BBSM Neutrino Course

Lecture XVII

LMU

Spring 2020

$$\boxed{\text{Symmetry breaking: global vs local}}$$

- U(1) gauge GROUP

- matter = fermions

$\psi_L \ (Q=1)$, $\psi_R \ (Q=0)$

$$\psi_{L/R} \to e^{i\alpha Q} \psi_{L,R} \qquad \alpha = \alpha(x) \ (1)$$

$$Q\psi_L = 1, \quad Q\psi_R = 1$$

$$\cancel{m \bar{\psi}_L \psi_R} \Rightarrow \phi \in \mathbb{C}$$

$$\boxed{\mathcal{L}_Y = y \bar{\psi}_L \phi \psi_R + h.c.}$$

$$Q\phi = \phi \qquad \phi \to e^{iQ\alpha} \phi \ (2)$$

$$\mathcal{L} = i\,\bar{\Psi}_L \gamma^\mu D_\mu \psi_L + i\,\bar{\psi}_R \gamma^\mu D_\mu \psi_R$$

$$+ \frac{1}{2}(D_\mu \phi)^\dagger (D^\mu \phi) - \mathcal{L}_Y$$

$$- V(\Phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$+ \mathcal{L}_{gf}$$

$$D_\mu = \partial_\mu - i g\, Q\, A_\mu$$

$$A_\mu \longrightarrow A_\mu + \frac{1}{g}\partial_\mu \alpha(x)$$

(3)

$$V = \frac{\lambda}{4}\left(\phi^*\phi - v^2\right)^2$$

$$\mathcal{M}_0 = \{ \phi_0 : V(\phi_0) = V_{min} = 0 \}$$

$$= \{ |\phi_0|^2 = v^2 \} = S_1$$

$$\boxed{\Phi_R = (v + h + iG)}$$

$\boxed{1961}$

Higgs

Goldstone

(Nambu)

$$V = \frac{\lambda}{4}\left[(v+h)^2 + G^2 - v^2\right]^2$$

$$= \frac{\lambda}{4}\left[2vh + h^2 + G^2\right]^2$$

$$= + \cdots G^4$$

$$\boxed{m_G = 0}$$

Global case: Nambu – Goldstone

$$\boxed{m_h^2 = 2\lambda v^2}$$

Higgs field

$$\boxed{m_h \propto v}$$

generic

$$D_\mu \phi = \partial_\mu h + i \partial_\mu G - i g A_\mu \left[(v+h) + i G \right]$$

$$\frac{1}{2} (D_\mu \phi)^\dagger (D^\mu \phi) = \cdots + \frac{1}{2} g^2 A_\mu A^\mu \times (v+h)^2$$

$$\Rightarrow \boxed{m_A = g v}$$

$$g m_A \, h \, A^\mu A_\mu$$

global

$$\boxed{\phi_{exp} = e^{i G / v} (Q = 1) (v + h)}$$

local

$$= v + h + i G + \cdots$$

$$|\phi_{exp}|^2 = (v+h)^2 \Rightarrow m_G = 0$$

$\underline{U(1)}$: $G \to G + v\alpha$

$\Leftrightarrow \quad \phi \to e^{i\alpha Q}\,\phi$

• $\underline{\text{global}}$ $\propto (\partial_\mu G)$

$$\phi_{exp} = e^{iG/v}(v+h)$$

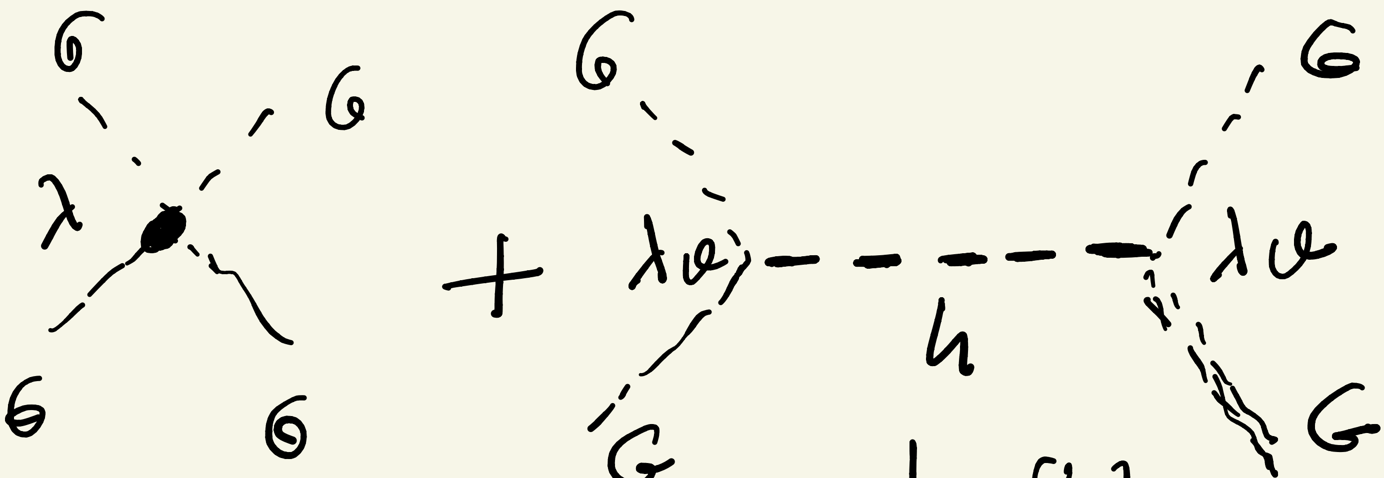
$$\phi_{exp} \to e^{-iG/v}\,\phi_{exp} = v+h$$

$$\partial_\mu \phi \to \partial_\mu G/v$$

all G out.

linear $\quad \phi = v + h + iG$

$$\lambda G^4 \quad + \lambda v h G^2$$



(a) (b)

$$\downarrow \quad \frac{1}{q^2 - m_h^2}$$

$$(b) \quad \lambda^2 v^2 \frac{1}{m_h^2} \simeq \frac{\lambda^2 v^2}{\lambda v^2}$$

$q \to 0$

$$\simeq \lambda$$

$$\Rightarrow \boxed{a + b = 0} \quad \boxed{\text{Prove}}$$

$$\frac{1}{q^2 - m_W^2} \simeq \frac{1}{m_W^2}\left(1 + \frac{q^2}{m_W^2} + \dots\right)$$

- - - - - -

cancel

gauge case

$$\Phi_{exp} = e^{iG/v}(v+h)$$

$$\boxed{\begin{aligned} &G \longrightarrow G + v\,\alpha(x) = 0 \\ &\alpha(x) \;\therefore\; G' = 0 \end{aligned}}$$

$\boxed{m_A \neq 0 \quad 3 \text{ dof}}$

initially $\quad A\ (d=2) \quad \phi \in \mathbb{C}$

$m_A = 0 \qquad d=2$

finally $\quad A\ (d=3) \quad h \in \mathbb{R}$

$m_A = qv$

$$3+1 = 2+2$$

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unitary picture $\quad \Phi = v + h$

$m_A = qv$

$$\boxed{\Delta_{\mu\nu}(A) \propto \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{m_A^2}}{k^2 - m_A^2} \quad \text{(bad)} \qquad \text{no } G}$$

$\boxed{\text{keep } G}$ Renormalizable picture

$\updownarrow$

QED : 4 $A_\mu \longrightarrow$ 2 physical

PROCA : 5 $(A_\mu, G) \rightarrow$ 3 physical
(Stuckelberg)

$$\mathcal{L}_{gf} = \frac{1}{2\xi}\left(\partial_\mu A^\mu + \xi m_A G\right)^2$$

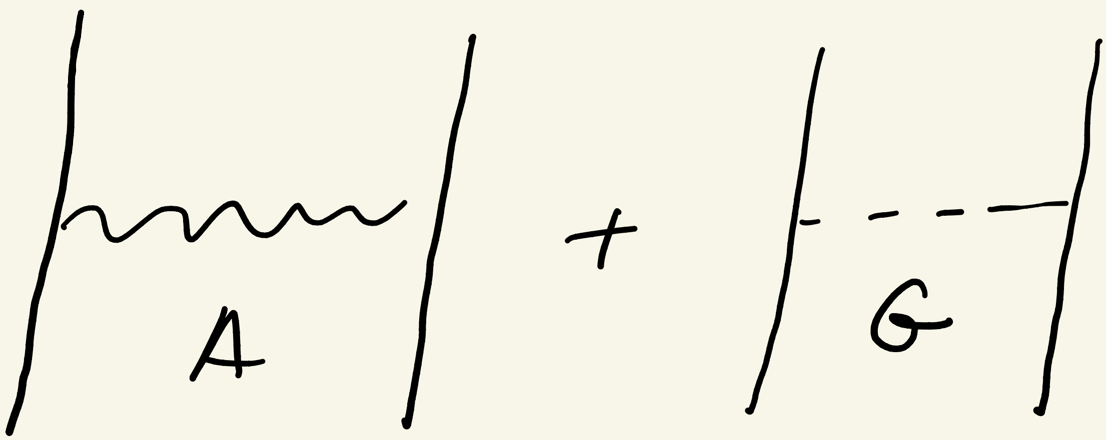
$$\Downarrow$$

$$\Delta_{\mu\nu}(A) \propto \frac{g_{\mu\nu} + (\xi - 1)\dfrac{k_\mu k_\nu}{k^2 - \xi m_A^2}}{k^2 - m_A^2} \qquad (4)$$

$$D(G) \propto \frac{1}{k^2 - \xi m_A^2} \qquad m_G = \sqrt{\xi}\, m_A$$

(5)

<u>NOT</u> a particle



= ξ - independent

$$\boxed{\Delta_{\mu\nu}(A) = \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{m_A^2}}{k^2 - m_A^2}} + \frac{f(\xi)\frac{k_\mu k_\nu}{m_A^2}}{k^2 - m_A^2}$$

$$= \frac{g_{\mu\nu} + (\xi - 1)\dfrac{k_\mu k_\nu}{k^2 - \xi m_A^2}}{k^2 - m_A^2} \quad (\xi)$$

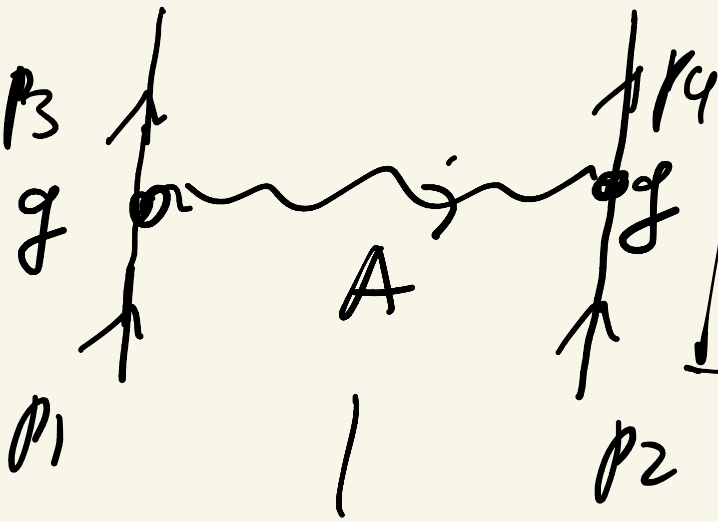
$$\Downarrow$$

$$\boxed{f(\xi) = ?}$$

$$\boxed{f(\xi)\ \text{piece} + G = 0}$$

$$\boxed{\xi\ \text{cancels}}$$

$$\boxed{\text{gauge invariance}}$$



$\boxed{\text{only } L}$

$\boxed{\text{unitary}}$

$$k = p_1 - p_3 = p_4 - p_2$$

$$\bar{\psi}_L \gamma^\mu D_\mu \psi_L + \cancel{L \leftrightarrow R} \qquad \boxed{Q\psi_R = 0}$$

$$\Downarrow$$

$$g\, \bar{\psi}_L A_\mu \gamma^\mu \psi_L \propto g\, \bar{\psi} A_\mu \gamma^\mu (1+\gamma_5)\psi$$

$$\Downarrow$$

$$\propto \bar{\Psi}(p_3)\gamma^\mu(1+\gamma_5)\Psi_1 \quad \bar{\Psi}(p_4)\gamma^\nu(1+\gamma_5)\Psi(p_2)$$

$$\times \frac{\overset{(i)}{g_{\mu\nu}} - \overset{(ii)}{\dfrac{(p_1-p_3)_\mu(p_4-p_2)_\nu}{m_A^2}}}{k^2 - m_A^2}$$

(ii)

$$\bar{\Psi}(p_3)\gamma^\mu(p_1-p_3)_\mu(1+\gamma_5)\Psi(p_1) =$$

$$= \bar{\Psi}(p_3)\gamma^\mu(p_1-p_3)_\mu\Psi(p_1) + \quad (a)$$

$$+ \bar{\Psi}(p_3)\gamma^\mu(\ldots)_\mu\gamma_5\Psi(p_1) \quad (b)$$

$$\Rightarrow \quad (a) = (m-m)\bar{\Psi}\Psi = 0$$

$$(b) = m\bar{\Psi}\gamma_5\Psi - (-)m\bar{\Psi}\gamma_5\Psi$$

$$\bar{\Psi}\gamma^\mu p_{1\mu}\gamma_5\Psi = -\bar{\Psi}\gamma_5\gamma^\mu p_{1\mu}\Psi$$

$$= 2m\bar{\Psi}\gamma_5\Psi$$

$$\Updownarrow$$

$$\boxed{k^\mu\bar{\Psi}\gamma_5\gamma_\mu\Psi \propto m\bar{\Psi}\gamma_5\Psi}$$

$$\boxed{k^\mu\bar{\Psi}\gamma_\mu\Psi = 0 \quad \text{conserved}}$$

$$m = 0 \rightarrow k^\mu\bar{\Psi}\gamma_5\gamma_\mu\Psi = 0$$

ii $\quad g^2 \dfrac{m_f^2}{m_A^2} (\bar{\psi}\gamma_5\psi)^2 \dfrac{1}{k^2 - m_A^2}$

$+ \; - - -$

$f\text{-}f$ scattering $M \propto$

U(1) gauge theory

no G field

local

global case $\Rightarrow$ there is G

Global $\Longleftrightarrow g = 0$

$$D_\mu = \partial_\mu - i g A_\mu Q$$

$$\uparrow$$

$$g \to 0 \Rightarrow D_\mu \to \partial_\mu$$

NO gauge

$$\mathcal{L}_{kin} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\alpha \neq \alpha(x)$$

Amplitude $= (i) + (ii)$

$$\Updownarrow$$

$$(ii) = g^2 \frac{m_f^2}{m_A^2} (\bar{\psi}\gamma_5\psi)^2 \frac{1}{k^2 - m_A^2}$$

$$(i) = g^2 (\bar{\psi}\gamma^\mu L \psi)^2 \frac{1}{k^2 - m_A^2}$$

$$\boxed{\text{global}: \; g \to 0}$$

$$m_A = g v$$

$g \to 0$

$(i) \longrightarrow 0$

(ii)

$$\downarrow$$

$$\boxed{\frac{m_f^2}{v^2} (\bar{\psi}\gamma_5\psi)^2 \frac{1}{k^2}}$$

NG

Amplitude $\longleftrightarrow$ exchange of

$m_G = 0$ boson!

$$\frac{\partial_\mu G}{v} \bar{\psi} \gamma^\mu \frac{1+\gamma_5}{2} \psi \Rightarrow$$

$$\rightarrow \frac{G}{v} k^\mu \bar{\psi} \gamma_\mu \frac{1+\gamma_5}{2} \psi$$

$$= G/v \, \bar{\psi} \gamma_5 \psi \, m_f$$

$$\Downarrow$$

$$\boxed{\text{exchange of NG boson!}}$$

SM

$SU(2)_L \times U(1)$

$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \Big| \quad \begin{matrix} u_R \\ d_R \end{matrix}$

wall

$\Phi \to U\Phi$

$\begin{pmatrix} u \\ d \end{pmatrix}_L \to U \begin{pmatrix} u \\ d \end{pmatrix}_L$

$\Updownarrow$

U(1) example

Minimal theory

$\Downarrow$

$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \Big| \; e_R$

$$\boxed{m_\nu = 0}$$

$\boxed{\text{if } \underline{P} \text{ is good}}$

$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} u \\ d \end{pmatrix}_R$

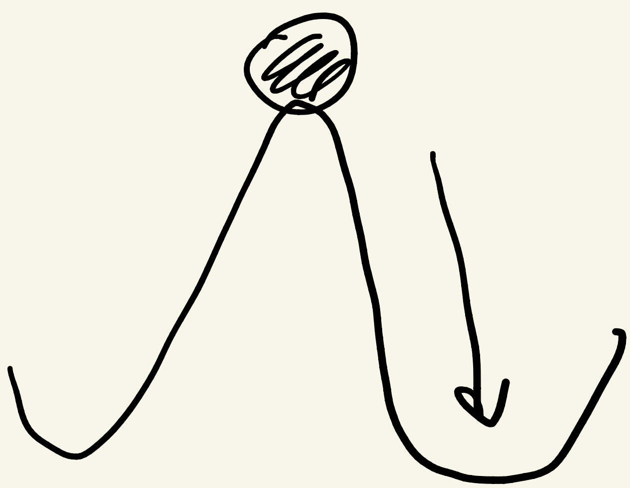
$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu \\ e \end{pmatrix}_R \Rightarrow \nu_R$

$$\boxed{m_\nu \neq 0}$$

$\not{P}$ maximal for me, --

$\underline{P}$ good for w,

$\not{\underline{P}}$ spontaneously



Minimal theory

$$G_{LR} = SU(2)_L \times SU(2)_R (\times P)$$

$$Q_{em} = T_{3L} + T_{3R}$$

$L \longleftrightarrow R$

symmetry

$\pm 1/2$ wrong!

$\Downarrow$

$$\boxed{G_{LR} = SU(2)_L \times SU(2)_R \times U(1)}$$

Remark '57 Schwinger

$SU(2)$ - ew

$\downarrow$

$$\boxed{W^+, W^-, A}$$

'61 Glashow $\Rightarrow$ $U(1)$

LR

$$Q = T_{3L} + \underbrace{T_{3R} + \frac{Y'}{2}}_{Y/2}$$

$\boxed{\text{matter}}$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$\Downarrow$

$$\boxed{m_\nu \neq 0}$$

$\begin{pmatrix}u\\d\end{pmatrix}$ $\quad Y' = 1/3 = Y \quad$ (SM)

$$\Big\updownarrow \text{LR}$$

$\begin{pmatrix}u\\d\end{pmatrix}_R$ $\quad Y' = 1/3 = Y$

$\begin{pmatrix}\nu\\e\end{pmatrix}_L$ $\quad Y' = Y = -1$

$$\Big\updownarrow \text{LR}$$

$\begin{pmatrix}\nu\\e\end{pmatrix}_R$ $\quad Y' = Y = -1$

$Y' = 1/3$ for quarks

$Y' = -1$ for leptons

$$\Downarrow$$

$$\boxed{Y' = B - L}$$

$\boxed{\text{gauged } B-L}$

SM $\Rightarrow$ global B, L

baryon   lepton

$\cancel{B+L}$ by axial anomaly

$B-L$ __no__ anomaly

$\Downarrow$

$$\boxed{G_{LR} = SU(2)_L \times SU(2)_R \times U(1)_{B-L}}$$

$\Downarrow$

$\boxed{\text{Higgs ???}}$