BBSM Neutrino Course

Lecture XVIII

LMU

Spring 2020

LR Symmetric Theory

$\Downarrow$ Parity is broken spontaneously

- $G_{LR} = SU(2)_L \times SU(2)_R \times U(1)_{\boxed{B-L}}$

$$g_L = g_R = g \,, \quad \bar{g}$$

- matter = q, l

$$f_L \begin{cases} q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \\[2ex] l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \end{cases} \qquad \left. \begin{matrix} q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R \\[2ex] l_R = \begin{pmatrix} \nu \\ e \end{pmatrix}_R \end{matrix} \right\} f_R$$

$\Downarrow$

$$\boxed{\text{main prediction!} \qquad m_\nu \neq 0}$$

$\boxed{SM}$

$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \phi \quad \begin{matrix} u_R \\ d_R \end{matrix}$

doublet

LR

$$\bar{f}_L \quad \bar{\Phi} \quad f_R = \text{inv.}$$

$\Uparrow$

$$f_L \longrightarrow \underset{\substack{\uparrow \\ SU(2)_L}}{U_L} f_L \qquad f_R \longrightarrow \underset{\substack{\uparrow \\ SU(2)_R}}{U_R} f_R$$

gauge sym.

$$\bar{f}_L U_L^{\dagger} \Phi' U_R f_R$$

$$\boxed{\Phi' = U_L \Phi U_R^{\dagger}}$$

$$\bar{f}_L \cancel{U_L^{\dagger} U_L} \Phi \cancel{U_R^{\dagger} U_R} f_R$$

$$= \bar{f}_L \Phi f_R \Leftarrow \text{inv.}$$

matrix

$$\boxed{\text{bi-doublet}}$$

$$\Phi = \begin{pmatrix} \phi & \tilde{\phi} \end{pmatrix}$$

$SU(2)_L$ doublet

$SU(2)_L$ anti-doublet

$$\boxed{\tilde{\phi} = i\sigma_2 \phi^*}$$

$$\boxed{\mathcal{L}_Y = \overline{f_L}\, Y\, \Phi\, f_R + h.c.}$$

$$\phi = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}, \quad \tilde{\phi} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \phi^{0*} \\ \phi^+ \end{pmatrix}$$

$$= \begin{pmatrix} \phi^+ \\ -\phi_0^* \end{pmatrix}$$

$$\Phi = \begin{pmatrix} \phi^0 & \phi^+ \\ \phi^- & -\phi_0^* \end{pmatrix}$$

$$\langle \Phi \rangle = \begin{pmatrix} v & 0 \\ 0 & -v \end{pmatrix} \Leftarrow \text{the same (11) and (22)}$$

$$\Downarrow$$

$$\boxed{|m_u| = |m_d|}$$

$$\Downarrow \boxed{\text{generalize to } 2\ \phi\text{'s}}$$

$$\boxed{\Phi = \left(\phi_1 \quad \tilde{\phi}_2\right)} \quad \text{realistic}$$

$$\Downarrow$$

$$\boxed{\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & -\phi_2^{0*} \end{pmatrix}}$$

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & -v_2 \end{pmatrix}$$

$$\Downarrow$$

$$\left.\begin{aligned} M_u &= Y v_1 \\ M_d &= -Y v_2 \end{aligned}\right\} \Rightarrow \boxed{M_u \propto M_d}$$

SM $\quad \phi \,, \quad \tilde{\phi} \equiv i\sigma_2 \phi^*$

$$\mathcal{L}_Y = (\bar{u}\,\bar{d})_L Y_d \phi d_R + (\bar{u}\,\bar{d})_L Y_u \tilde{\phi} u_R + h.c.$$

LR

$$\boxed{\begin{aligned} &\Phi = \text{bi-doublet} \\ &\qquad\Downarrow \\ &\tilde{\Phi} \equiv \sigma_2 \Phi^* \sigma_2 \end{aligned}}$$

bi-doublet

$$\Downarrow$$

$$\mathcal{L}_Y = \bar{f}_L \left(Y_1 \Phi + Y_2 \tilde{\Phi} \right) f_R + h.c.$$

$f = q, \ell$ doublets

$Y_1 \equiv Y, \quad Y_2 \equiv \tilde{Y}$

$$\Updownarrow$$

SM: Y_u, Y_d

$\boxed{SM}$ ϕ $M_W = M_Z \cos\theta_W$

$e = g \sin\theta_W$

$\tan\theta_W = g'/g$

$\downarrow$

$\phi_i \quad i = 1, 2, \dots, \infty$

$\Downarrow$

$\langle \phi_i \rangle = v_i \Rightarrow$

$$M_W^2 = \left(\frac{g}{2}\right)^2 \left[v_1^2 + v_2^2 + \cdots\right]$$

$$M_Z = M_W / \cos\theta_W$$

$\Downarrow$

$$\boxed{LR: \; M_W = \frac{M_Z}{\cos\theta_W}}$$

G $\quad SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

$\downarrow U(1)_Y \; (g')$

$\downarrow \bar{g}$ (*)

which Higgs ?

$SU(2)_R$: $\pm 1/2 \leftrightarrow T_{3R}$

$SU(2)_R \not\to U(1)_Y$

$$\boxed{Y \neq T_{3R}}$$

(*) $\boxed{\text{SM} = \text{high precision}}$

Build: MLRSM

Minimal $\quad$ Symmetric

• φ_R : $\langle \varphi_R \rangle = v_R$:

$$SU(2)_R \times U(1)_{B-L} \xrightarrow[v_R]{} U(1)_Y$$

$P = LR$ gets broken

$$\boxed{\varphi_L \longleftrightarrow \varphi_R}$$

$$\boxed{\begin{array}{c}\langle \varphi_L \rangle = 0, \quad \langle \varphi_R \rangle = v_R \neq 0 \\ \text{MUST}\end{array}}$$

$\Downarrow$ φ_L, φ_R = scalars (real)

$$V(\varphi_L, \varphi_R) = -\frac{\mu^2}{2}(\varphi_L^2 + \varphi_R^2)$$

$$+ \frac{\lambda}{4}(\varphi_L^4 + \varphi_R^4) + \frac{\lambda'}{2}\varphi_L^2\varphi_R^2$$

$$= \boxed{-\mu^2/2\,(\varphi_L^2 + \varphi_R^2) + \frac{\lambda}{4}(\varphi_L^2 + \varphi_R^2)^2}$$

(i)

$$+ \frac{\lambda' - \lambda}{2}\varphi_L^2\varphi_R^2$$

(ii)

(i) $\boxed{\text{flat direction}}$

$$\left.\frac{\partial^2 V}{\partial \varphi_L^2}\right|_{\varphi_L = 0} = -\mu^2 < 0 \quad \text{local maximum}$$

$$\Downarrow$$

$$\langle \varphi_L^2 + \varphi_R^2 \rangle = \mu^2/\lambda$$

(ii) $\lambda' - \lambda$ = decision maker

a) $\lambda' - \lambda < 0$

$$\langle \varphi_L \rangle \neq 0 \neq \langle \varphi_R \rangle$$

$$\Downarrow$$

$$\langle \varphi_L \rangle = \langle \varphi_R \rangle \quad (V_L = V_R)$$

b) $\lambda' - \lambda > 0$

$$\boxed{\langle \varphi_L \rangle = 0, \quad \langle \varphi_R \rangle \neq 0}$$

SM $\quad V = -\mu^2/2 \, \phi^+\phi + \frac{\lambda}{4}(\phi^+\phi)^2$

- $\lambda > 0$ — bounded from below
- $\mu^2 > 0$

($\mu^2 < 0 \Rightarrow M_W = M_Z = 0$)

$\Gamma(h \to f\bar{f})$, $\Gamma(h \to W^+W^-)$

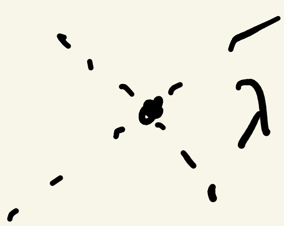
$\Gamma(h \to ZZ)$

· Z, W, t, b, τ $\Leftarrow$ mass from Higgs

$m_h = 2\lambda v^2$

$\frac{\lambda}{4} h^4$

measure



test of the Higgs

$$V = + \frac{1}{2} m_h^2 h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4$$

$M_W = \frac{g}{2} v$

$$\langle \bar{e}_L e_R \rangle \sim \langle \phi \rangle = v$$

$$\uparrow$$

$$\phi^+$$

$$\Lambda_{gr}^3 = \langle \underset{\uparrow \; 3/2}{\bar{e}_L} \; \underset{\uparrow \; 3/2}{e_R} \rangle$$

10^{18} GeV

$$\boxed{\Lambda_{gr}^3 \lesssim M_p^3 e^{-N}}$$

$\#$ of freedom

$$N = 124 \Rightarrow \boxed{\Lambda_{gr} \leq \text{GeV}}$$

Q. $\varphi_L, \varphi_R = ?$

under $SU(2)_L$, $SU(2)_R$?

1. D = doublet
2. T = triplet

1. $\boxed{\varphi_L, \varphi_R = D}$ doublets

new thing

$\bar{q}_L \, \Phi \, q_R$

bi-doublet

$$\boxed{\langle \Phi \rangle \simeq M_{W_L}}$$

$\varphi_L, \varphi_R \longleftrightarrow$ new Higgs

$\boxed{(\lambda' - \lambda > 0)}$

$\therefore$

$\langle \varphi_R \rangle = v_R \;\therefore\; \begin{matrix} M_{W_R} \\ M_{Z_R} \end{matrix} \propto v_R$

$\langle \varphi_L \rangle = 0$

1. $\varphi_L, \varphi_R = D \Rightarrow$ they do <u>not</u> couple to $f_{L,R}$

$$\cancel{\mathcal{L}_Y = \bar{q}_L \varphi_L d_R}$$

$$\Downarrow$$

$\begin{pmatrix} \nu \\ e \end{pmatrix}_{L,R} \Rightarrow \boxed{\begin{matrix} m_\nu \simeq 0 \\ m_e \neq 0 \end{matrix}}$

$$\boxed{m_{\nu_\tau} \leq 1\,eV} \qquad m_\tau \simeq GeV$$

$$\boxed{m_{\nu_\tau} \leq 10^{-9}\, m_\tau}$$

$$\boxed{\text{Why is neutrino light?}}$$

$$G \; (G_{LR},\; G_{GUT},\; \ldots)$$

$$\Big\downarrow \; M_{new} = v_{new} \; (= v_R)$$

$$G_{SM}$$

$\Longleftrightarrow$ SM particles have $m \sim M_W$

new particles have $m \sim M_{new}$

MLRSM (LR) $\Rightarrow$

$$\left(M_{W_R} , M_{Z_R} \propto v_R \right)$$

$$\boxed{\left(m_{\nu_R} \propto v_R \right)}$$

$$\Downarrow ?$$

φ_L, φ_R = triplets

2. $\varphi_L, \varphi_R \equiv \underbrace{\Delta_L, \Delta_R}$

$SU(2)_L$, $SU(2)_R$ triplets

$SU(2)_R \times U(1)_{B-L}$

$\downarrow$

$U(1)_Y$

Question: Can $\Delta_{L,R}$ be real?

A. NO

Proof: if $\Delta_R \in \mathbb{R} \Rightarrow$

$$SU(2)_R \to U(1)_R \quad (T_{3R})$$

$$\Rightarrow \boxed{M_{ZR} = 0}$$

WRONG

Q.E.D.

$$\Downarrow$$

$$\boxed{\begin{array}{c}\Delta_L, \Delta_R \text{ are complex triplets} \\ B-L=2\end{array}}$$

$$\Phi = (\phi_1 \; \tilde{\phi}_2)$$

- $\phi_1, \; \phi_2 \Rightarrow$ one connection

$$M_f = y_f \, v$$

$$\Downarrow$$

$$\phi = (v_1 \phi_1 + v_2 \phi_2) \frac{1}{\sqrt{v_1^2 + v_2^2}}$$

$$\phi' = (v_2 \phi_1 - v_1 \phi_2) \frac{1}{\sqrt{v_1^2 + v_2^2}}$$

$$\Downarrow$$

$$\boxed{\begin{aligned} \langle \phi \rangle &= \sqrt{v_1^2 + v_2^2} \equiv v \\ \langle \phi' \rangle &= 0 \end{aligned}}$$

$\langle \phi \rangle \longleftrightarrow \phi_{WS}$

$\langle \phi' \rangle \longleftarrow$ new

In general, ϕ and ϕ' are <u>not</u> physical

$SU(2)_R$ by v_R

$\hookrightarrow$ $\boxed{m_{\phi'} \propto v_R} \longrightarrow \boxed{10\ \text{TeV}}$

$$M_{W_R} \simeq \frac{g}{2} v_R \Rightarrow v_R \gtrsim 10\ \text{TeV}$$

$\hookrightarrow \gtrsim 5\ \text{TeV}$

SM Higgs Yukawa

$$+ \quad O\left(\left(M_{W_L} / M_{W_R} \right)^2 \leq 10^{-3} \right)$$

$$D_\mu = \partial_\mu - i g_L \hat{\vec{T}}_L \cdot \vec{A}_{\mu L} - i g_R \hat{\vec{T}}_R \vec{A}_{\mu R}$$

$$\boxed{g_L = g_R \equiv g} \quad -i\tilde{g}\,\frac{B-L}{2}\,C_\mu$$

gauge boson of $B-L$

$$D_\mu f_L = \partial_\mu - i g \frac{\sigma^i}{2} A^i_{\mu L}$$

$$- i\tilde{g}\,\frac{B-L}{2}\,C_\mu$$

$f = q \Rightarrow B-L = 1/3$

$f = \ell \Rightarrow B-L = -1$

$$D_\mu f_R = (L \to R)\, f_R$$

$$D_\mu \Phi = \;?$$

$$\boxed{(B-L)\,\Phi = 0}$$

$\Uparrow$

$$\Downarrow \qquad\qquad \bar{f}_L \, \Phi \, f_R$$

$$D_\mu \Phi = \partial_\mu - ig \frac{\sigma^i}{2} A^i_{\mu L} \Phi +$$

$$+ ig \Phi \frac{\sigma^i}{2} A^i_{\mu R}$$

$$\Leftrightarrow \quad \boxed{\Phi \to U_L \Phi U_R^+}$$

- $\boxed{\Delta_{L,R}}$ $\qquad \boxed{(B-L)(\Delta_{L,R}) = 2}$

$$\Downarrow \quad \boxed{\Delta_L \to U_L \Delta_L U_L^+}$$

$$D_\mu \Delta_L = \partial_\mu - ig \frac{\sigma_i}{2} A_{\mu i L} \Delta_L +$$

$$+ ig A_{\mu i L} \Delta_L \frac{\sigma_i}{2} - i g' C_\mu \Delta_L$$

$$\boxed{D_\mu \Delta_L = \partial_\mu - ig\left[\frac{\sigma_i}{2}, \Delta_L\right] A_{\mu i L} - i\tilde{g}\, c_\mu \Delta_L}$$

$$\boxed{D_\mu \Delta_R = L \leftrightarrow R}$$

- $(D_\mu \phi)^\dagger (D^\mu \phi)$ — SM Higgs

- $\mathrm{Tr}(D_\mu \Phi)^\dagger (D^\mu \Phi)$ — LR Higgs

$$\mathrm{Tr}\,(D_\mu \Delta_{L,R})^\dagger (D^\mu \Delta_{L,R})$$

$$\mathrm{Tr}\,\Phi^\dagger \Phi \qquad \Phi \to U_L \Phi U_R^\dagger$$

$$\Phi^\dagger \Phi \to U_R \Phi^\dagger U_L^\dagger U_L \Phi U_R^\dagger$$

$$= U_R \Phi^\dagger \Phi U_R^\dagger \neq \text{inv.}$$

$$\boxed{\begin{aligned} \operatorname{Tr} \Phi^\dagger \Phi &\to \operatorname{Tr} U_R \Phi^\dagger \Phi U_R^\dagger \\ &= \operatorname{Tr} \Phi^\dagger \Phi = \text{inv.} \end{aligned}}$$

$$\left[\begin{aligned} A^i_{\mu L} &\longleftrightarrow A^i_{\mu R} \\ f_L &\longleftrightarrow f_R \\ \Phi &\longleftrightarrow \Phi^\dagger \\ \Delta_L &\longleftrightarrow \Delta_R \end{aligned}\right]$$

$$- \quad g_L = g_R$$

$$\Downarrow$$

$$\boxed{\begin{aligned} V &= V(L \leftrightarrow R) \\ \mathcal{L}_Y &= \mathcal{L}_Y(L \leftrightarrow R) \end{aligned}}$$

$\boxed{\text{Crucial}}$ $\boxed{\Delta_{L,R} - \text{Yukawa}}$

$$\mathcal{L}_Y^{\Delta} = \underset{(D)}{l_L^T} \, i\sigma_2 \, \Delta_L \, C \, \underset{(D)}{l_L} + L \leftrightarrow R$$

B-L : -1 $\qquad\qquad\qquad -1$

$$\Downarrow$$

$$g_\Delta \, e_L^T \, e_L \, \delta_L^{++}$$

Q_{em}: -1 $\quad -1$

$$\boxed{\delta_L^{--} \rightarrow e + e}$$

$$SU(2)_L \times U(1)_Y$$

$$\downarrow$$

$$U(1)_{em}$$

$$\Downarrow$$

$$e = g \sin\theta_W$$

$$\tan\theta_W = g'/g$$

$$\longleftrightarrow$$

$$SU(2)_R \times U(1)_{B-L}$$

$$\downarrow$$

$$U(1)_Y$$

$$\Downarrow$$

$$g' \doteq g \sin\theta_{LR}$$

$$\tan\theta_{LR} = \bar{g}/g$$