BBSM Neutrino Course

Lecture XV

LMU

Spring 2020

$\boxed{\text{In praise of SM}}$

$$q_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \Bigg| \quad \begin{matrix} u_R \\ d_R \end{matrix}$$

Weinberg '67
Higgs doublet

$$G \,|\, SU(2)_L \times U_Y(1)$$

$$\mathcal{L}_Y = y_d^{ij} (\bar{u}\,\bar{d})_L^{0i}\, \Phi\, d_R^{0j} \quad +$$

$$\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$

$$+ y_d^{ij} \, (\bar{u}\,\bar{d})_L^{0i}\, i\sigma_2 \Phi^* \, u_R^{0j} \quad +$$

$$+ y_e^{ij} \, (\bar{u}\,\bar{d})_L^{0i}\, \Phi\, e_R^{0j} \quad + \ h.c.$$

$$i, j = 1, 2, \ldots, N_g \qquad (N_g = 3)$$

Rule: write down all gauge inv. interactions

$$\Phi_{un} = \begin{pmatrix} 0 \\ h + v \end{pmatrix}$$

Coleman

"Hidden symmetry"

vacuum expectation value (vev)

$$\boxed{\begin{aligned} M_W &= \frac{g}{2} v \\ M_Z \cos\theta_W &= M_W \end{aligned}}$$

$$\tan\theta_W = \frac{g'}{g}$$

$$e = g \sin\theta_W$$

$$g = g_W$$

$$\Downarrow$$

$$\mathcal{L}_Y = \bar{d}_L^{\,0} M_d \, d_R^{\,0} \left(1 + h/v\right) + h.c.$$

$$d^0_{L\,(R)} = \begin{pmatrix} d \\ s \\ b \\ \vdots \end{pmatrix}^0_{L\,(R)} \qquad M_d^{ij} = Y_d^{ij}\, v$$

Higgs

$$= \bar{d}_L \, \tilde{m}_d \, d_R \left(1 + h/v\right) + h.c.$$

$$\tilde{m}_d \equiv m_d = \mathrm{diag}(m_d, m_s, m_b \ldots)$$

$\in \mathbb{R}$

$$d^0_{L,R} = U_{L,R}\, d_{L,R} \quad (\text{column})$$

$$\Downarrow$$

$$\boxed{U_L^\dagger M_d U_R = \tilde{m}_d}$$

$$M_d M_d^\dagger = H_1 \qquad M_d^\dagger M_d = H_2$$

$$U_L^\dagger H_1 U_L = U_L^\dagger M_d M_d^\dagger U_L$$

$$= U_L^\dagger M_d U_R U_R^\dagger M_d^\dagger U_L$$

$$= \tilde{m}_d^2$$

$$U_R^\dagger H_2 U_R = \tilde{m}_d^2$$

$$\boxed{\begin{array}{c} U_L^\dagger U_L = U_R^\dagger U_R \\ \| \\ 1 \end{array}}$$

$d \to u \to e$

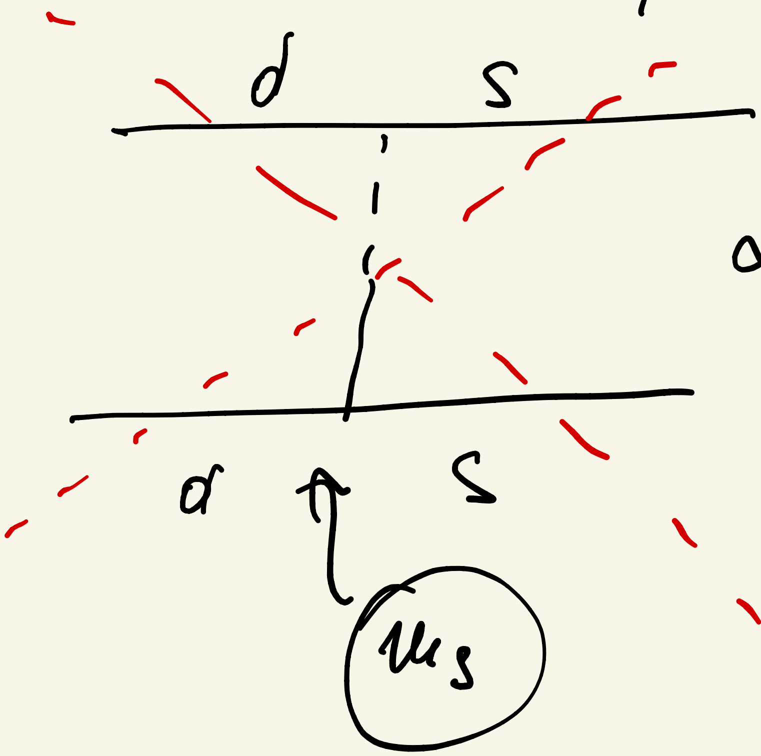
the same

$$\Downarrow$$

$$\mathcal{L}_y = \left(m_d \bar{d} d + m_s \bar{s} s + m_b \bar{b} b + \cdots\right)\left(1 + \frac{h}{v}\right)$$

Higgs = flavor diagonal

$$\underbrace{d,\ u\ ---\ e,\nu}_{\text{flavor}}$$



$$\propto \frac{m_s^2}{m_h^2}$$

$$\left(-\frac{1}{3}e\right)A_\mu\left(\bar{d}_L^0\gamma^\mu d_L^0 + \bar{s}_L^0\gamma^\mu s_L^0 + \bar{b}_L^0\gamma^\mu b_L^0 \cdots\right) + L\to R$$

$$=\left(-\frac{1}{3}e\right)A_\mu\left[\bar{d}_L^0\gamma^\mu d_L^0 + L\leftrightarrow R\right]$$

column

$$d_L = \begin{pmatrix} d\\ s\\ b\\ \vdots \end{pmatrix}_L$$

$$= \left(-\tfrac{1}{3} e A_\mu\right)\Big[\bar{d}_L U_L^\dagger \gamma^\mu U_L d_L \ \text{colour} + L \leftrightarrow R\Big]$$

$$U_L^\dagger U_L = 1$$

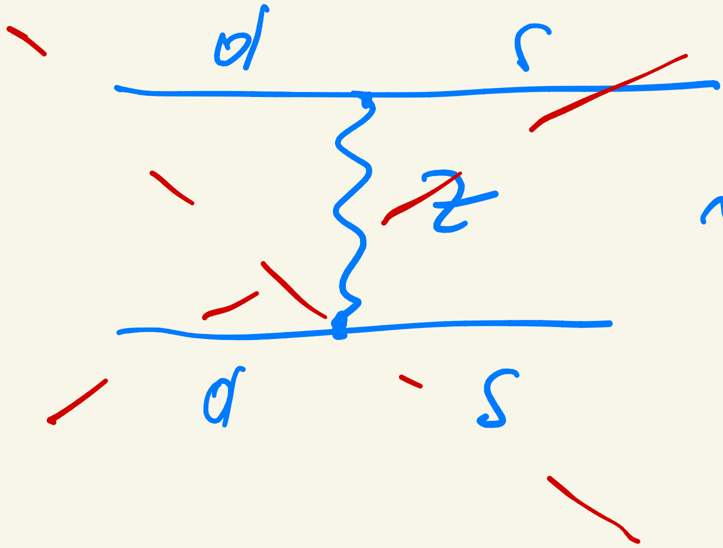
$$e J_\mu^{em} A_\mu \qquad J^\mu_{em} = \bar{f}\gamma^\mu Q_{em} f$$

$$\frac{g}{\cos\theta_W} J_\mu^Z Z_\mu \qquad J^\mu_Z = \bar{f}\gamma^\mu\left[T_3 - Q\sin^2\theta_W\right] f$$

gen. independent

$$\Downarrow$$

$\boxed{\text{flavor diagonal}}$



$$\sim \frac{g^2/\cos^2\theta_W}{M_Z^2}$$

$$\Downarrow$$

$$\frac{g}{\sqrt{2}} W_\mu^+ \left[\bar{u}_L^0 \gamma^\mu d_L^0 + \bar{c}_L^0 \gamma^\mu s_L^0 + \bar{t}_L^0 \gamma^\mu b_L^0 + \cdots\right]$$

$$= \frac{g}{\sqrt{2}} W_\mu^+ \left(\bar{u}\ \bar{c}\ \bar{t} \cdots\right)_L^0 \gamma^\mu \begin{pmatrix} d \\ s \\ b \\ \vdots \end{pmatrix}^0$$

$$= \frac{g}{\sqrt{2}} W_\mu^+ \left(\bar{u}\ \bar{c}\ \bar{t}\right)_L U_{Lu}^\dagger \gamma^\mu U_{Ld} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L +$$

$$\boxed{U_{Lu}^\dagger U_{Ld} \equiv V_{CKM}}$$

Cabibbo; Kobayashi, Maskawa

$$\boxed{V_{CKM} \equiv V}$$

N^2 elements

$\Uparrow$

$U^{+} U = 1$

$$N^2 = \underbrace{\frac{N(N-1)}{2}}_{\text{Euler angles = rotation}} + \underbrace{\frac{N(N+1)}{2}}_{\text{phases}}$$

$\Updownarrow$

$$U \in \mathbb{R} \Rightarrow U = O : OO^T = 1$$

$\Downarrow$

$\frac{N(N-1)}{2}$ elements

$\boxed{\text{phases} \Longleftrightarrow \text{CP violation}}$

~~$N=1 \Rightarrow 2$ phases~~

~~$N=2 \Rightarrow 3$~~

not true!

KM: let there be $N_g = 3$!

$\boxed{\text{And there was 3 gen}}$

$\boxed{\#\text{ of phases??}}$

$$\bar d d\, m_d = m_d \left(\bar d_L d_R + \bar d_R d_L\right)$$

real, positive #

$$d_L \to e^{i\alpha_d} d_L, \quad d_R \to e^{i\alpha_d} d_R$$

$$m_d \left(\bar{d}_L \cancel{e^{-i\alpha_d}}\, \cancel{e^{i\alpha_d}}\, d_R + h.c.\right)$$

$$= m_d \bar{d} d$$

$$\boxed{\cancel{\#\text{ of phases} = \frac{N(N+1)}{2} - 2N}}$$

$N=2 \Rightarrow$ # of phases $= -1$

$N=3 \Rightarrow$ −||− $= 0$

$$\Downarrow$$

$$u \to e^{i\alpha_u} u = \boxed{e^{i\alpha_u}} u$$

$$c \to e^{i\alpha_c} c = \boxed{e^{i\alpha_u}} e^{i(\alpha_c - \alpha_u)} c$$

$$t \to e^{i\alpha_t} t = \boxed{e^{i\alpha_u}} e^{i(\alpha_t - \alpha_u)} t$$

$$\vdots \qquad\qquad \vdots$$

$$d \to \boxed{e^{i\alpha_u}} e^{i(\alpha_d - \alpha_u)} d$$

$$s \to \boxed{e^{i\alpha_u}} e^{i(\alpha_s - \alpha_u)} s$$

$$b \to \boxed{e^{i\alpha_u}} e^{i(\alpha_b - \alpha_u)} b$$

$$\vdots$$

$$\frac{g}{\sqrt{2}} W_\mu^+ (\bar{u}\ \bar{c}\ \bar{t})_L \gamma^\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L +$$

$$\to \frac{g}{\sqrt{2}} W_\mu^+ (\bar{u}\bar{c}\bar{t}) \cancel{e^{-i\alpha_u}} \gamma^\mu \cancel{V e^{i\alpha_u}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

$$\Downarrow$$

$$\boxed{\#\text{ of phases} = \frac{N(N+1)}{2} - (2N-1)}$$

$N = 2 \Rightarrow 0\,!$

$N = 3 \Rightarrow 1$ KM phase

V_{CKM} :

$\theta_1 = \theta_{12}$ $(1 \leftrightarrow 2\text{ gen})$

$\theta_2 = \theta_{23}$ $(2-3 - " -)$

$\theta_3 = \theta_{13}$ $(1-3 - " -)$

$\theta_{12} = \theta_c$

Cabibbo

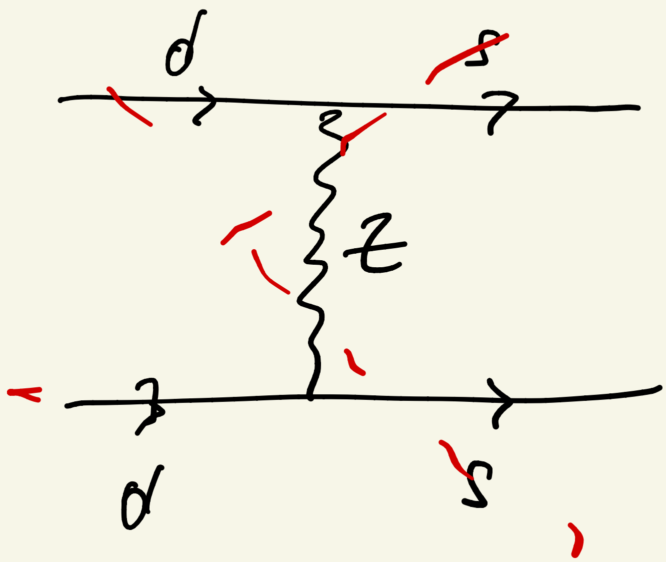
$\simeq 13^\circ$

$\delta \simeq 45^\circ$

$\theta_2 \simeq \theta_{23} \simeq 4\times 10^{-2}$

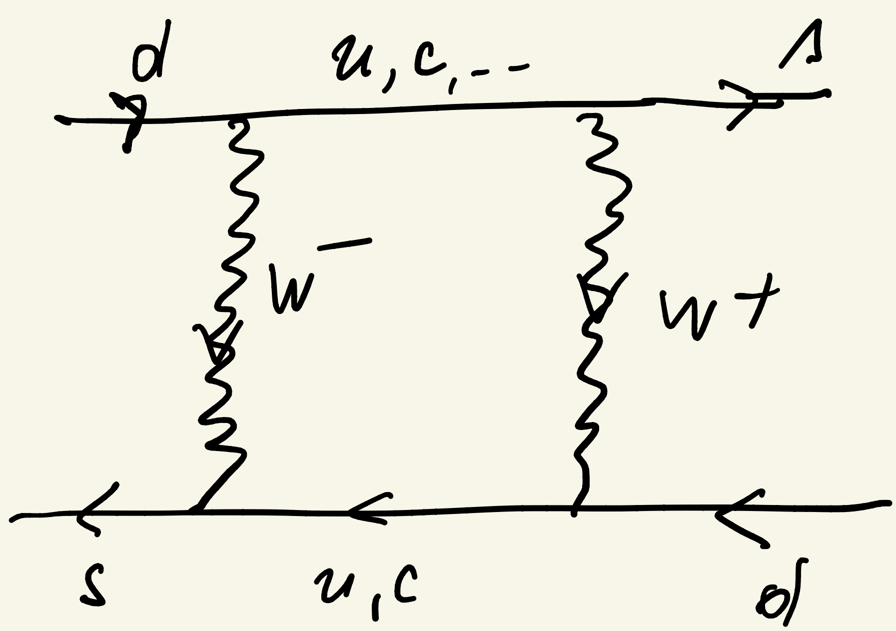
$$\boxed{\theta_3 \simeq \theta_{13} \simeq 4\times 10^{-3}}$$

$K^0 \to \pi^+\pi^-$ $\qquad$ $\boxed{K^0 \leftrightarrow \bar{K}^0}$



$$\simeq \frac{g^2}{\cos^2\theta_W M_Z^2} = \frac{g^2}{M_W^2}$$

$$\simeq G_F$$



GIM '69

c

$\begin{pmatrix} u \\ d \end{pmatrix}^0$ $\qquad$ $\begin{pmatrix} c \\ s \end{pmatrix}_0$ ← complete doublet

$$\simeq G_F\left(\frac{\alpha}{4\pi}\right)\sin^2\theta_c \times \boxed{\text{small} \simeq 10^{-3}}$$

$1/10$

$$\underbrace{\qquad}_{10^{-3}}$$

$$\boxed{\frac{m_c^2-m_u^2}{M_W}}$$

$\boxed{2\text{ gen}}$

$$V=\begin{pmatrix}\cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c\end{pmatrix}$$

$$\frac{g}{\sqrt{2}} W_\mu^{+}\left[\bar{u}\gamma^\mu d\cos\theta_c + \bar{u}\gamma^\mu s\sin\theta_c + \right.$$

$$\left. + \bar{c}\gamma^\mu d(-\sin\theta_c) + \bar{c}\gamma^\mu s\cos\theta_c\right]$$

$\cdot\ m_u = m_c = 0$

$m_c \simeq \text{GeV}$

$m_u \simeq \text{few MeV}$

$$\Downarrow$$

$$\boxed{U_{Lf}^\dagger M_f V_{Rf} = \tilde{m}_f \ (\equiv m_f)}$$

diagonal

imagine $\boxed{\tilde{m} \equiv m = m_0 \mathbb{1}}$

$$m_0 \left[\bar{u}_L u_R + \bar{c}_L c_R \right] + h.c. \qquad 2\text{ gen}$$

$$= m_0 \begin{pmatrix} \bar{u}_L & \bar{c}_L \end{pmatrix} \begin{pmatrix} u_R \\ c_R \end{pmatrix} + h.c.$$

$$\begin{pmatrix} u \\ c \end{pmatrix}_{L,R} \to V_{L,R} \begin{pmatrix} u \\ c \end{pmatrix}_{L,R}$$

$$m_0 \begin{pmatrix} \bar{u} & \bar{c} \end{pmatrix}_L \underbrace{V_L^\dagger V_R}_{1} \begin{pmatrix} u \\ c \end{pmatrix}_R + h.c.$$

$V_L = V_R$

$$W_\mu^+ (\bar{u}\,\bar{c}\,\bar{t})_L\, \gamma^\mu\, \underbrace{U_{Lu}^+ U_{Ld}}_{V_{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

$$\to W_\mu^+ (\bar{u}\,\bar{c}\,\bar{t})\, \gamma^\mu\, V_L^+ \underbrace{U_{Lu}^+ U_{Ld}}_{V_{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

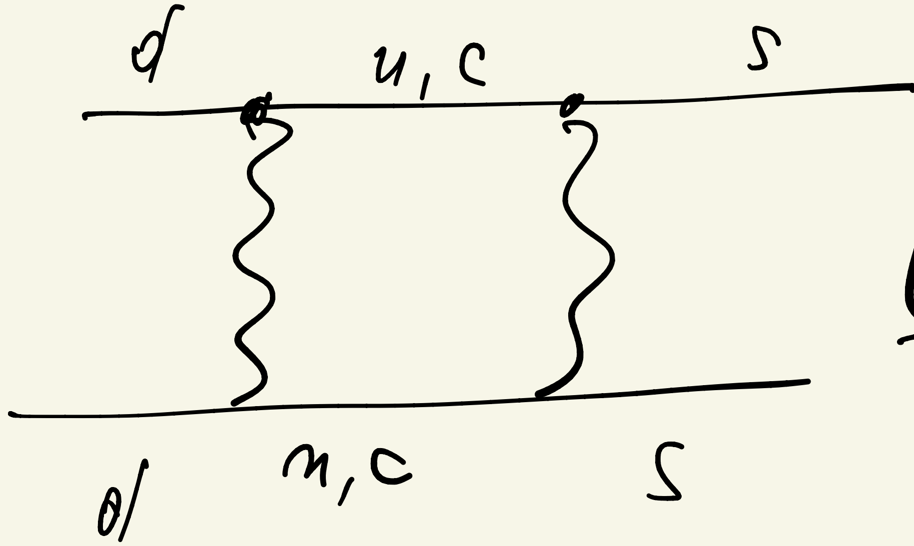
$$\bar{V}_{CKM} \to \bar{V}_L^+ \bar{V}_{CKM}$$

$$V_L = V_R = V_{CKM}$$

$$\Downarrow$$

$$\boxed{\bar{V}_{CKM} \to \mathbb{1}}$$

physics the same



$\boxed{\text{GIM}}$

$$\sim G_F \frac{\alpha}{4\pi} \sin^2\theta_c \frac{m_c^2}{M_W^2}$$

$\boxed{\text{predict } m_c!}$

Gaillard, Lee '74

$\boxed{m_c \simeq 1.5\,\text{GeV}}$

$$+ G_F \frac{\alpha}{4\pi}\left[\theta_{13}^2 \frac{m_t^2}{M_W^2}\theta_{23}^2\right]$$

negligible

small!

$$\boxed{\epsilon_{\cancel{CP}} \simeq 10^{-3}}$$

$$\cancel{CP} \simeq G_F \, \epsilon_{\cancel{CP}}$$

$\boxed{\text{SM}}$

1. Z boson – neutral current
2. $\exists$ c (G/M)
3. NFC – natural flavor conservation
   – in neutral currents

- Z, A, h :: preserve flavor
- W - violates flavor V_{CKM}
- GIM :: FV (neutral)

$$\simeq m_c^2 / M_W^2$$

- $\boxed{\text{KM} : \exists\ t, b}$

$\boxed{\exists \text{ Higgs-Weinberg}}$

$$\boxed{h \frac{m_f}{v} \bar{f} f = h \frac{g}{2} \frac{m_f}{M_W} \bar{f} f}$$

mass = dynamical

to every $m_f \to$ associated process

$h \to f\bar{f}$ predicted

$$\boxed{\Gamma(h \to f\bar{f}) = \frac{m_h}{8\pi}\left(\frac{g}{2}\frac{m_f}{M}\right)^2}$$

$$\boxed{\begin{array}{c} t,\ b,\ t,\ W,\ Z \\ \text{tested} \end{array}}$$

$$\mathcal{L}_y = (\bar{u}_L\ \bar{d}_L)^{i}_{o}\left[\, Y^{ij}_{1d}\,\Phi_1 + Y^{ij}_{2d}\,\Phi_2 \,\right] d^{j}_{R\,o}$$

$+h.c.$

Two Higgs

$$M_\phi = Y_1 v_1 + Y_2 v_2$$

$$\Phi_\alpha = \begin{pmatrix} 0 \\ v_\alpha + h_i \end{pmatrix} \quad \alpha = 1,2$$

$$U_L^\dagger M_\alpha U_R = \text{diagonal} = \tilde{m}_\alpha$$

<u>SM</u> $\bar{f} M f \left(1 + h/v\right)$

diagonal $M \Leftrightarrow$ diagonal Y

$\Rightarrow h \, m_f \bar{f} f$!!!

2 doublets $M = \text{diag} \not\Rightarrow Y_\alpha$ diag.

$\boxed{\text{LOSE: NFC}}$

$\boxed{\text{LOSE: prediction } h \to f\bar{f}}$

What if $\underline{P}$ was good?

$\Downarrow$

disaster

$\underline{P}$ Conserving world

$\Downarrow$

$$\overline{q}_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d \end{pmatrix}_R \equiv q_R$$

$$\downarrow$$

$$\mathcal{L}_Y = \overline{q}_L \left(\underline{M} + y_T \, \underline{T} \right) q_R + h.c.$$

singlet $\downarrow$ $\qquad$ $\downarrow$

$m_u = m_d \qquad \langle T \rangle = \begin{pmatrix} v_T & 0 \\ 0 & -v_T \end{pmatrix}$

vev

$$\Downarrow$$

$$\boxed{\begin{aligned} \underline{M}_u &= \underline{M} + y_T v \\ \underline{M}_d &= \underline{M} - y_T v \end{aligned}}$$

$\boxed{\underline{M}_Z = 0}$ $\boxed{\text{disparte}}$

$$T = \text{Adjoint} = \text{vector} = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix}$$

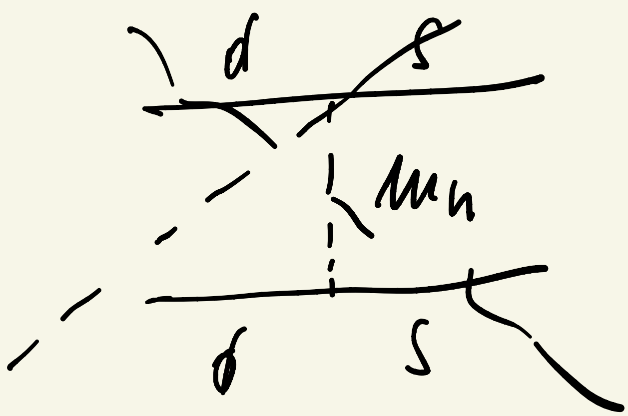
$$\langle T \rangle = \boxed{\begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}} \quad \boxed{SO(2) = U(1)}$$

- $M_d \rightarrow$ diagonal

$$\Rightarrow U_{Ld}^{\dagger} M_d U_R = \text{diagonal}$$

$$\Downarrow$$

$$Y_T = \text{diagonal} \qquad \boxed{\text{NFC}}$$



$m_h \simeq 125\,\text{GeV}$

but

$$\boxed{\begin{pmatrix}\nu\\ e\end{pmatrix}_L \xleftrightarrow{\;P\;} \begin{pmatrix}\nu\\ e\end{pmatrix}_R} \; ?$$

$$\Downarrow$$

$$\exists\, \nu_R \longleftrightarrow \nu_L$$

$$\Downarrow$$

$$\boxed{M_\nu \neq 0}$$

- 3, ℓ masses $\longleftrightarrow$ $\cancel{P}$
- $P \Rightarrow m_\nu = 0$

$$\boxed{\text{Catch 22 situation}}$$

$\not{P}$ for charged fermions

P for neutrino