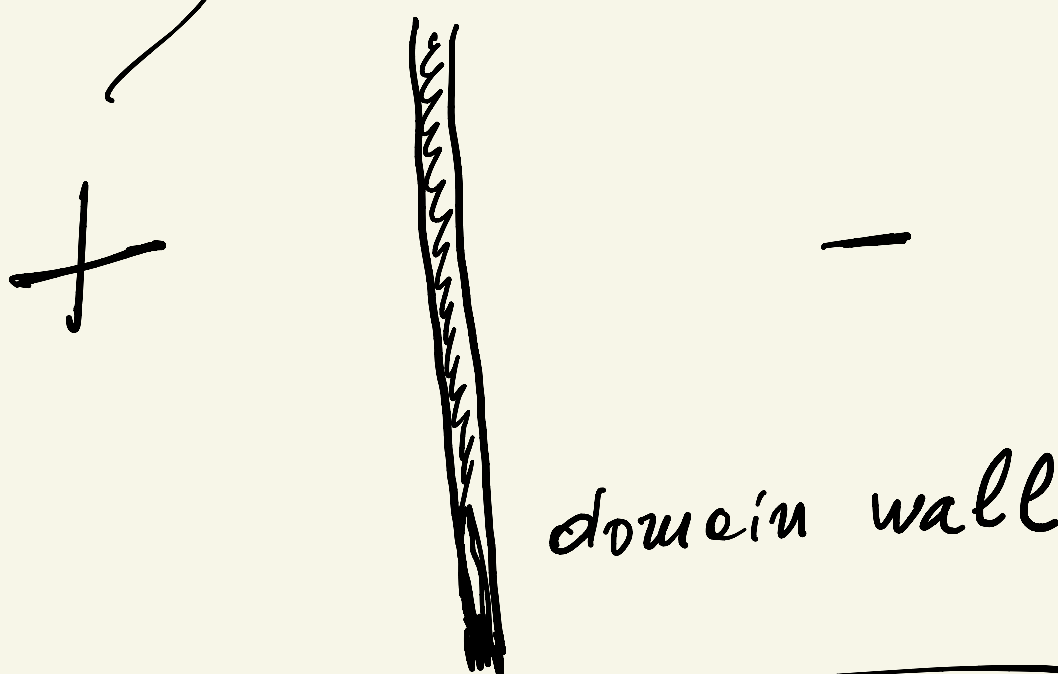
$$\boxed{D: \quad \phi \in R, \quad \phi \to -\phi} \quad \oplus$$

$$G_{LR}: \quad \Delta_L \leftrightarrow \Delta_R$$

$$\circledast \quad V(\phi) = \frac{\lambda}{4}\left(\phi^2 - v^2\right)^2$$

$$\phi_0 = \pm v$$

$$\phi_0^{6} = +v \qquad \phi_0^{0} = -v$$



$$\boxed{\begin{array}{c}\text{domain walls, strings,} \\ \text{magnetic monopoles} \\ \| \\ \text{topological defects}\end{array}}$$

$$\boxed{\Delta_R = \begin{bmatrix} \delta^+ & \delta^{++} \\ v + h_R + iG & -\delta^+ \end{bmatrix}_R}$$

$$\Delta_L = \begin{bmatrix} \delta^+ & \delta^{++} \\ \delta^0 & -\delta^+ \end{bmatrix}_L$$

$$G \xrightarrow[\Phi_0]{} H$$

$$T_a \Phi_0 \neq 0 \implies$$

(i) G_a, $m_{G_a} = 0$ for global G

(ii) G_a "eaten" by a gauge field (massive)

Our case:

$$SU(2)_R \times U(1)_{B-L} \to U_Y(1)$$

$$3 + 1 \quad - 1 = 3$$

broken generators

$$G_{LR} \longrightarrow G_{SM}$$

$\Leftrightarrow$ $SU(2)_L \times U_Y(1)$ <u>not</u> broken

$\Leftrightarrow$ <u>no</u> splitting in $SU(2)_L$ multiplet

$$\Downarrow$$

$\delta_R^{\pm} \longleftrightarrow$ eaten by $W_R^{\pm}$

$G_R \longleftrightarrow$ eaten by Z_R

h_R = Higgs at G_{LR} breaking

$$m_{h_R} \propto v_R$$

$$v_R^2 = \frac{\mu_\Delta^2}{f(\lambda, \lambda')}$$

$$\boxed{m_{\delta_R^{++}} \propto v_R , \quad m_{\nu_R} \propto v_R \qquad m_{\Delta_L} \propto v_R}$$

$\boxed{SM}$ $\quad SU(2)_L \times U(1)$

(1) • $\Phi \leftarrow$ Doublet

• fermion mass $\Rightarrow Y_\Phi = 1$

$$Q = T_3 + \frac{Y}{2}$$

$\Downarrow$

$$\boxed{\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \longleftrightarrow \qquad l = \begin{pmatrix} \nu \\ e \end{pmatrix}_L}$$

def. of Y $\Longleftrightarrow$ one neutral in Φ

$\Rightarrow$ there is a charged field

$$Q = T_3 + Y/2 \qquad [Y, T] = 0$$

$$\boxed{\Delta Q = \Delta T_3 = \pm 1}$$

Triplet $\boxed{\Delta \;\therefore\; \exists\, \delta_0}$

$$\Rightarrow \quad \Delta Q = \Delta T_3$$

$$\Rightarrow \text{ also } \exists\, \delta^+$$

Doublet: $\quad Y_\Phi = 10$

$$Q = T_3 + \frac{1}{2}\,\frac{1}{10}\,Y$$

$$\boxed{SU(5) \quad \text{vs} \quad U(5)}$$

G = simple non-Abelian

$$\boxed{Q \in G}$$

$\underline{G = SU(2)}$ $\quad Q = T_3$

charge quantization

$G = SU(5)$ $\quad \mathrm{Tr}\, T_a = 0$

$\det U = 1$

$$Q = \sum_a c_a T_a$$

$$\Rightarrow \mathrm{Tr}\, Q = 0 \leftarrow \text{correct charge}$$

$$SU(5) \Downarrow$$

$$\boxed{Q_e = 3\, Q_d}$$

- $SO(3) = SU(2)$

$$\Downarrow \text{structure}$$

$$\mathrm{Tr}\, Q = 0 \Rightarrow \boxed{Q = n\, 1/2}$$

- $SU(2) \times U(1)$ — anomaly

$$\Rightarrow \mathrm{Tr}\, Q = 0$$

$$\Downarrow$$

<u>singlet</u> D_L, D_R new d-quark

<u>no</u> anomaly

$$\boxed{q_D = \text{arbitrary}}$$