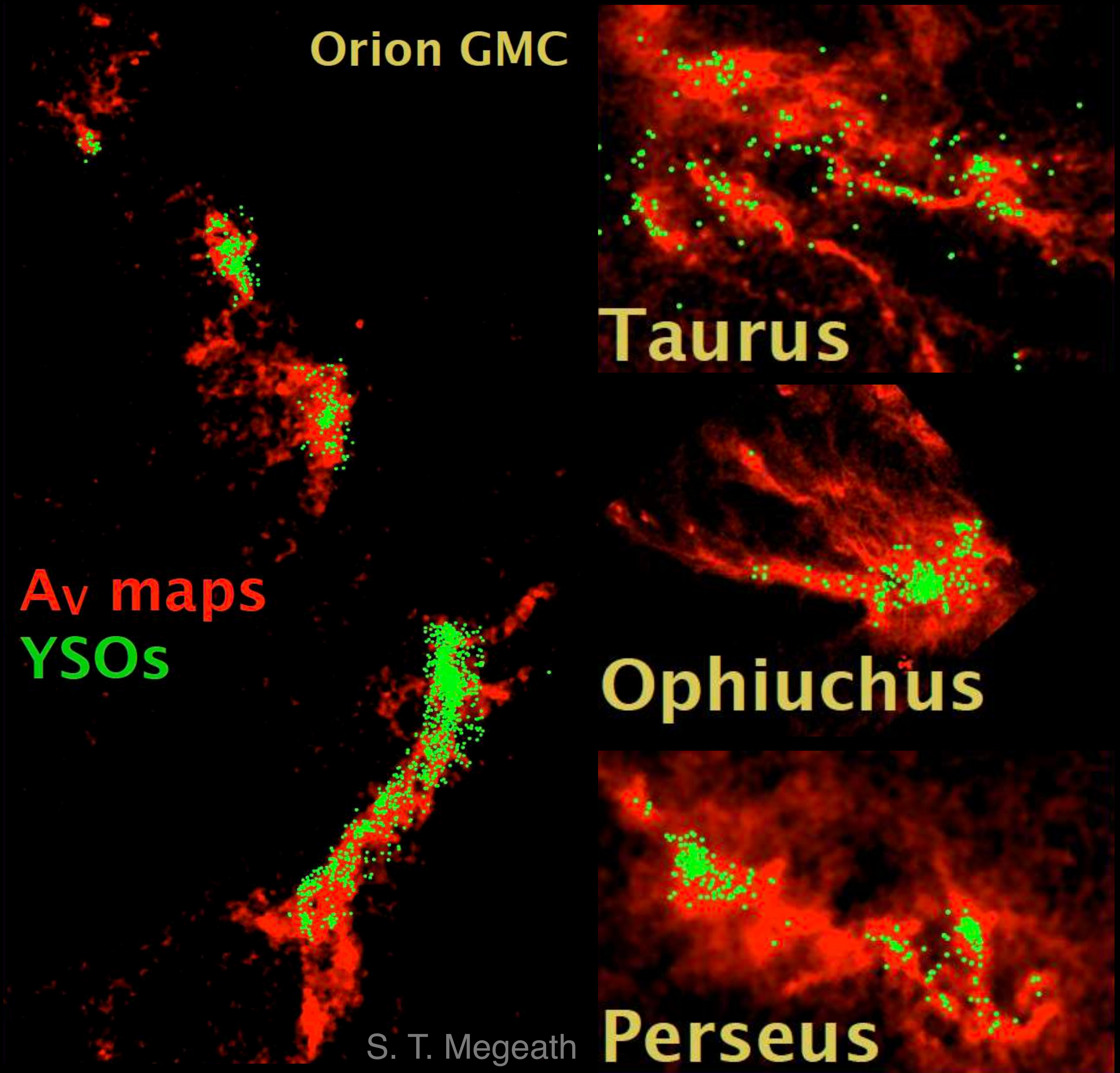
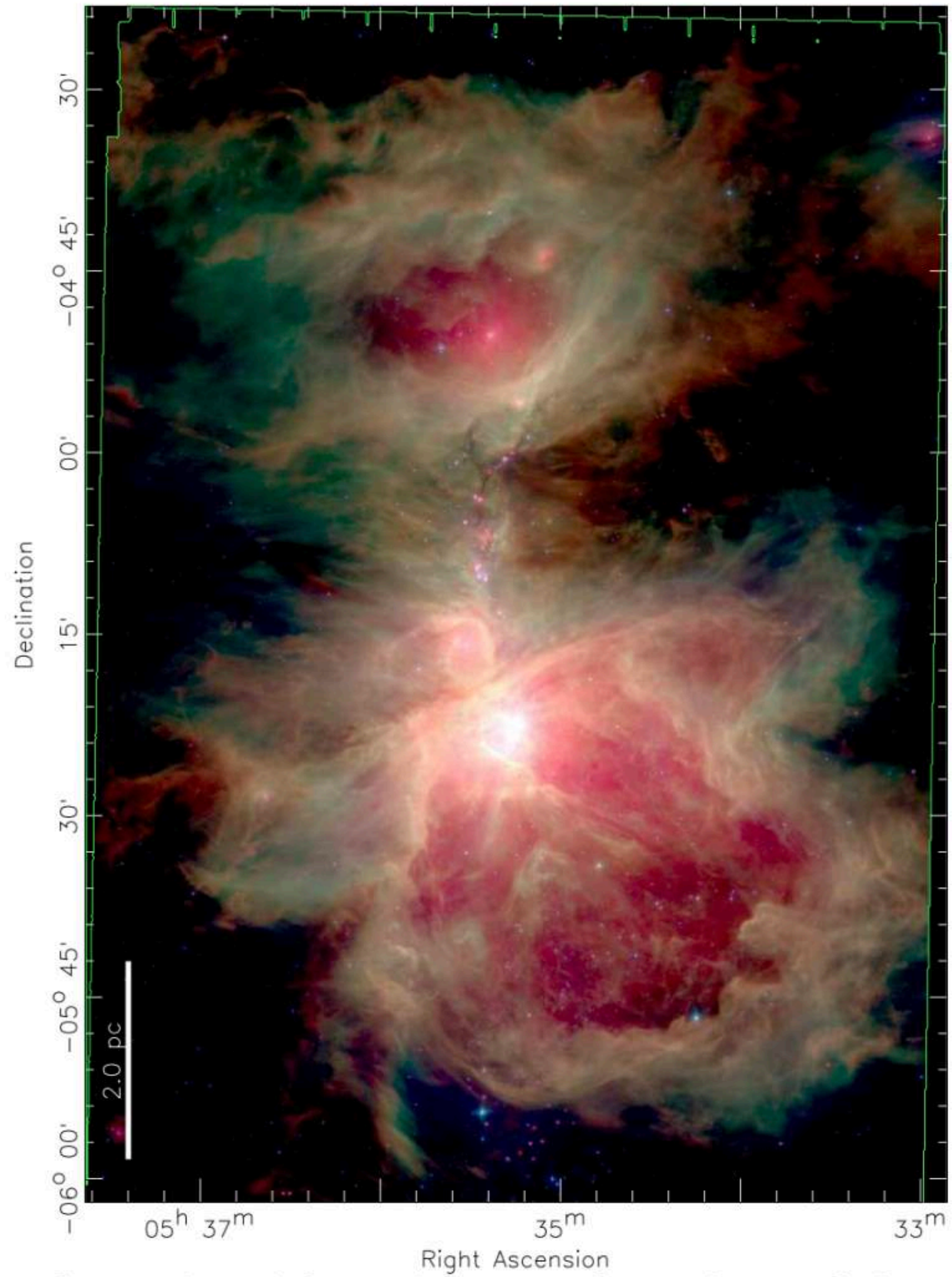
FROM UNIVERSE

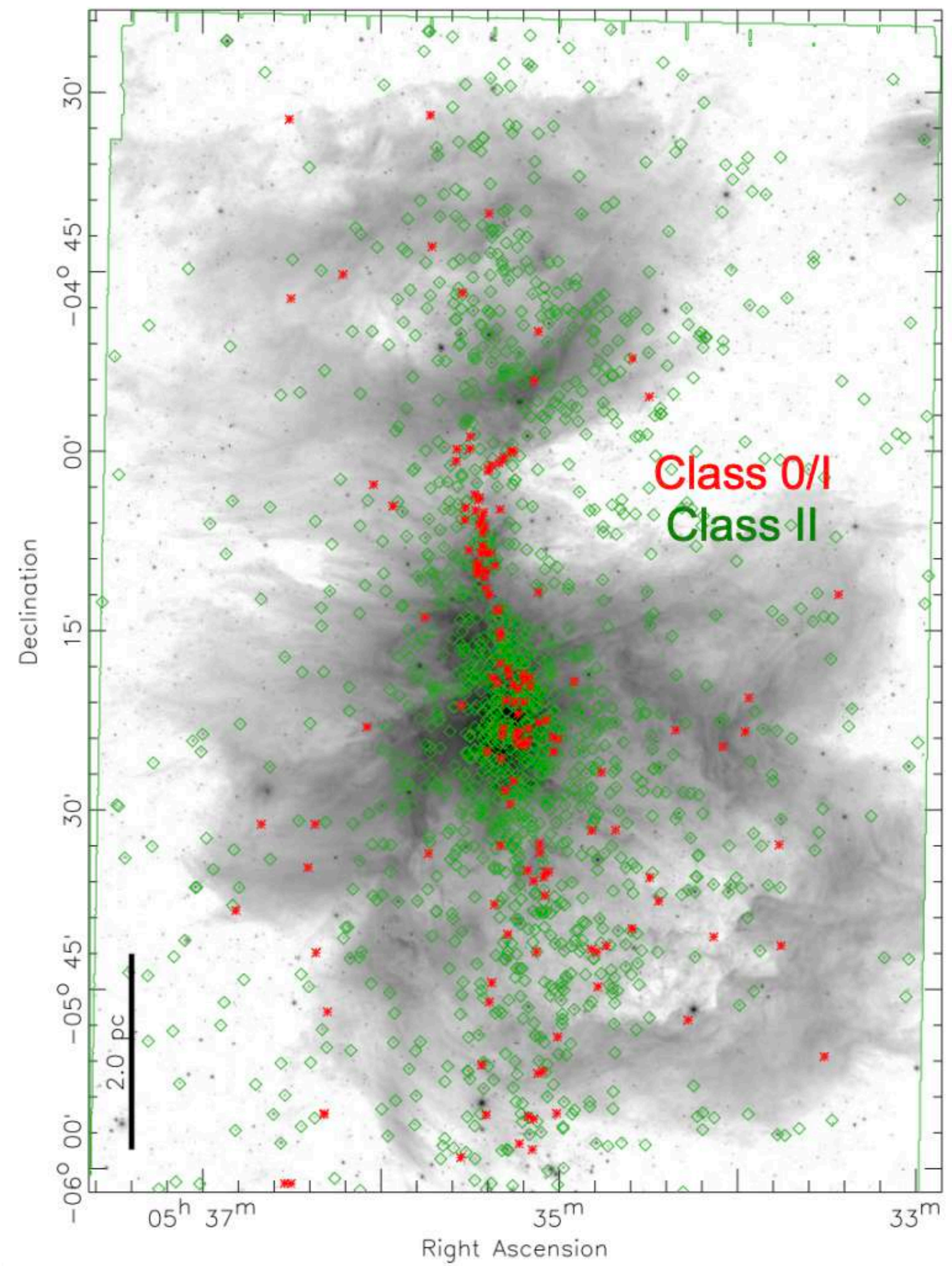
TO PLANETS

LECTURE 2

REVIEW: PROTOSTARS FORM IN COLD DARK DUSTY POCKETS







Spitzer 4.5, 5.8, +24 μm image of Northern Orion A

Megeath et al. (2006)

EMBEDDED SOURCES REVEALED IN INFRARED (NIR AND MIR)

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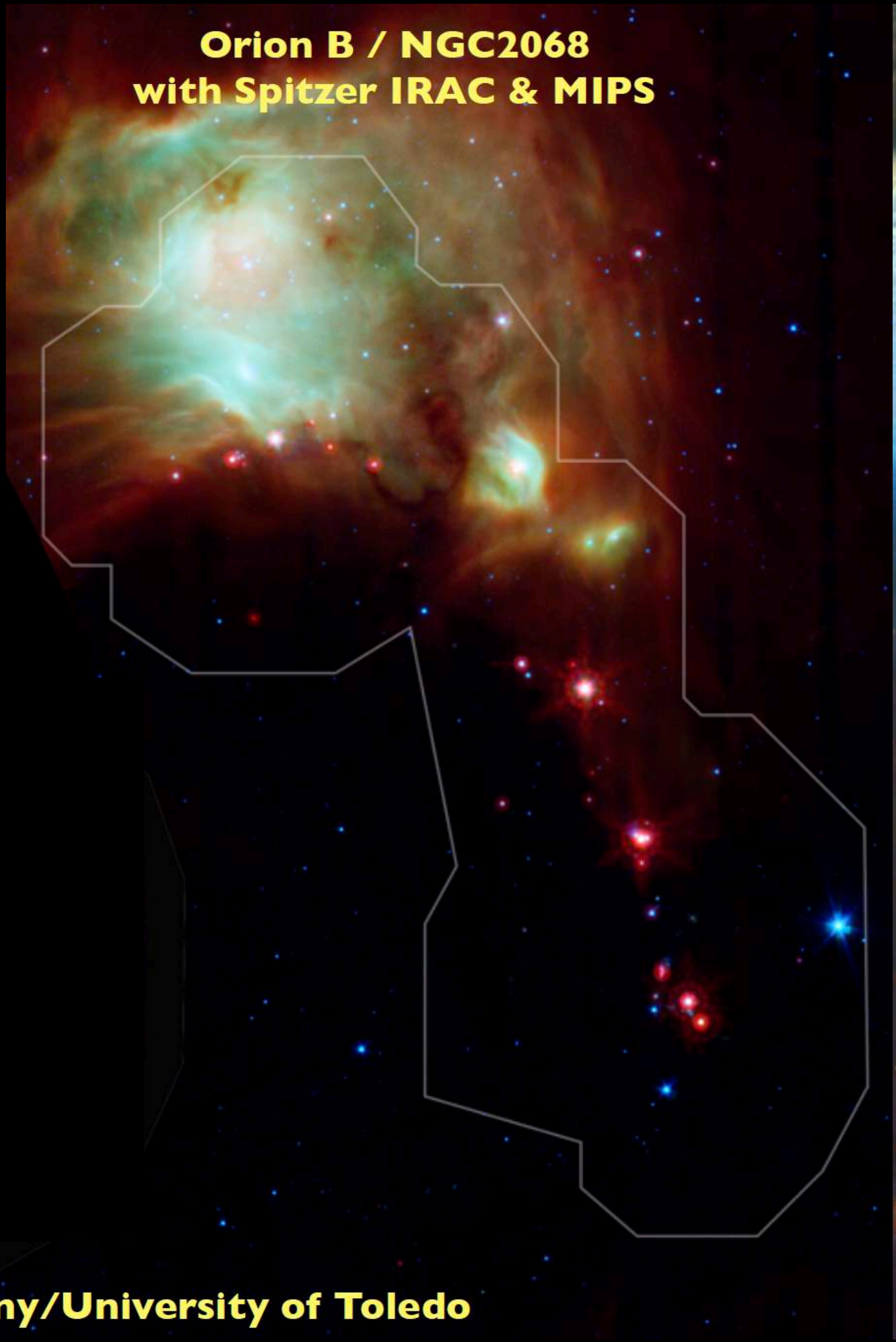
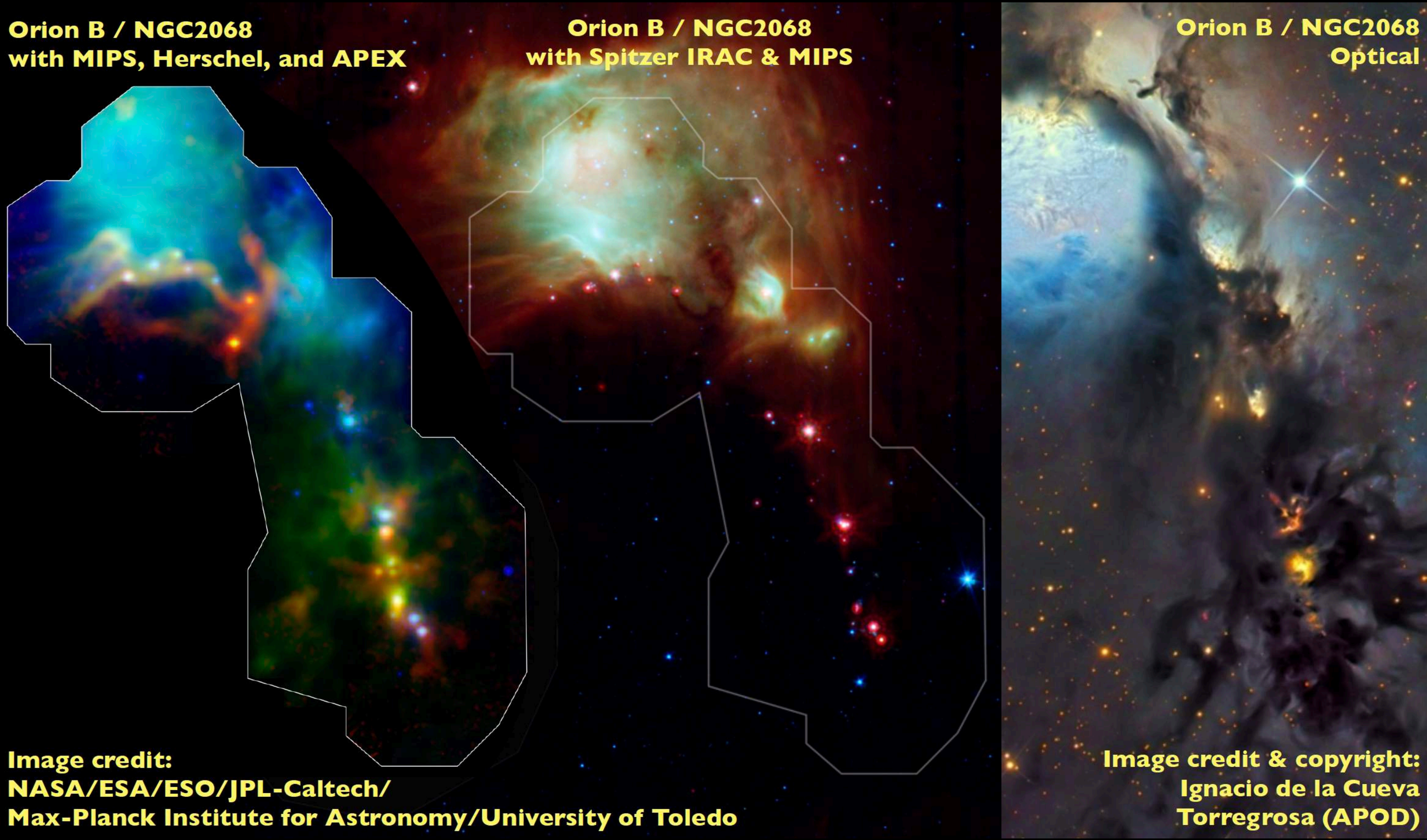


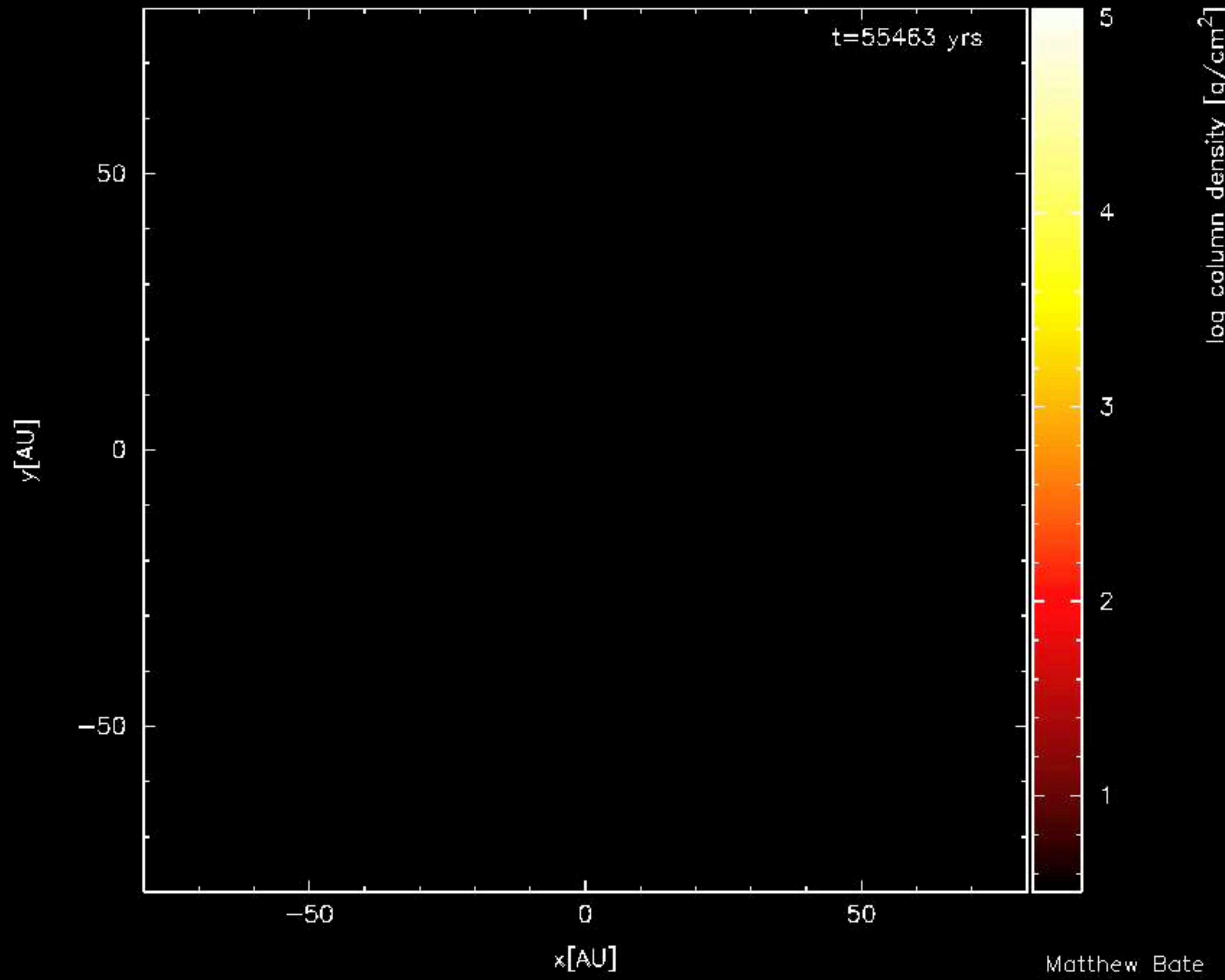
Image credit:
NASA/ESA/ESO/JPL-Caltech/
Max-Planck Institute for Astronomy/University of Toledo

EMBEDDED SOURCES REVEALED IN INFRARED (NIR AND MIR)



CORE + DISC FORMATION

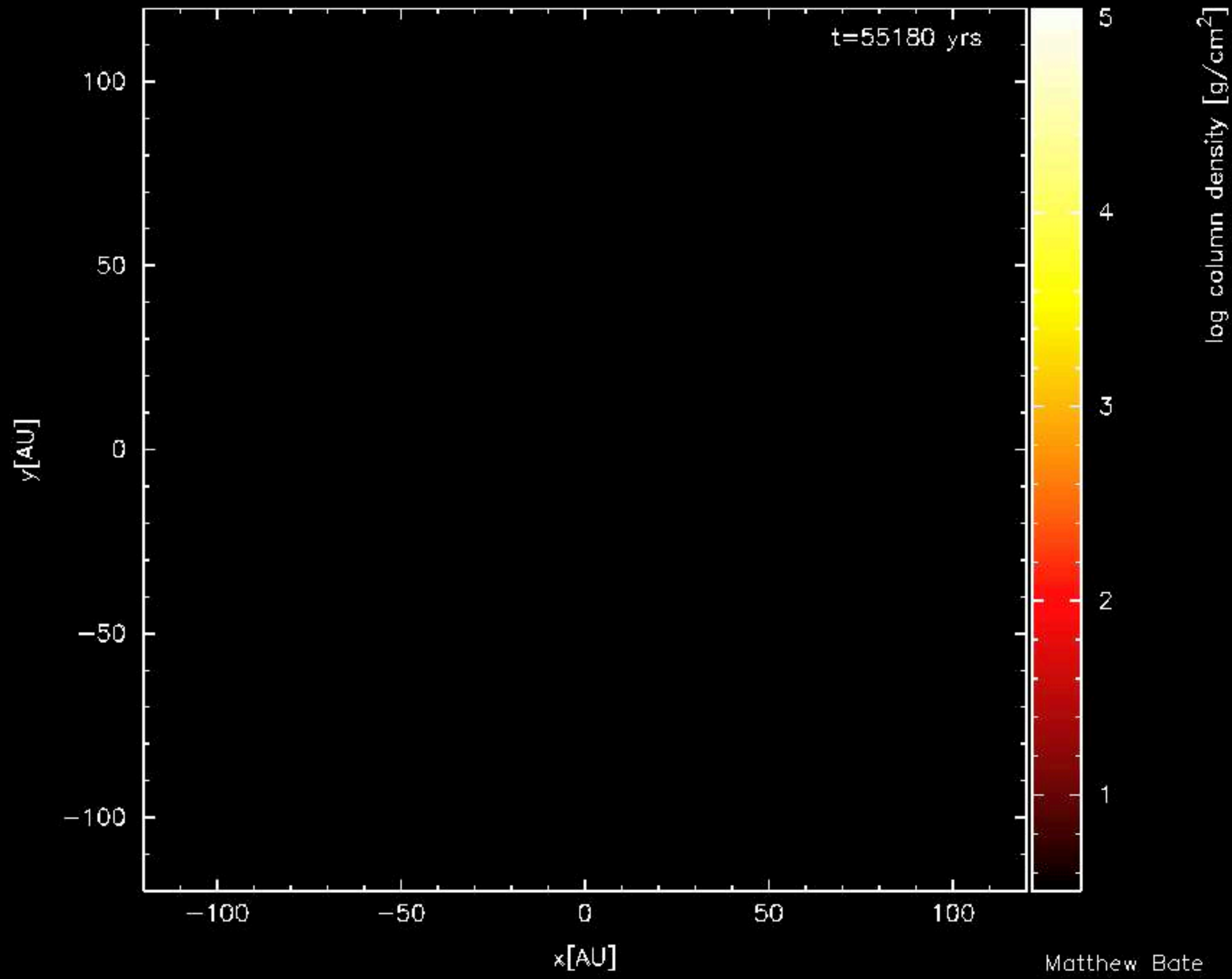
$\beta = E_{rot} / E_{grav} = 0.005$



- [illegible]

CORE + DISC FORMATION

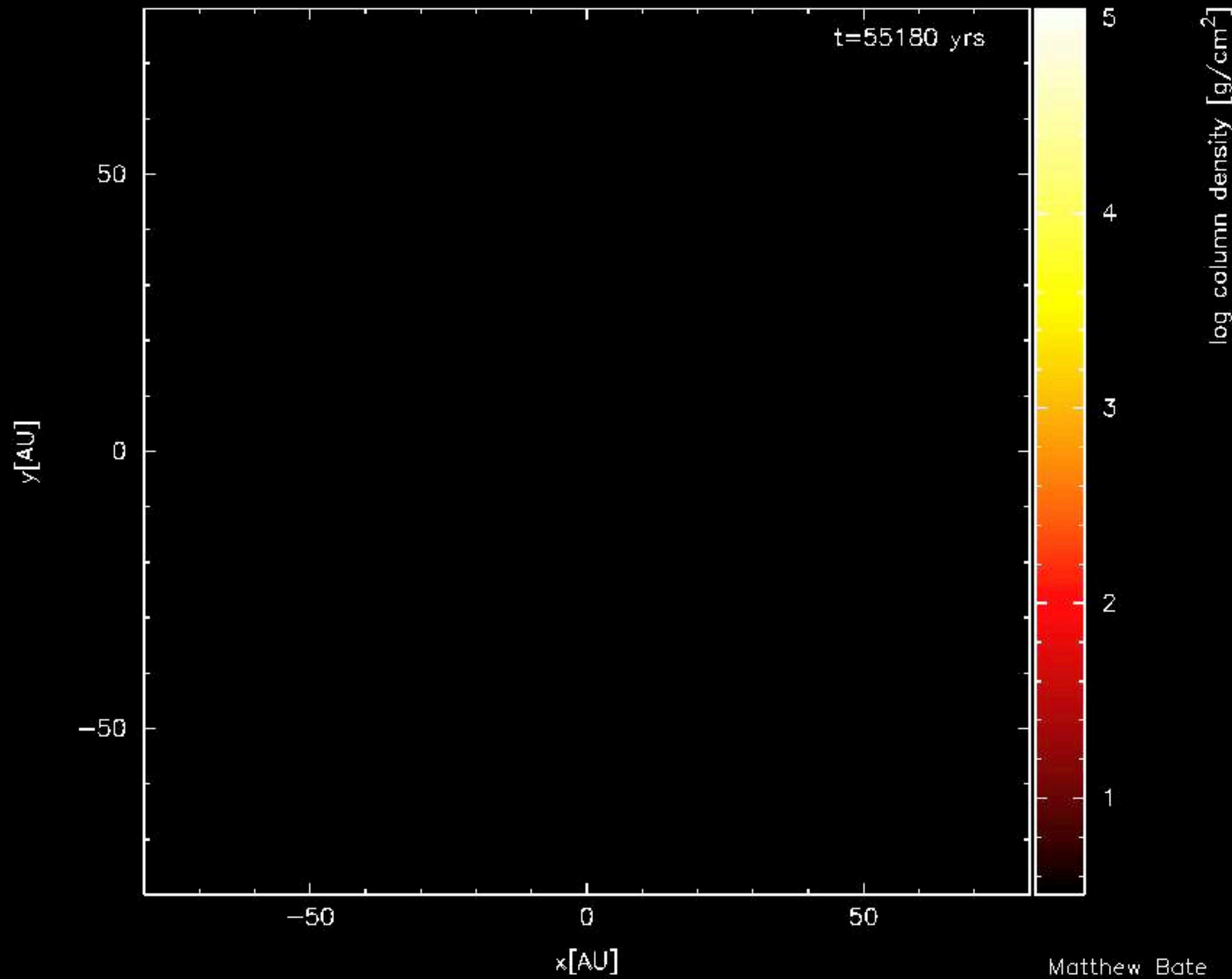
$\beta = E_{rot} / E_{grav} = 0.01$



- [illegible] arms manages to collect enough gas to form a self-gravitating fragment.

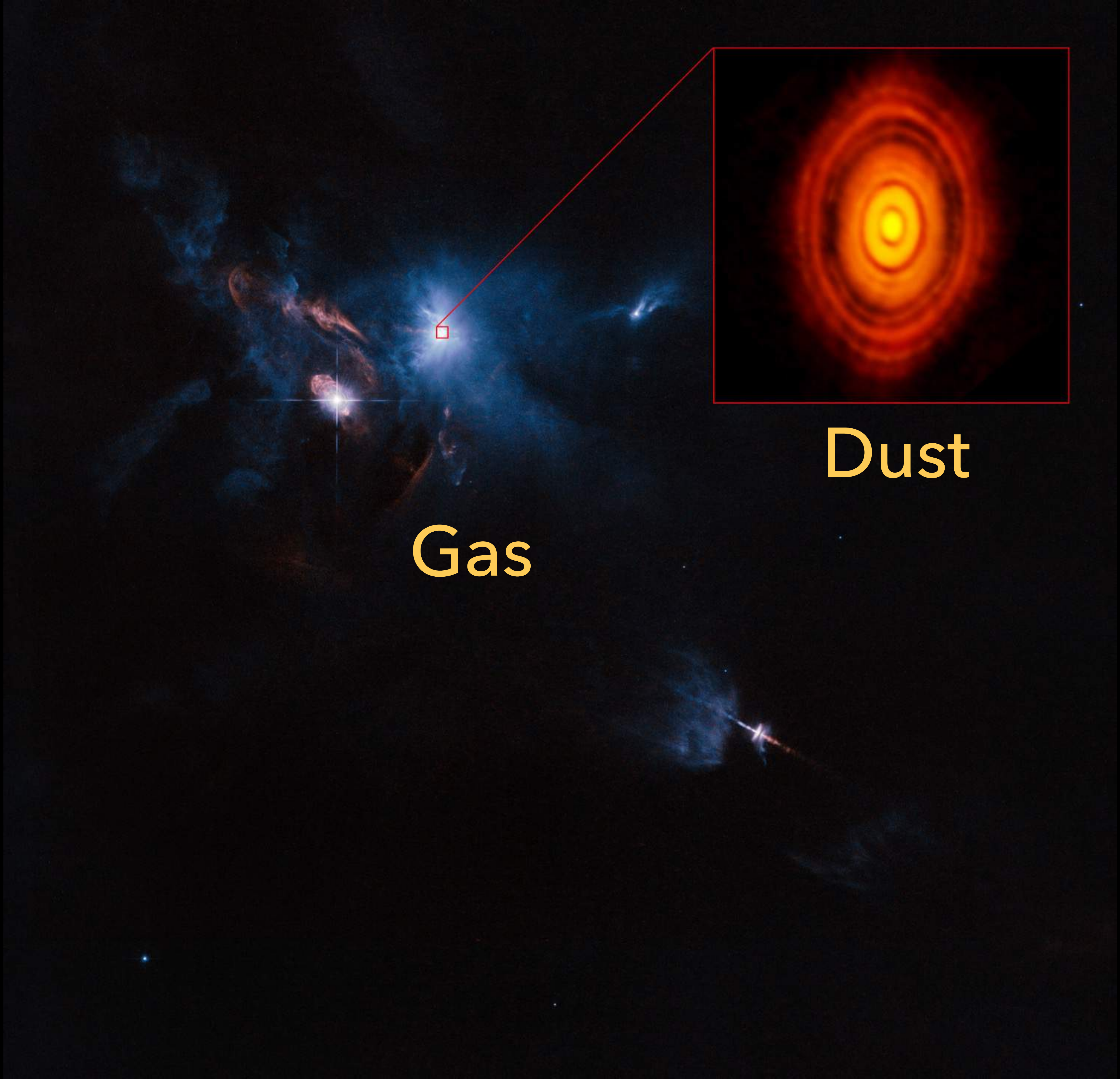
CORE + DISC FORMATION

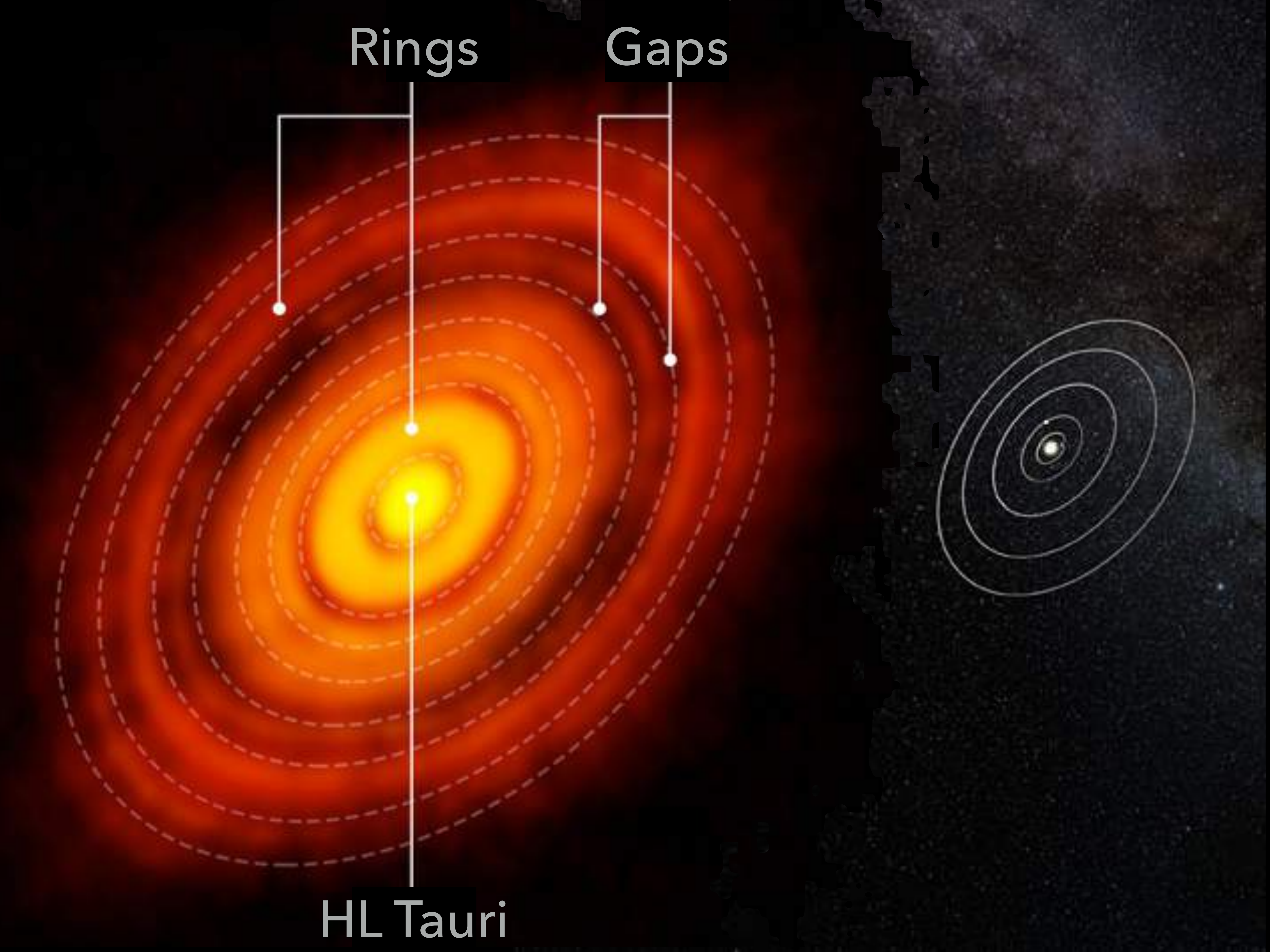
$\beta = E_{rot} / E_{grav} = 0.001$

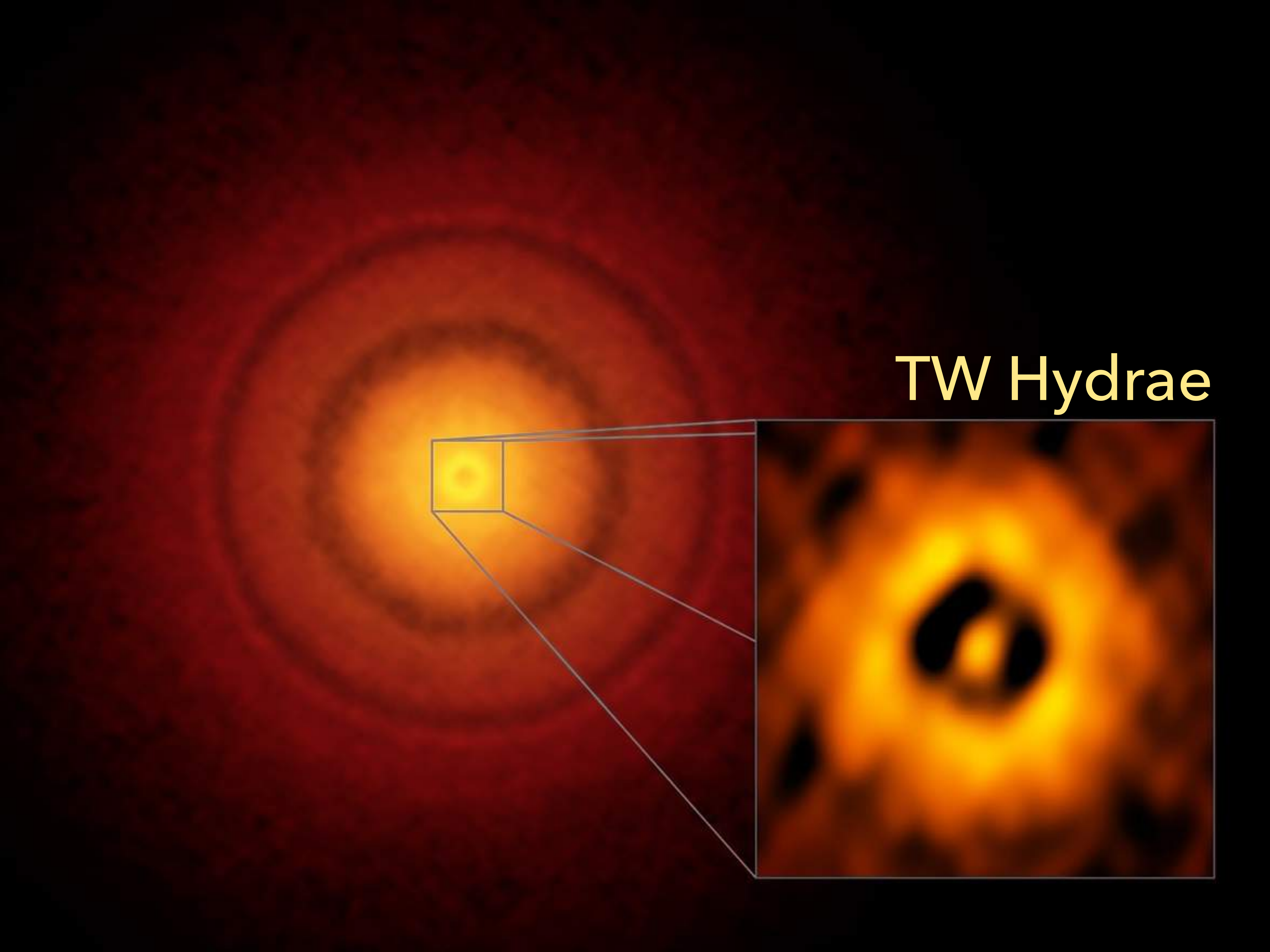


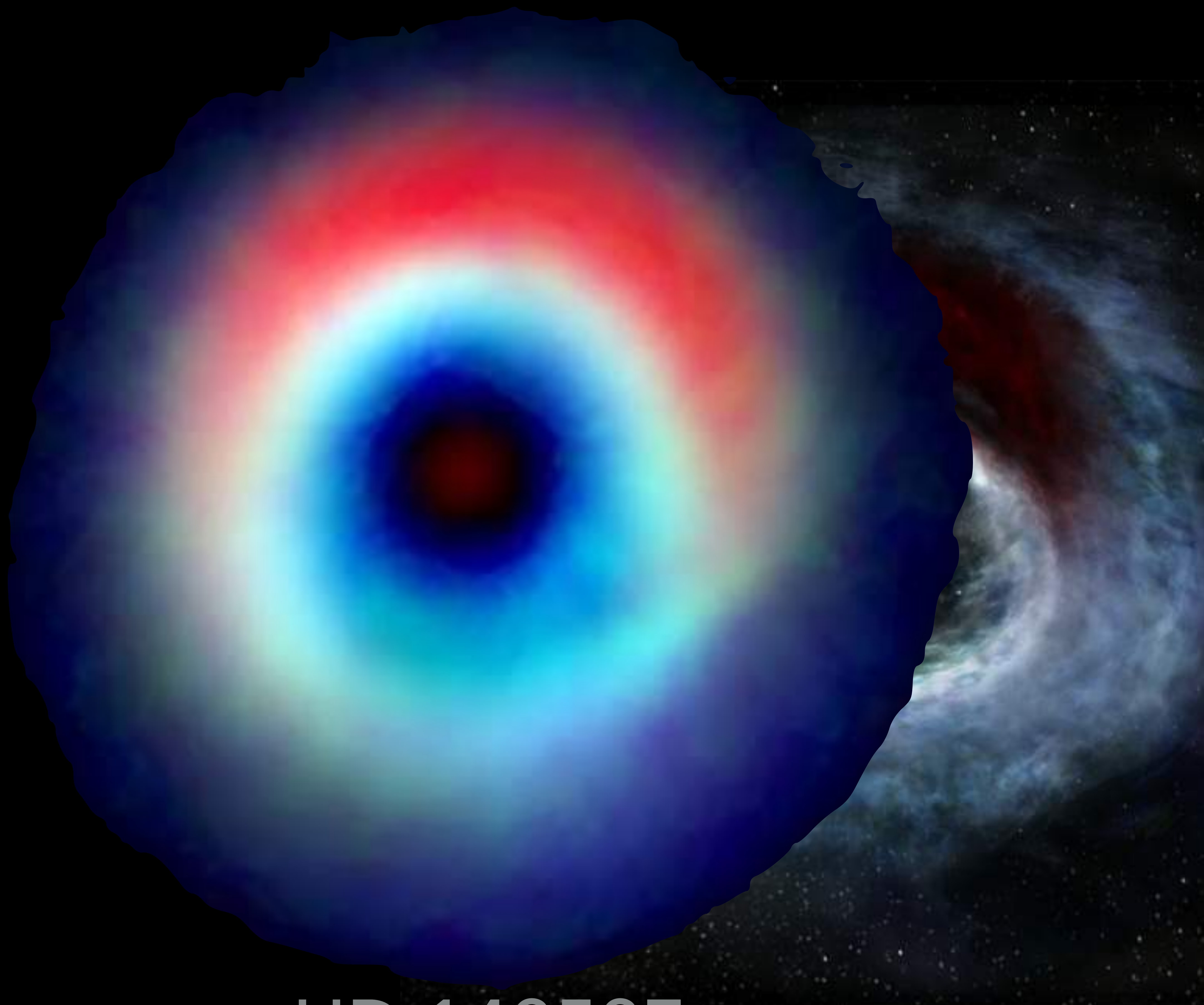
- Slow rotation allows the first hydrostatic core to remain (almost) axisymmetric.
- There is only a very weak bar mode instability visible, and the radius of the first core only grows to approximately 10 AU.

PROTOPLANETARY DISCS


Dust
Gas

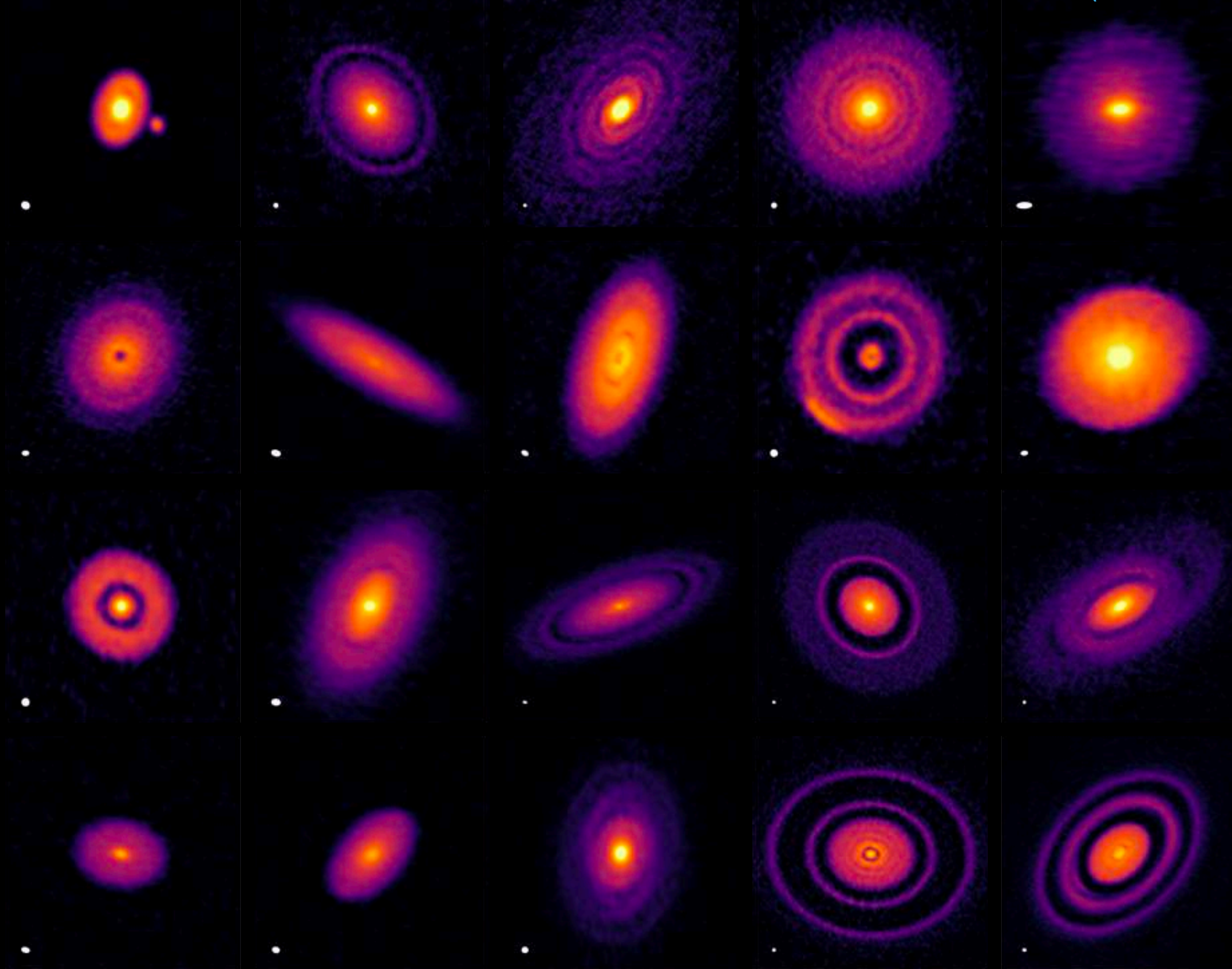

Rings
Gaps
HL Tauri


TW Hydrae


HD 142527

HK Tauri

DISK SUBSTRUCTURES AT HIGH ANGULAR RESOLUTION PROJECT (DSHARP)



BASIC PROPERTIES (UNCERTAIN, BUT IMPROVING)

- Masses: ~ 10^{-3}–10^{-1} $M_\odot$
- Radii: ~ 100 au
- Accretion rates: ~ 10^{-10}–10^{-7} $M_\odot$ / yr
- Lifetimes: ~ 1–15 Myr
- Relevant information for planet formation:
  - Structure – rotation, density, temperature, and chemical composition.
  - Early evolution and disc lifetimes – strength and nature of turbulence.
  - Dust dynamics – radial drift, vertical settling (we'll discuss growth and fragmentation next time).

FROM UNIVERSE

TO PLANETS

LECTURE 2.1: DISC STRUCTURE

ACTIVE VS PASSIVE DISCS

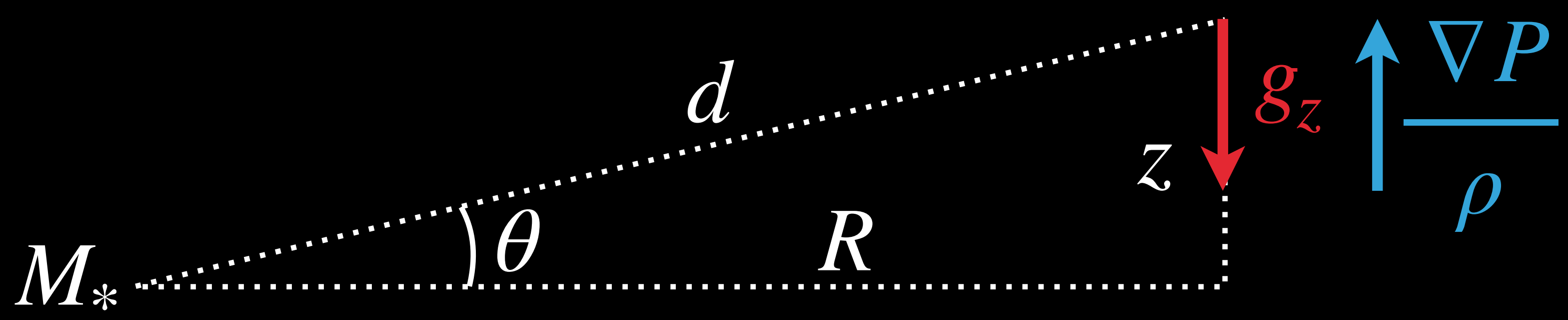
- Active: most of their luminosity comes from the release of gravitational energy as material flows inwards.
- Passive: luminosity comes from reprocessed starlight.
- Critical Accretion rate can be estimated by assuming the disc is flat and intercepts 1/4 of the stellar flux (we will verify this later):

$$\frac{1}{4}L_* = \frac{GM_*\dot{M}}{2R_*}$$

  - Solving for $\dot{M}$ we find: $\dot{M} \approx 3 \times 10^{-8} M_\odot \, \mathrm{yr}^{-1}$
- Accretion rates are higher for younger objects, so young disks are generally active, while older object are dominated by reprocessed radiation (passive).

PASSIVE DISCS: VERTICAL STRUCTURE

- Consider hydrostatic equilibrium with pressure gradient:



$$\frac{\nabla P}{\rho} = \frac{1}{\rho}\frac{dP}{dz} = -\,g_z$$

- For $M_{\rm disc} \ll M_*$ and $z \ll R$: $\quad g_z = \dfrac{GM_*}{d^2}\sin\theta \approx \Omega_{\rm K}^2 z$
  - Where Keplerian angular frequency: $\quad \Omega_{\rm K} \equiv \sqrt{\dfrac{GM_*}{R^3}}$
- Equation of state for an isothermal disc: $\quad P = \rho c_{\rm s}^2$
- Equation of hydrostatic equilibrium: $\quad \dfrac{1}{\rho}\dfrac{dP}{dz} = c_{\rm s}^2 \dfrac{d\ln\rho}{dz} = -\,\Omega_{\rm K}^2 z$

$$\longrightarrow \qquad \rho(z) = \rho_0 \exp\left[-\frac{1}{2}\left(\frac{z}{H}\right)^2\right] \qquad \text{where} \qquad H \equiv \Omega_{\rm K}/c_{\rm s}$$

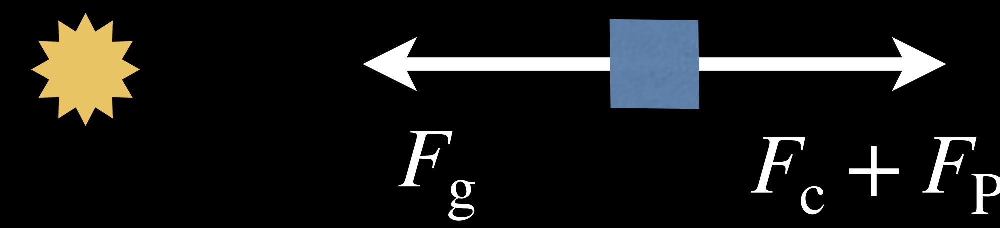
PASSIVE DISCS: VERTICAL STRUCTURE

- Often convenient to use vertically averaged quantities, such as surface density:

$$\Sigma = \int_{-\infty}^{\infty} \rho(z)\, dz = \sqrt{2} H \rho_0 \int_{-\infty}^{\infty} \mathrm{e}^{-x}\, dx = \sqrt{2\pi} H \quad \longrightarrow \quad \rho_0 = \frac{\Sigma}{\sqrt{2\pi} H}$$

- The Minimum Mass Solar Nebula (MMSN) is a protoplanetary disk that contains the minimum amount of solids necessary to build the planets of the solar system.
    - An aspect ratio $h \equiv H/R \sim 0.05$ gives a mid-plane density (ρ_0) of about 10^{-9} g cm^{-3} at 1 au.
- If we assume: $T \propto R^{-q}$ then $c_s \propto R^{-q/2}$ and $h \propto R^{-(q-1)/2}$
    - Flared discs will have a T power-law index $q < 1$

PASSIVE DISCS: RADIAL STRUCTURE



- In the radial direction a parcel of gas in the disc feels:
  - Gravity from the star (non self-gravitating case)
  - Centrifugal force
  - Pressure force

$$\frac{v^2}{R} = \frac{GM_*}{R^2} + \frac{1}{\rho}\frac{dP}{dR}$$

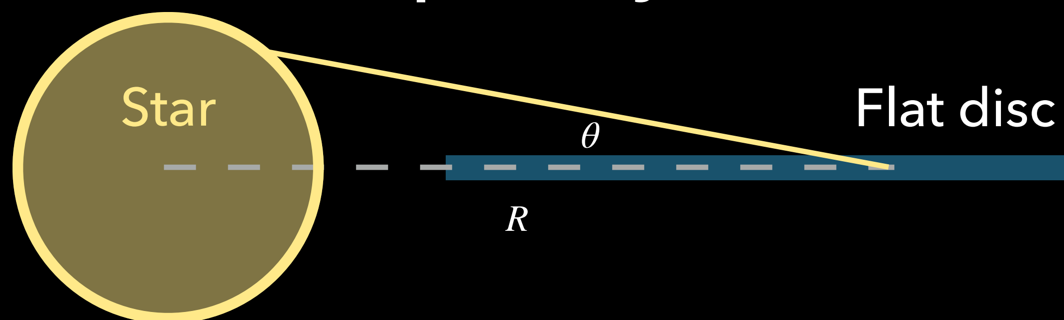
- Pressure decreases with radius, so gas rotates slightly slower than solids at the same radius (sub-Keplerian).

$$\frac{v^2}{R} \approx \Omega_K^2 R - \frac{c_s^2}{R} = \Omega_K^2 R \left(1 - \frac{c_s^2}{R^2 \Omega_K^2}\right) \longrightarrow v = v_K \left[1 - \left(\frac{H}{R}\right)^2\right]$$

- $H/R \ll v_K$ so we say the disc is in Keplerian motion, but this difference is crucial for understanding dust dynamics.

PASSIVE DISCS: RADIAL TEMPERATURE PROFILE

- Simplest case is a flat thin disk in the equatorial plane.
- All stellar radiation is absorbed and re-emitted as a single temperature blackbody.
  - Perfect absorber/emitter, frequency spectrum depends only on T, emits radiation isotropically.
- The star has a uniform intensity I_*.



- $F_{\rm inc}$ is the incident stellar flux on the upper surface at radius R:

$d\Omega = \sin\theta\, d\theta\, d\phi$ (solid angle)

$$F_{\rm inc} = \int I_* \sin\theta \cos\phi\, d\Omega = I_* \int_{-\pi/2}^{\pi/2} \cos\phi\, d\phi \int_0^{\sin^{-1}(R_*/R)} \sin^2\theta\, d\theta$$

PASSIVE DISCS: RADIAL TEMPERATURE PROFILE

- Integrating: $$F_{\rm inc} = I_* \left[\sin^{-1}\left(\frac{R_*}{R}\right) - \frac{R_*}{R}\sqrt{1 - \left(\frac{R_*}{R}\right)^2} \right]$$
- Stefan-Boltzmann law relates the flux radiated from a black body in terms of its effective temperature: $F = \sigma T^4$
- Intensity (i.e. brightness): $I_* = \frac{1}{4\pi}\sigma T_*^4$
- One sided disc emission: $F_{\rm disc} = \frac{1}{2}\sigma T_{\rm disc}^4$
- Equating $F_{\rm inc} = F_{\rm disc}$, we find a temperature profile:

$$\left(\frac{T_{\rm disc}}{T_*}\right)^4 = \frac{1}{\pi}\left[\sin^{-1}\left(\frac{R_*}{R}\right) - \frac{R_*}{R}\sqrt{1 - \left(\frac{R_*}{R}\right)^2} \right]$$

PASSIVE DISCS: RADIAL TEMPERATURE PROFILE

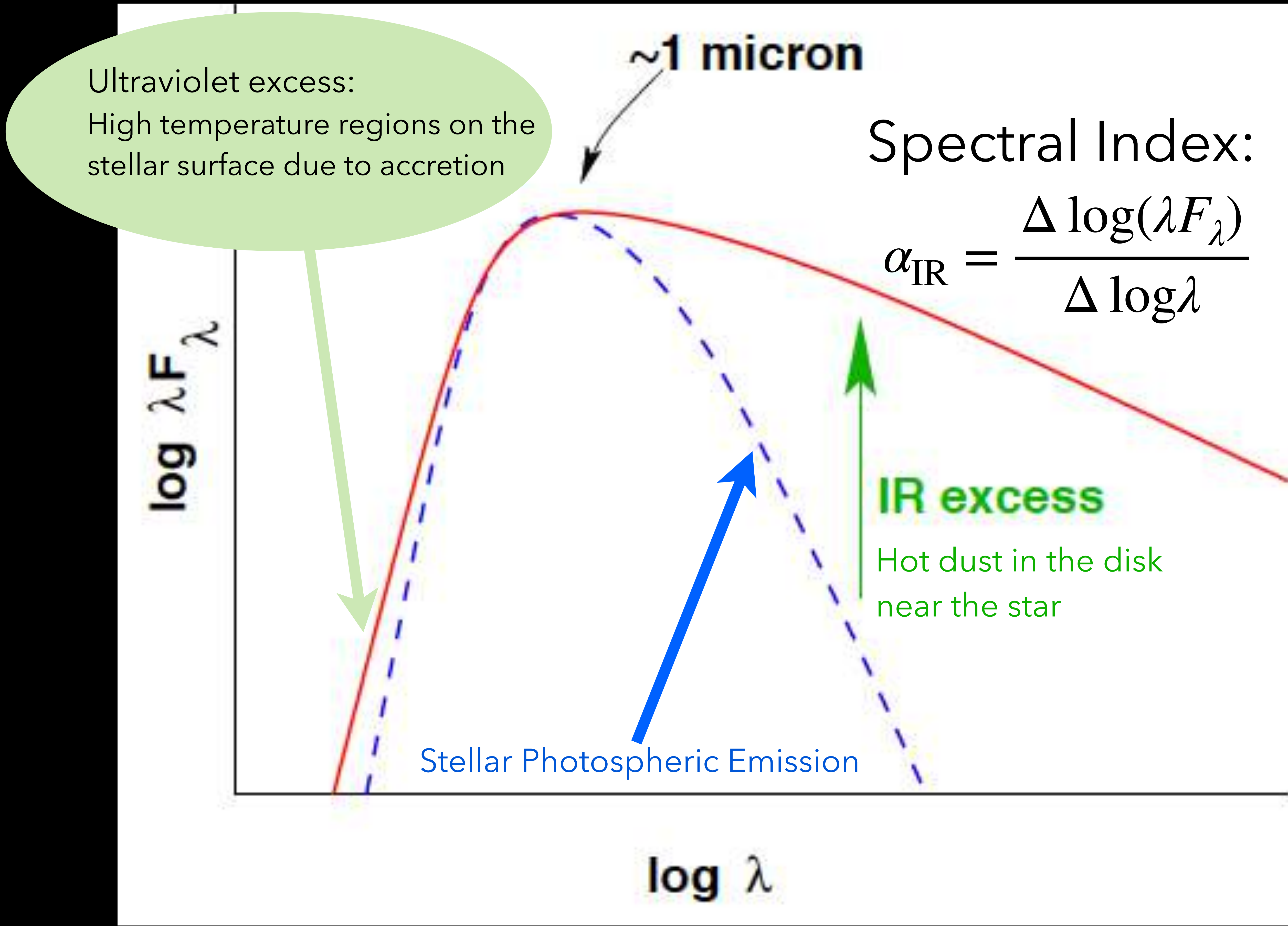
- This isn't very clear…if we assume $R_* \ll R$ and expand this in a Taylor series, we find $q = 3/4$, which is about the largest value one can expect.

- If the disk is flared (i.e. h increases with R), then the outer regions intercept more stellar flux leading to a higher temperature and a shallower exponent

- Integrating over all radii, we find the disc has 1/4 of the stellar luminosity (validating our earlier assumption):

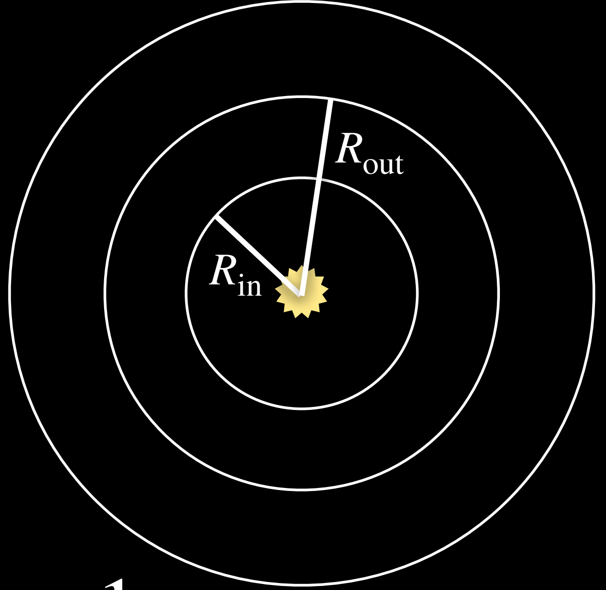
$$L_{\rm disc} = 2\int_{R_*}^{\infty} 2\pi R\sigma T_{\rm disc}^4 \, dR = \pi R_*^2 \sigma T_*^4 = \frac{1}{4} L_*$$

PASSIVE DISCS: SPECTRAL ENERGY DISTRIBUTION (SED)



$$\alpha_{\rm IR} = \frac{\Delta \log(\lambda F_\lambda)}{\Delta \log \lambda}$$

PASSIVE DISCS: SED



- We break the disc into cylindrical rings and treat each ring as a black body radiator.

$$F_\lambda \propto \int_{R_{\rm in}}^{R_{\rm out}} 2\pi R\, B_\lambda(T)\, dR \qquad \text{where} \quad B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_{\rm B} T} - 1}$$

- At long wavelengths we have the Rayleigh-Jeans limit:

$$\lambda F_\lambda \propto \lambda^{-3} \qquad \text{if} \quad \lambda \gg hc/k_{\rm B}T(R_{\rm out})$$

- At short wavelengths we have an exponential cut-off:

$$\lambda F_\lambda \propto \lambda^{-4} e^{-hc/\lambda k_{\rm B} T(R_{\rm in})} \qquad \text{if} \quad \lambda \ll hc/k_{\rm B}T(R_{\rm in})$$

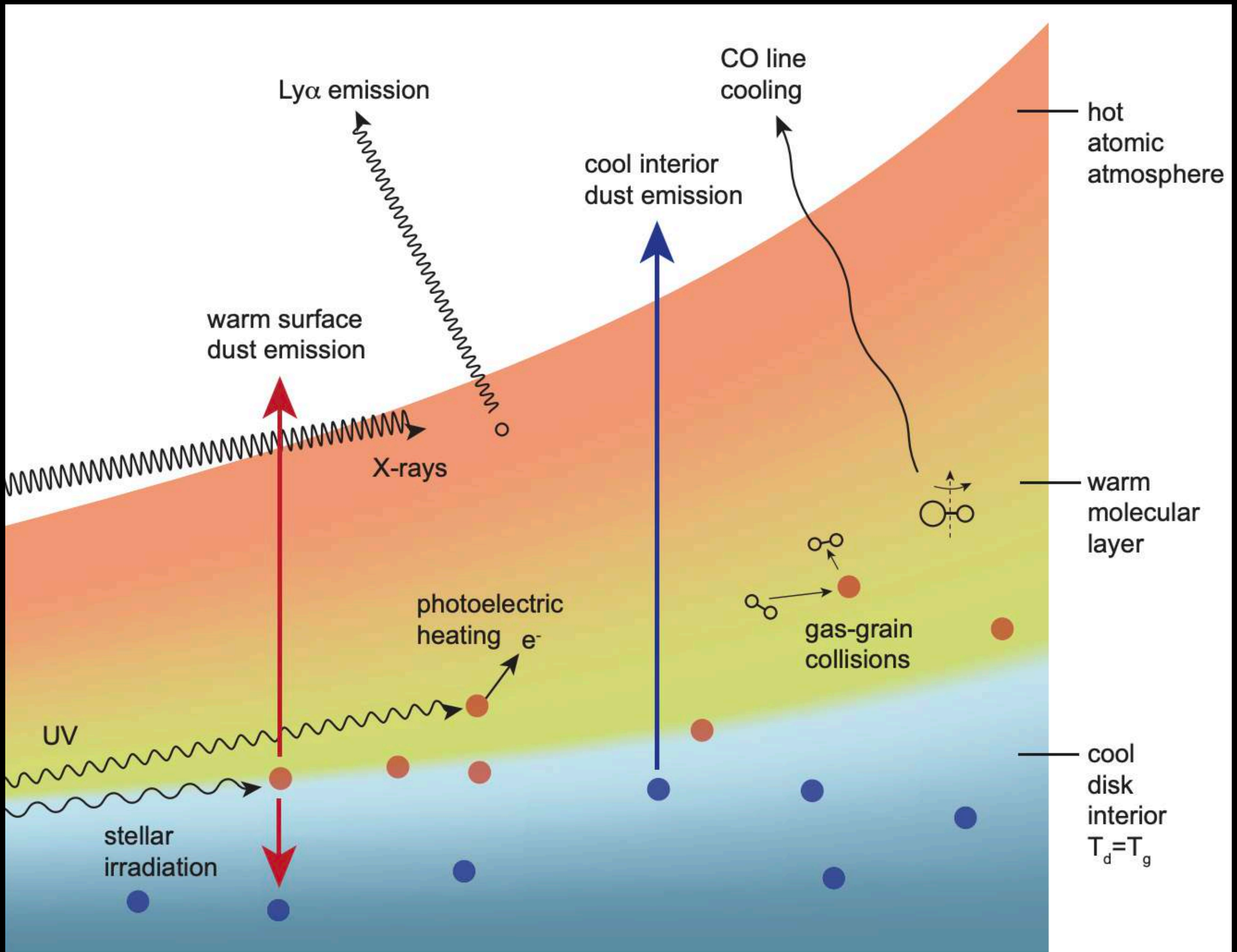
- At intermediate wavelengths, the behaviour depends on the value of q. In our flat disc scenario (i.e. $q = 3/4$):

$$\lambda F_\lambda \propto \lambda^{-4/3} \qquad \text{if} \quad \frac{hc}{k_{\rm B}T(R_{\rm in})} \ll \lambda \ll \frac{hc}{k_{\rm B}T(R_{\rm out})}$$

PASSIVE DISCS: SED

- Of course we are oversimplifying…discs are flared, not flat. More realistic to assume $T_{\rm disc} \propto R^{-1/2}$.

- Discs are also not single T black bodies. Dust in the upper layers absorbs stellar radiation more efficiently than it emits IR radiation.



PASSIVE DISCS: SED

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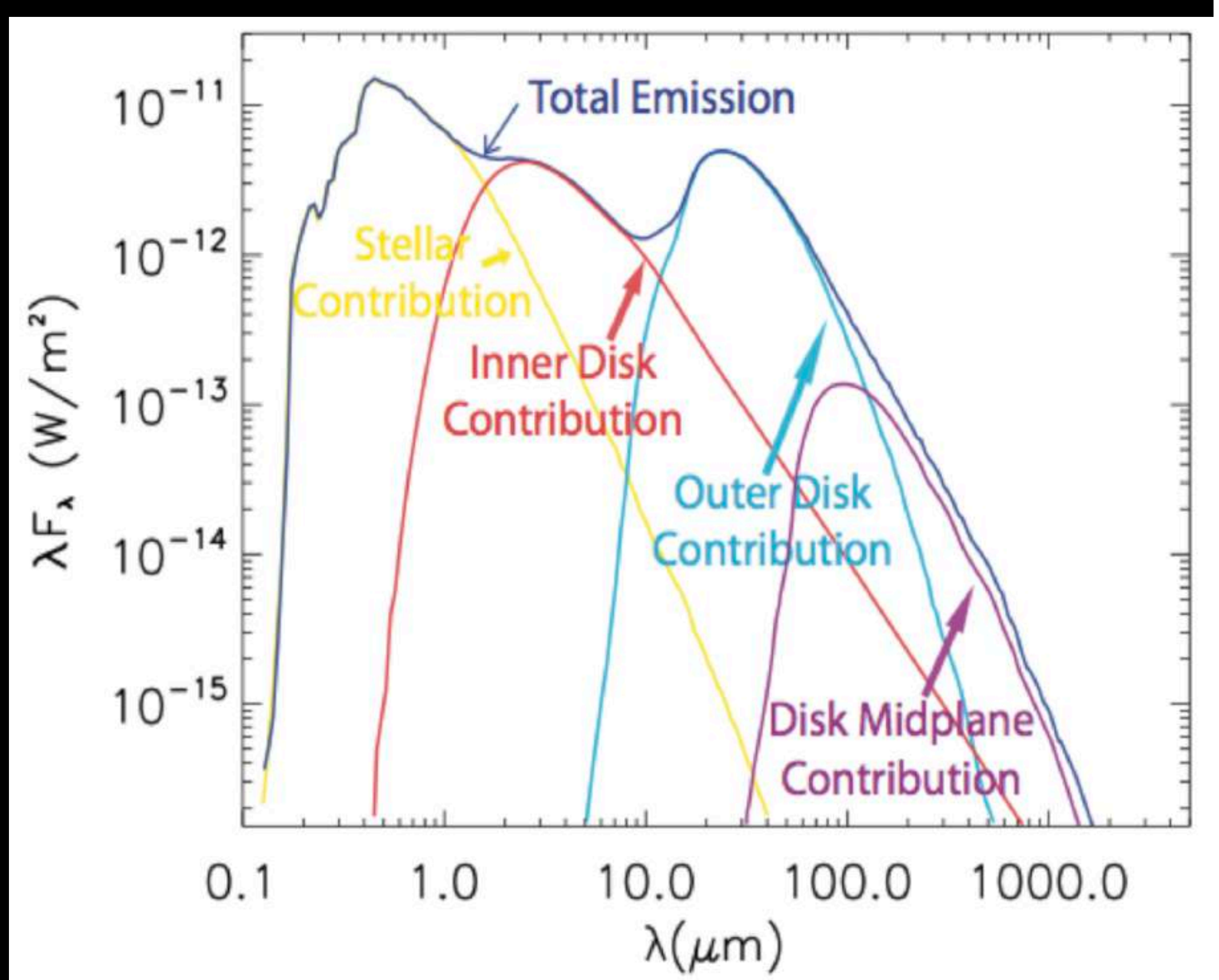
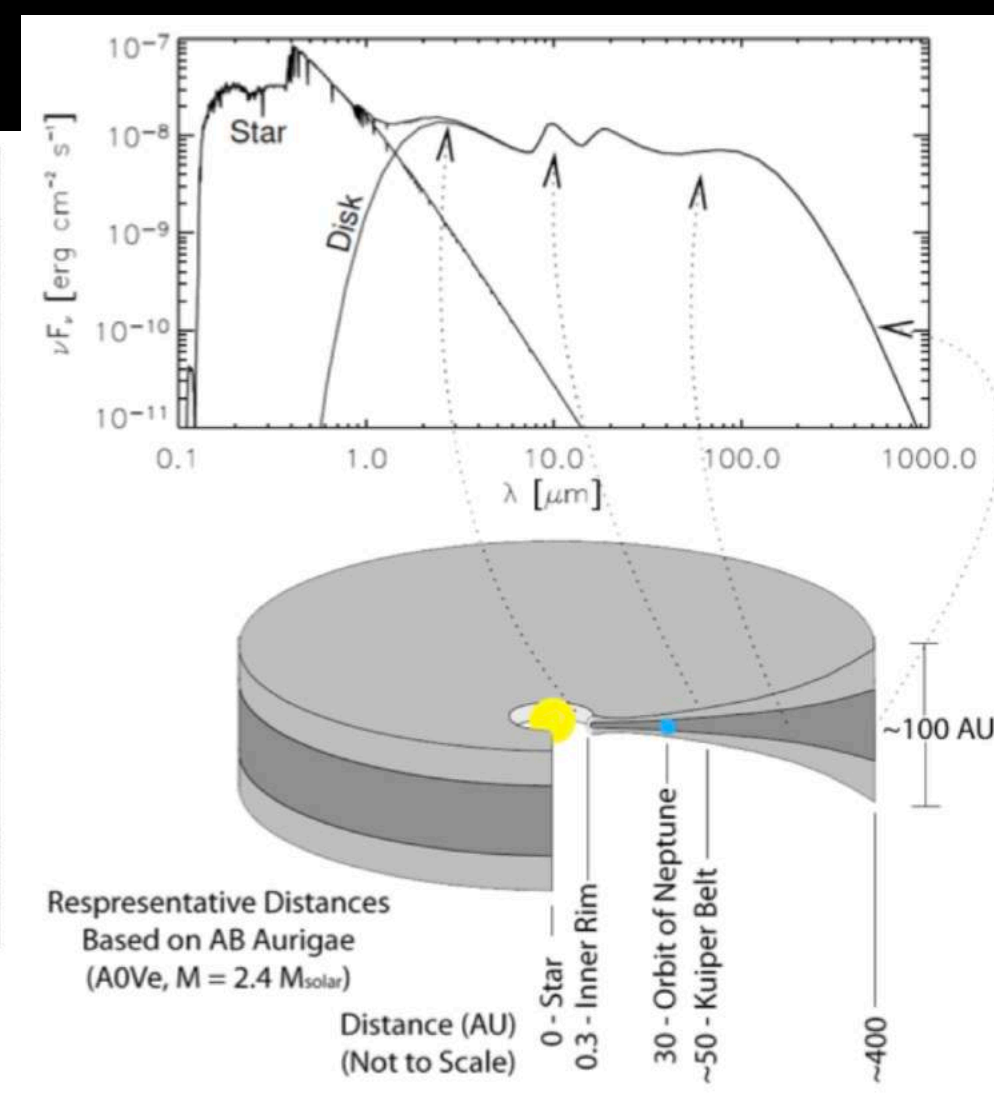
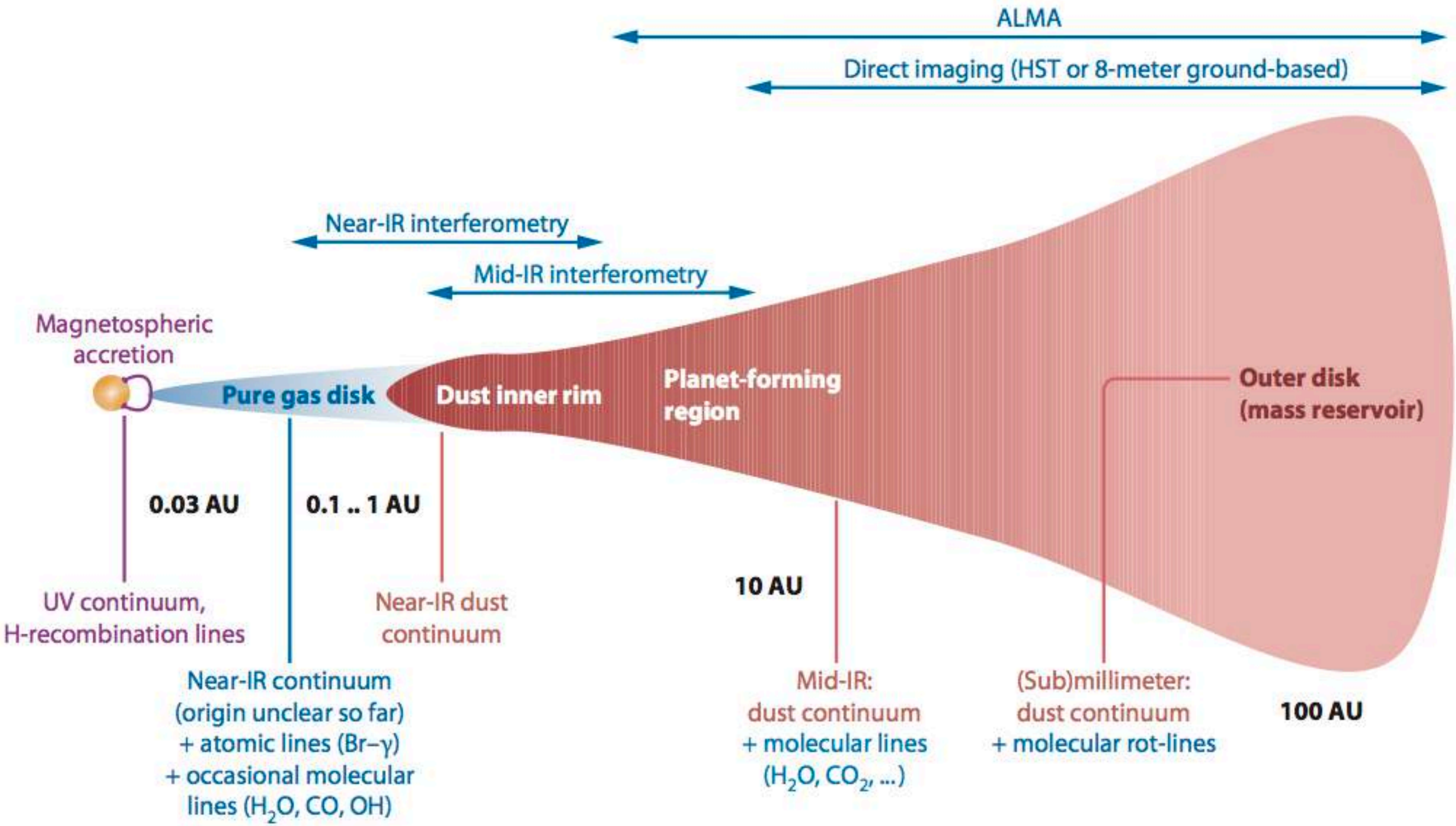
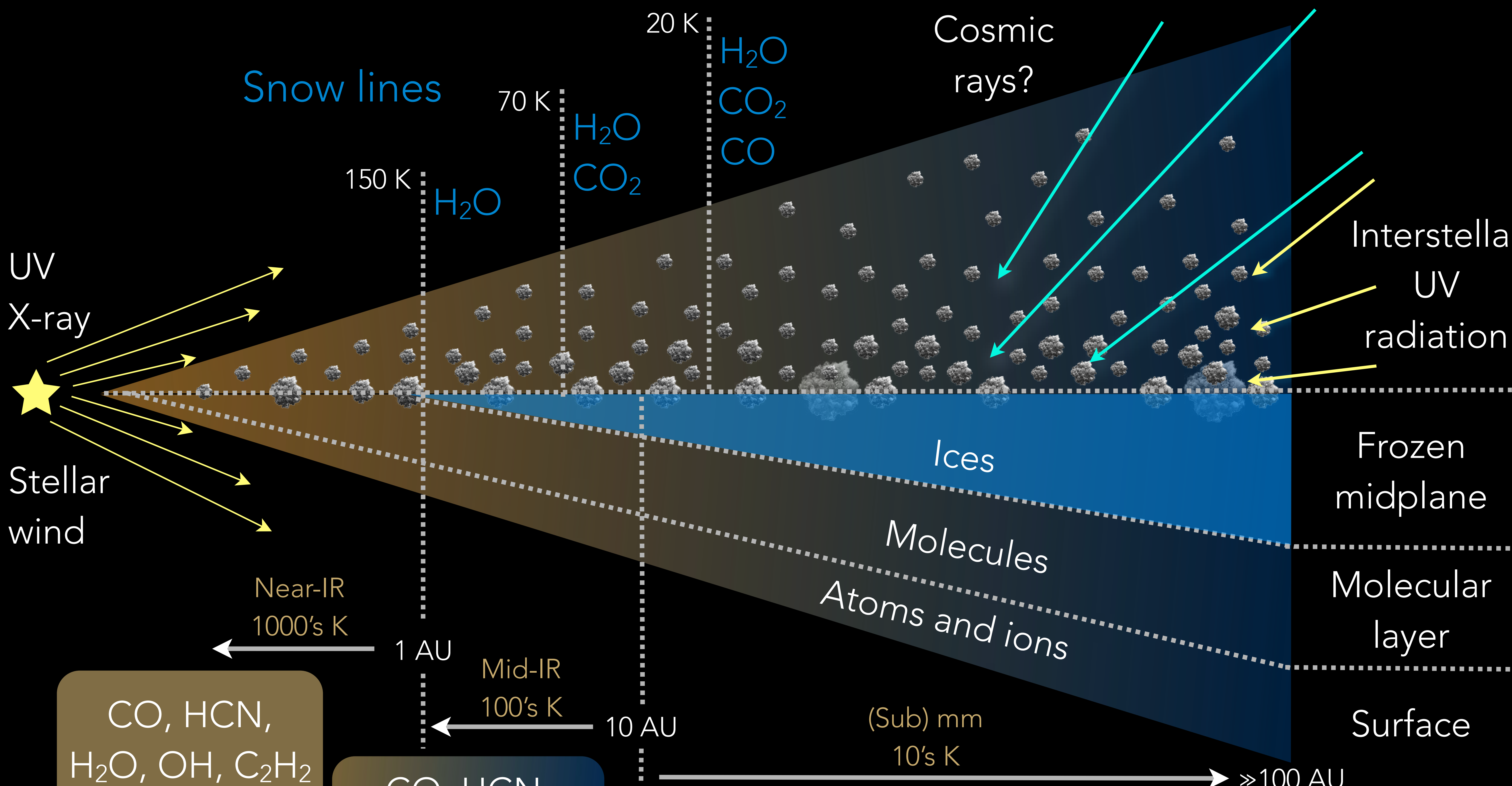




COMPOSITION



COMPOSITION



CO, HCN, H_2O, OH, C_2H_2

CO, HCN, H_2O, OH, C_2H_2 CH_4, CO_2

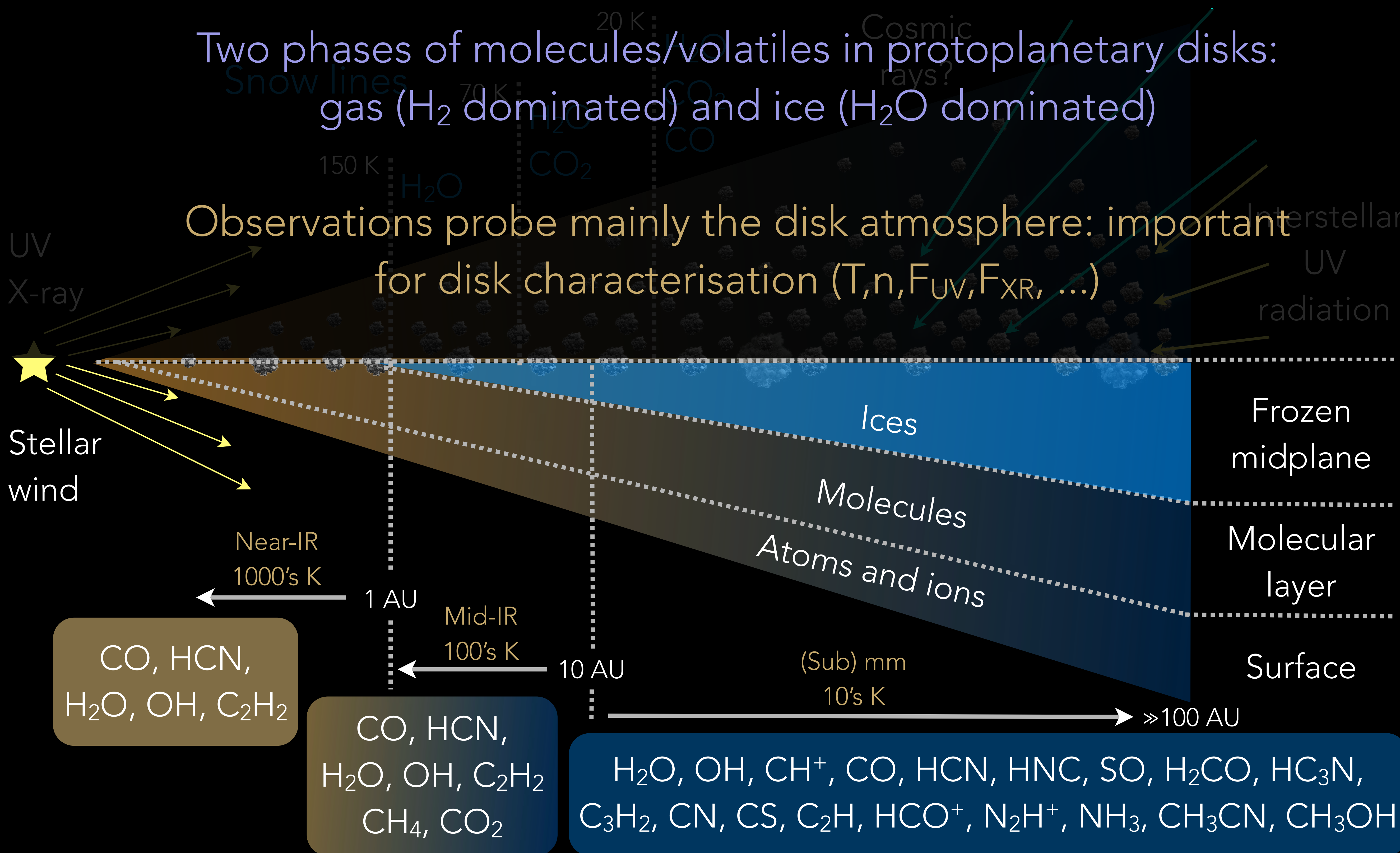
H_2O, OH, CH^+, CO, HCN, HNC, SO, H_2CO, HC_3N, C_3H_2, CN, CS, C_2H, HCO^+, N_2H^+, NH_3, CH_3CN, CH_3OH

Pontoppidan+, Gibb+, Salyk+, van Dishoeck+, Dutrey+, Chapillon+, Qi+, Oberg+, Kastner+, Thi+, Carr+, Najita+, Hogerheijde+, Fedele+, Meeus+

COMPOSITION

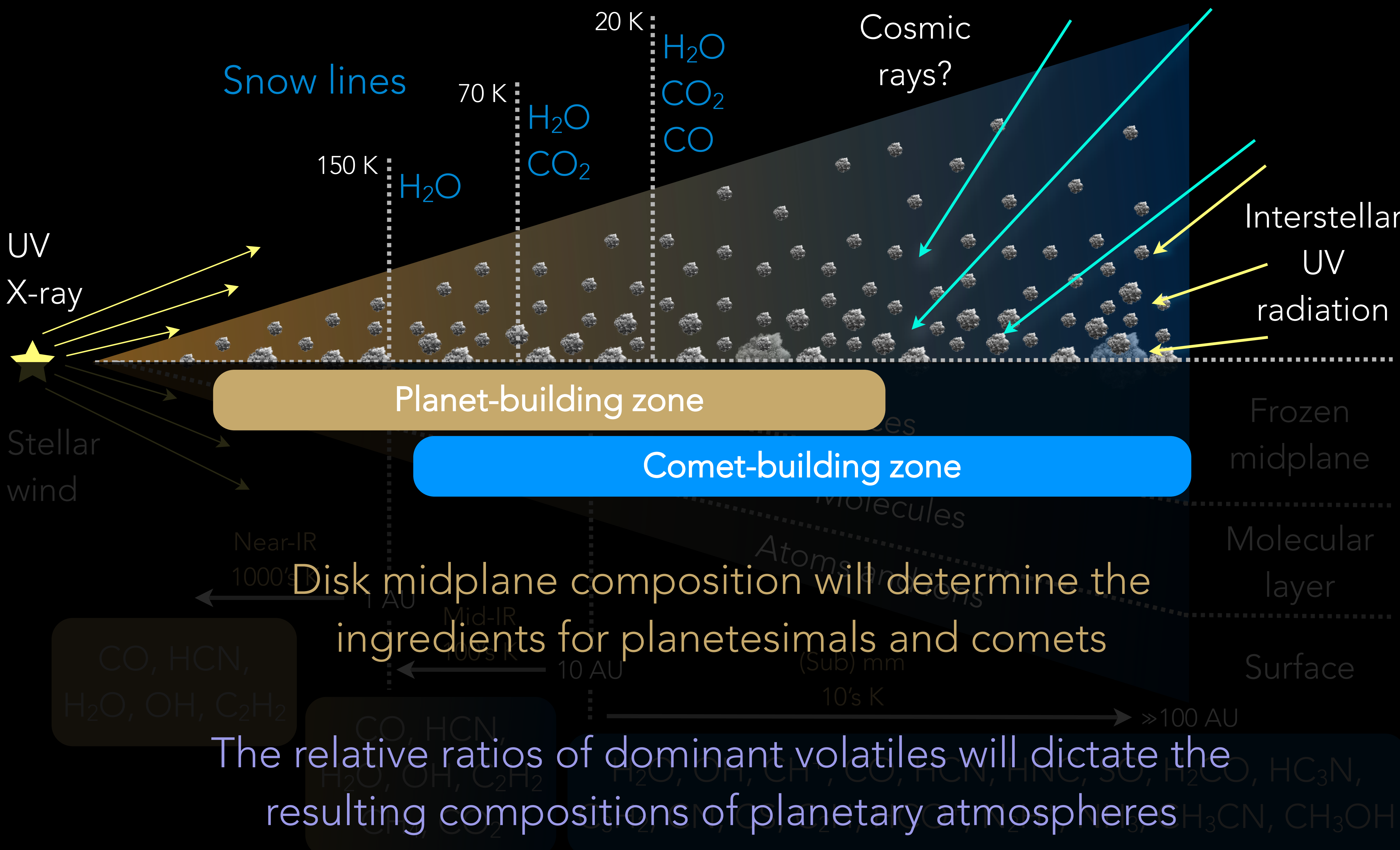
Two phases of molecules/volatiles in protoplanetary disks: gas (H_2 dominated) and ice (H_2O dominated)

Observations probe mainly the disk atmosphere: important for disk characterisation (T,n,F_{UV},F_{XR}, ...)



Pontoppidan+, Gibb+, Salyk+, van Dishoeck+, Dutrey+, Chapillon+, Qi+, Oberg+, Kastner+, Thi+, Carr+, Najita+, Hogerheijde+, Fedele+, Meeus+

COMPOSITION



Disk midplane composition will determine the ingredients for planetesimals and comets

The relative ratios of dominant volatiles will dictate the resulting compositions of planetary atmospheres

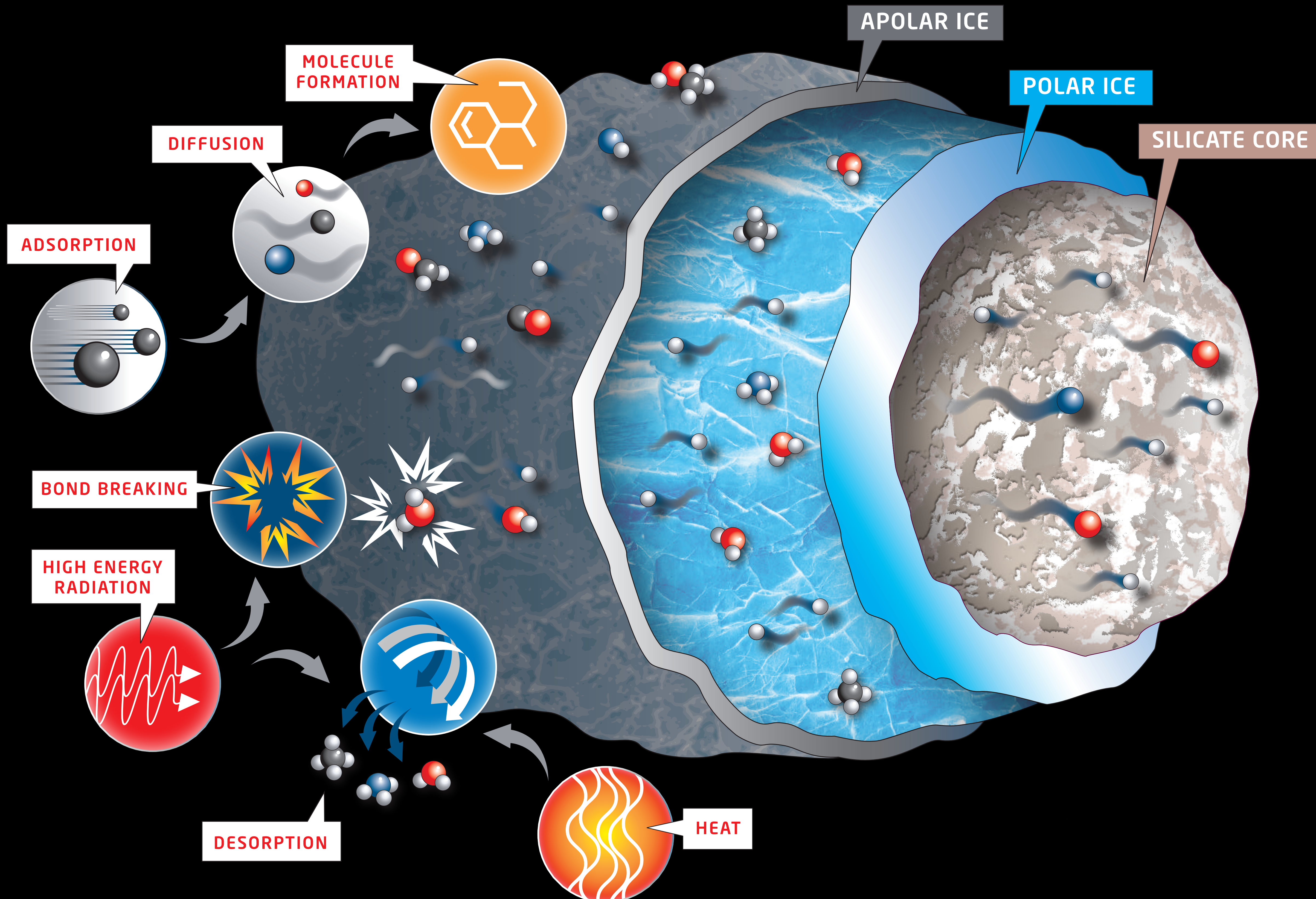
Pontoppidan+, Gibb+, Salyk+, van Dishoeck+, Dutrey+, Chapillon+, Qi+, Oberg+, Kastner+, Thi+, Carr+, Najita+, Hogerheijde+, Fedele+, Meeus+

COMPOSITION

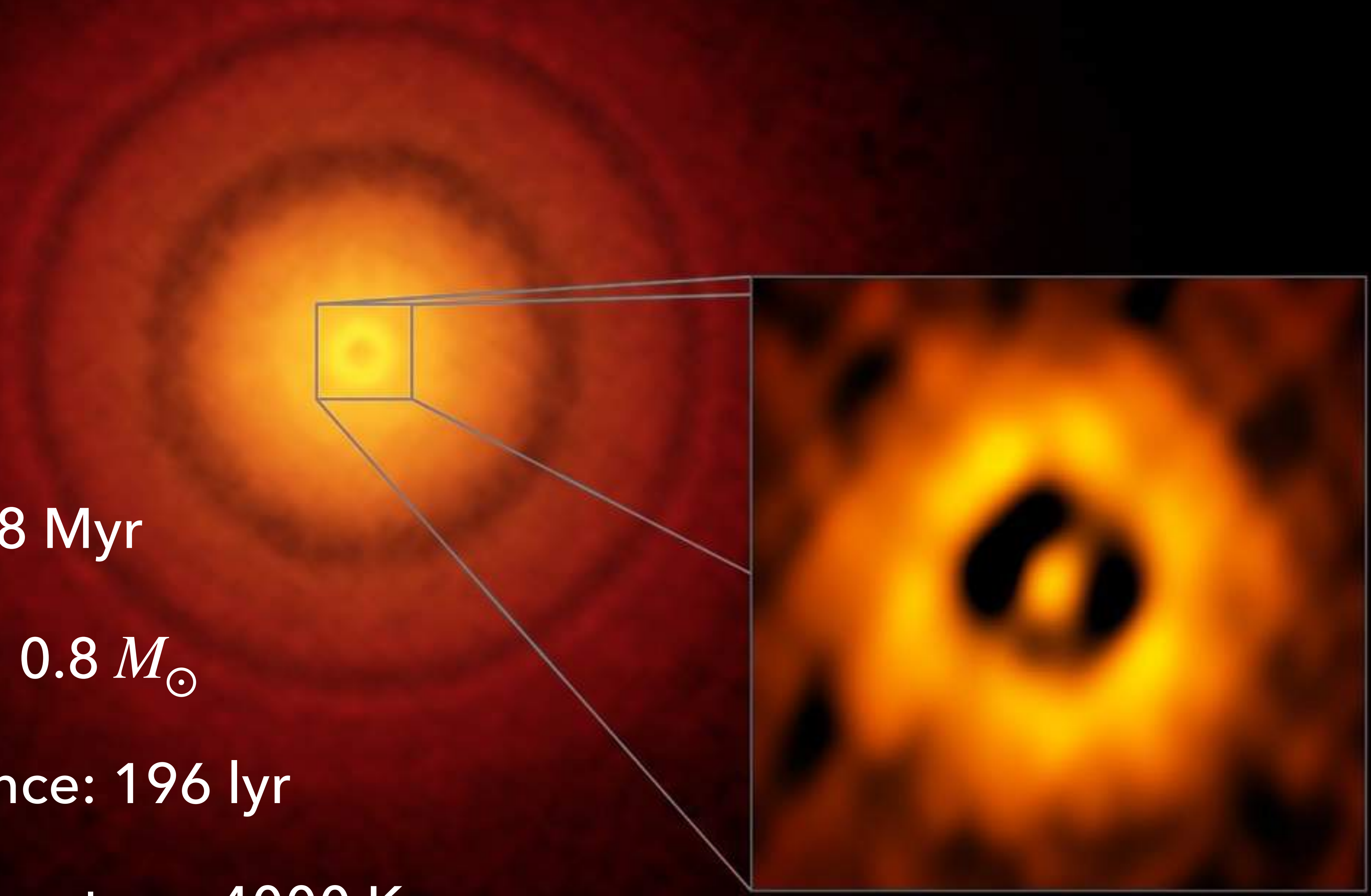
- Many molecules form on grain surfaces (small grains dominate surface area). Provide reservoir for atoms/molecules, allowing them to easily react and form new, more complex molecules.
- Atoms and molecules adsorb (freeze-out) on their surfaces and form icy mantles. These mantles consist of a water-dominated layer (polar) and a water-poor layer (apolar).
- Molecules/atoms diffuse (move around) and react, forming more complex species. Energetic radiation impinging on grains further affect chemistry by breaking bonds or desorbing (released due to heat) molecules from the grain surfaces into the gas phase.
- Small grains are incorporated into comets and eventually may end up atmospheres of planets.

COMPOSITION

Interstellar Grain Surface Chemistry



TW HYDRAE CASE STUDY

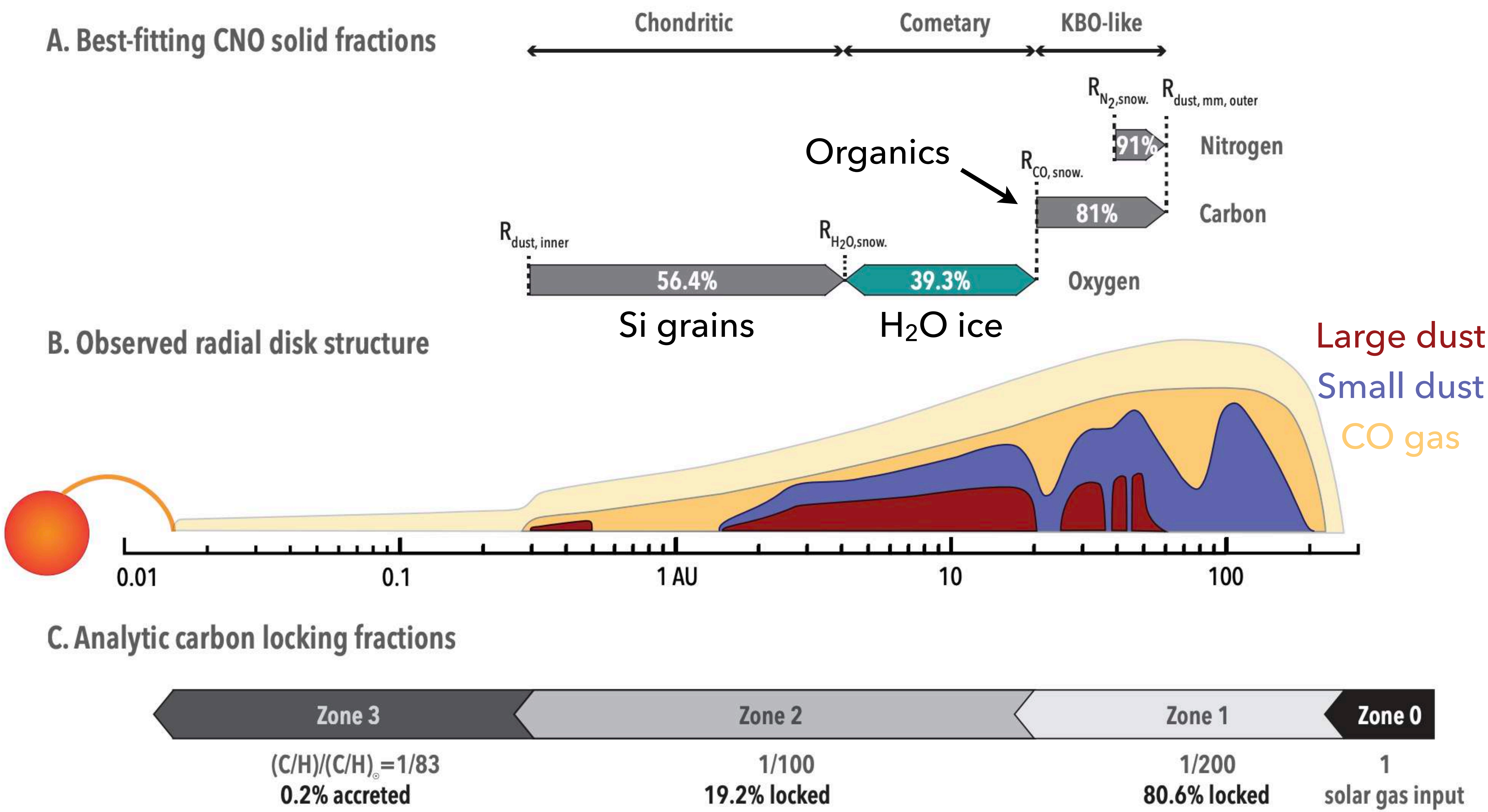


- Age: 8 Myr
- Mass: 0.8 $M_{\odot}$
- Distance: 196 lyr
- Temperature: 4000 K

TW HYDRAE CASE STUDY

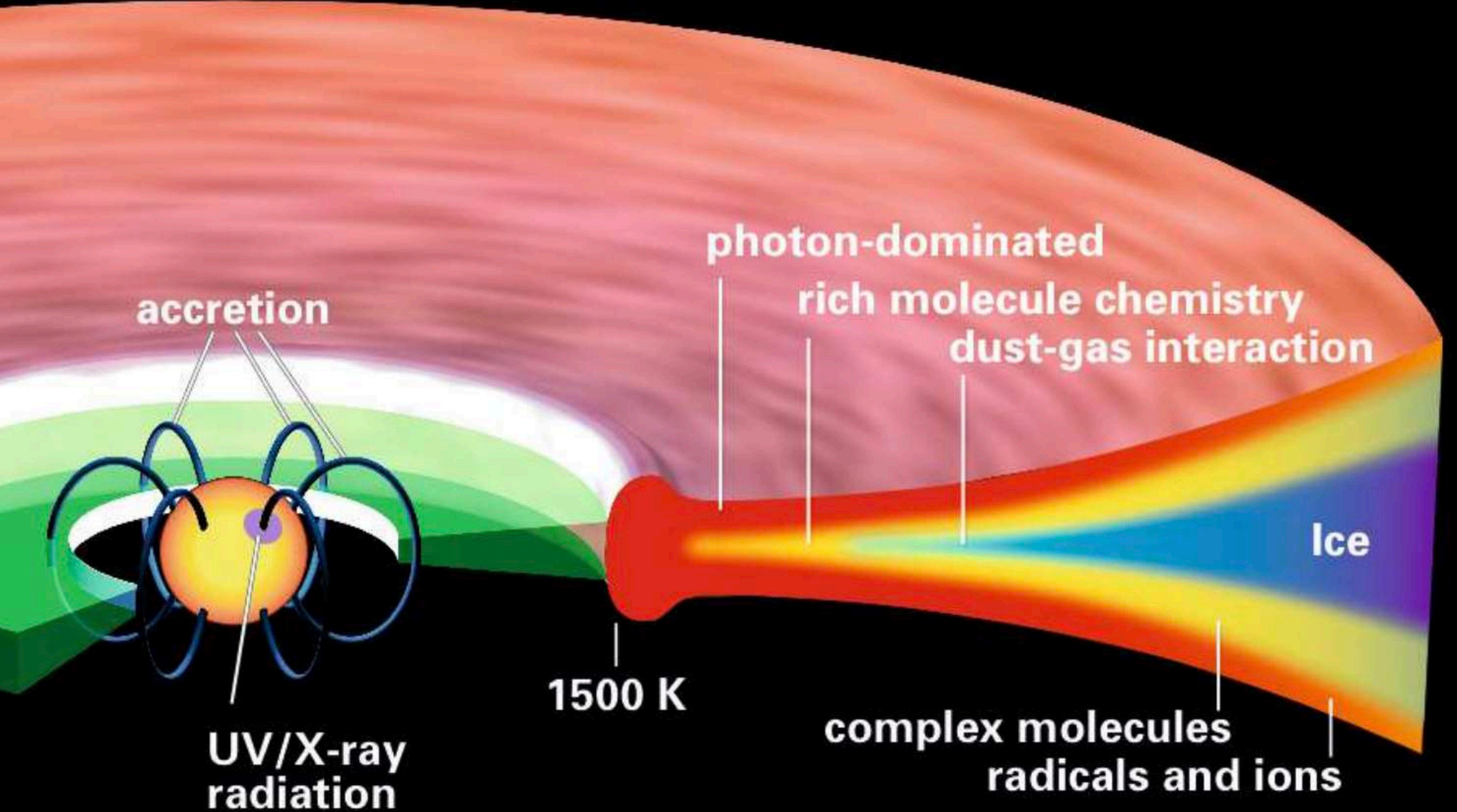
- Midplane ice reservoir inferred from gas depletions of CO (~100 ×) and H_2O (~800 ×), relative to the ISM value.
    - "Missing" volatiles possibly locked in large icy particles. Cannot reach surface layers for ice to be photodesorbed.
- Measure CO/H_2 ratio at different radii with ALMA and assume all gas phase C is in CO $\longrightarrow$ estimate gas C/H ratio. NIR atomic carbon emission lines $\longrightarrow$ C/H ratio in dust-free inner disc.
    - Compare C/H ratios to get C mass locked in disc solids.
- Any terrestrial planets forming in TW Hya from the remaining solids will be relatively "dry" and carbon poor, similar to those in our solar system.

TW HYDRAE CASE STUDY

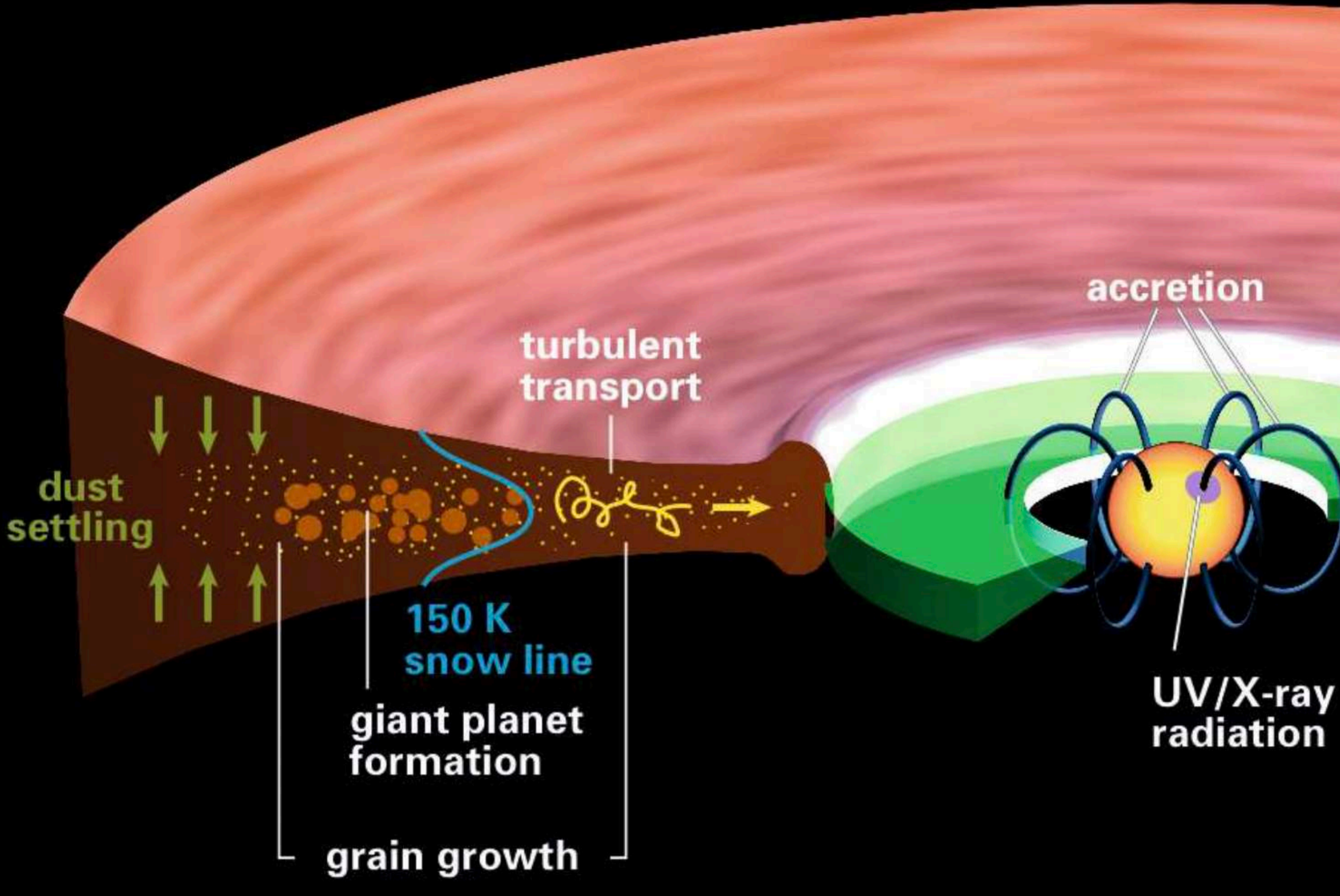


McClure & Dominik (2019).

STRUCTURE AND COMPOSITION



EVOLUTION AND LIFETIME

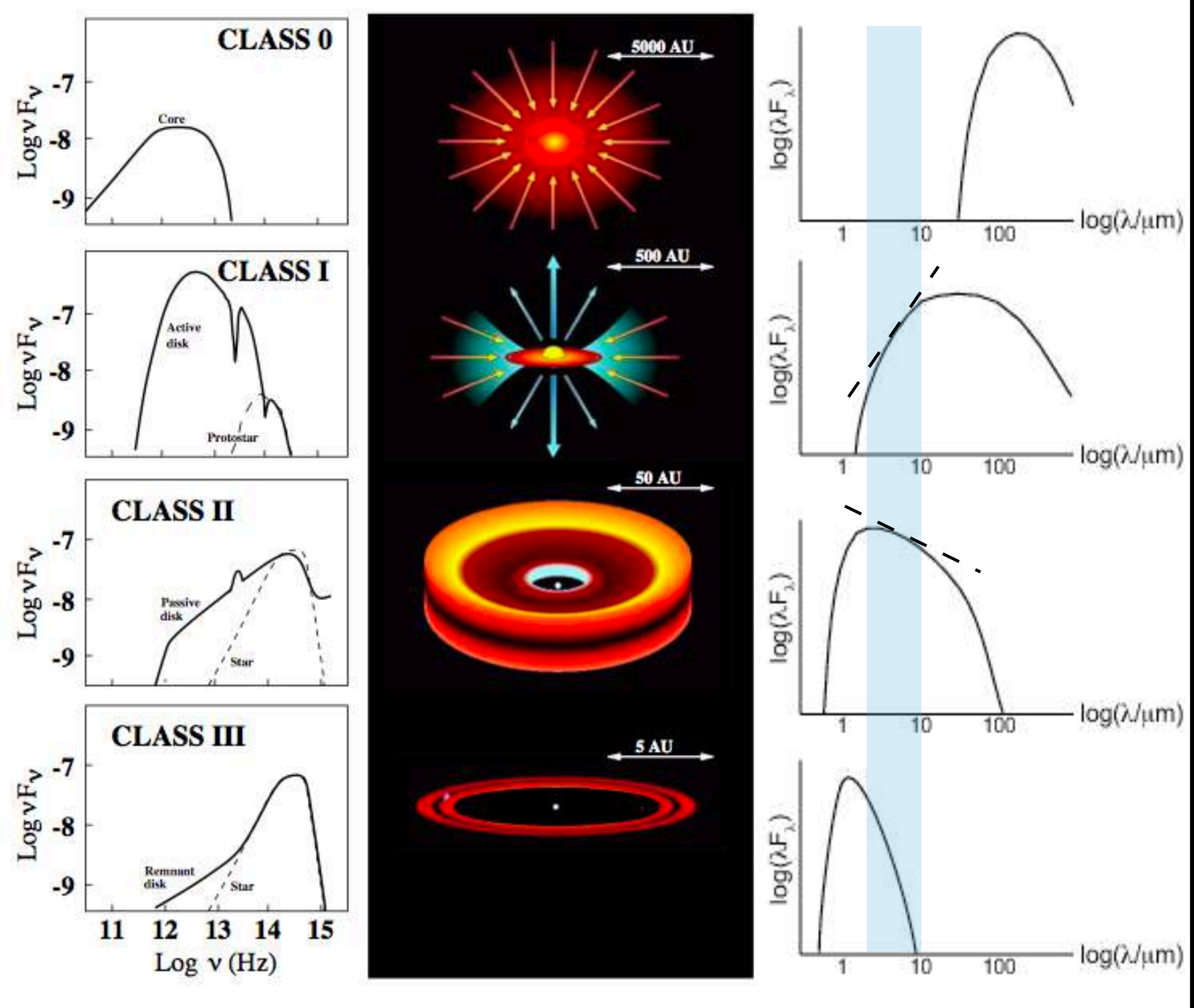


FROM UNIVERSE

TO PLANETS

LECTURE 2.2: DISC EVOLUTION/LIFETIME

STAGES OF EVOLUTION

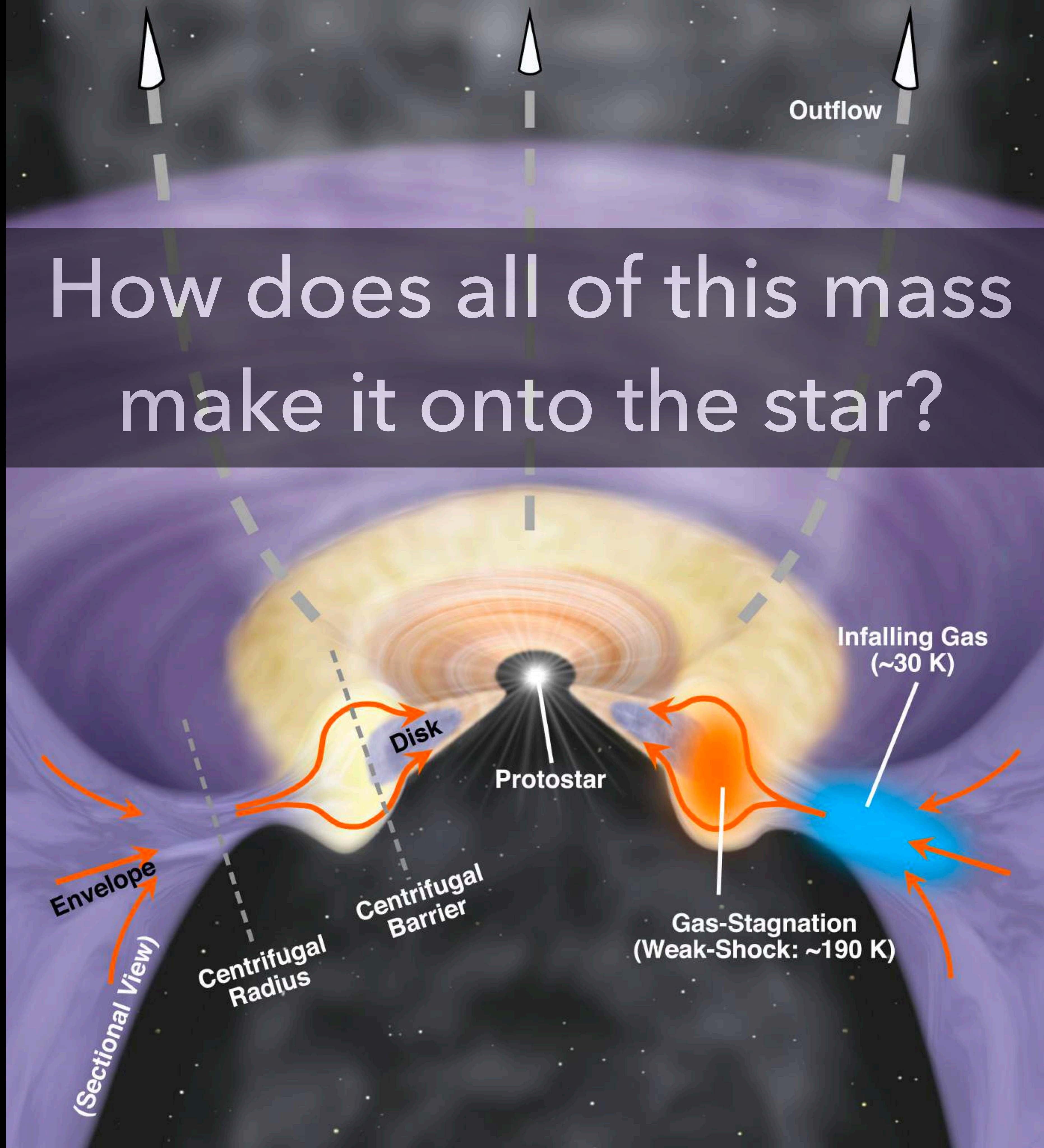


- SED peaks in the FIR or mm (no NIR flux)
- Flat or rising SED into MIR $\alpha_{IR} > 0$
- Falling SED into MIR $-1.5 < \alpha_{IR} < 0$
- Little or no excess in the IR

STAGES OF EVOLUTION

- Class 0 sources are the youngest stage, here the protostar rapidly accretes the bulk of its mass (main accretion phase) and is surrounded by a massive envelope and a disc.
- Class I sources are slowly accreting the rest of the final stellar mass (late accretion phase). The young stellar object (YSO) is still surrounded by a remnant envelope and massive disc.
- Class II sources no longer have an envelope, but still have an accretion disc producing the observed excess infrared emission. Most T Tauri stars (classical & some weak-line) belong to this class.
- At the Class III stage finally, the star is basically free from circumstellar material, evolving towards the main sequence. Most weak-line, but no classical T Tauri stars.

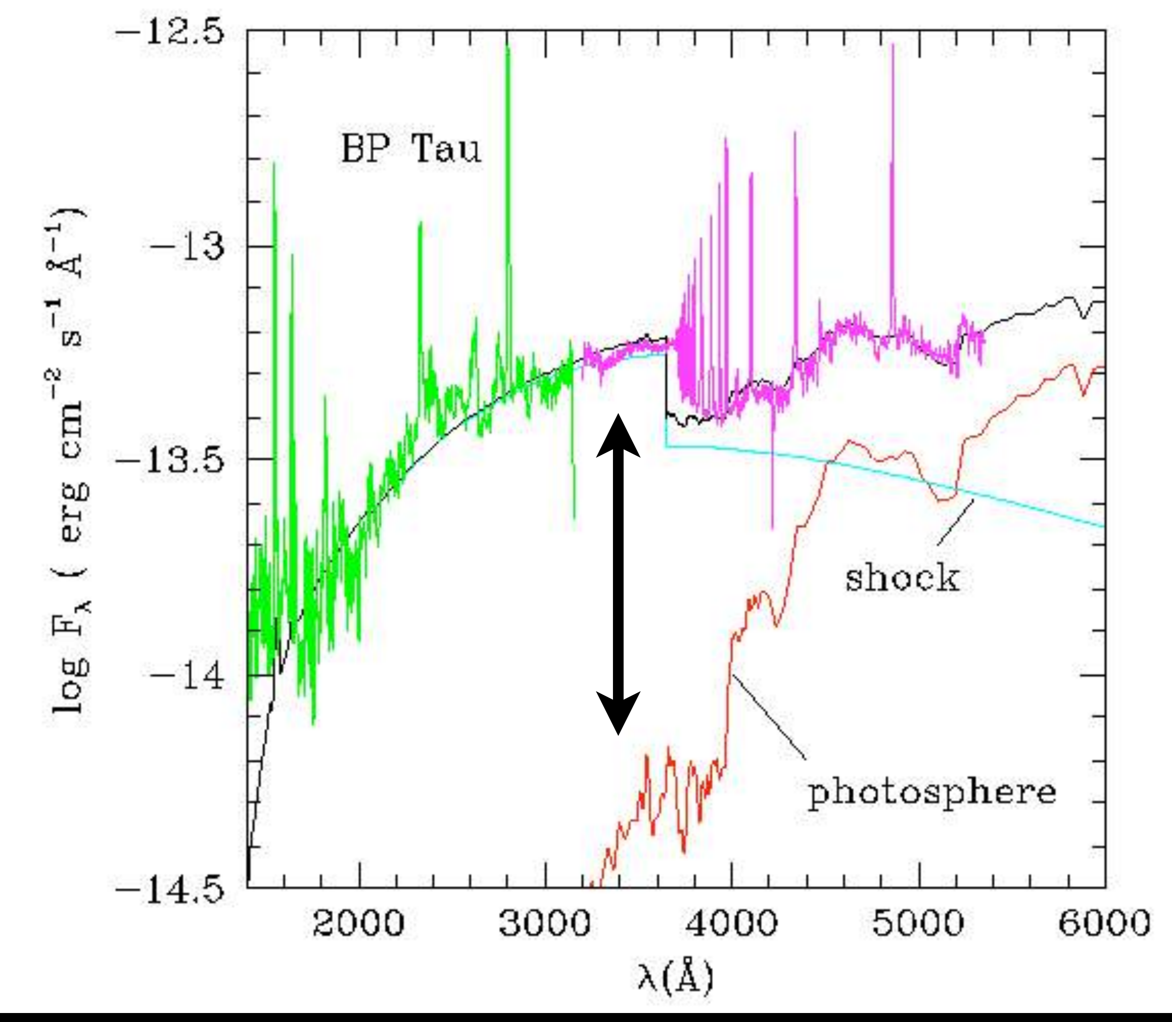
How does all of this mass make it onto the star?

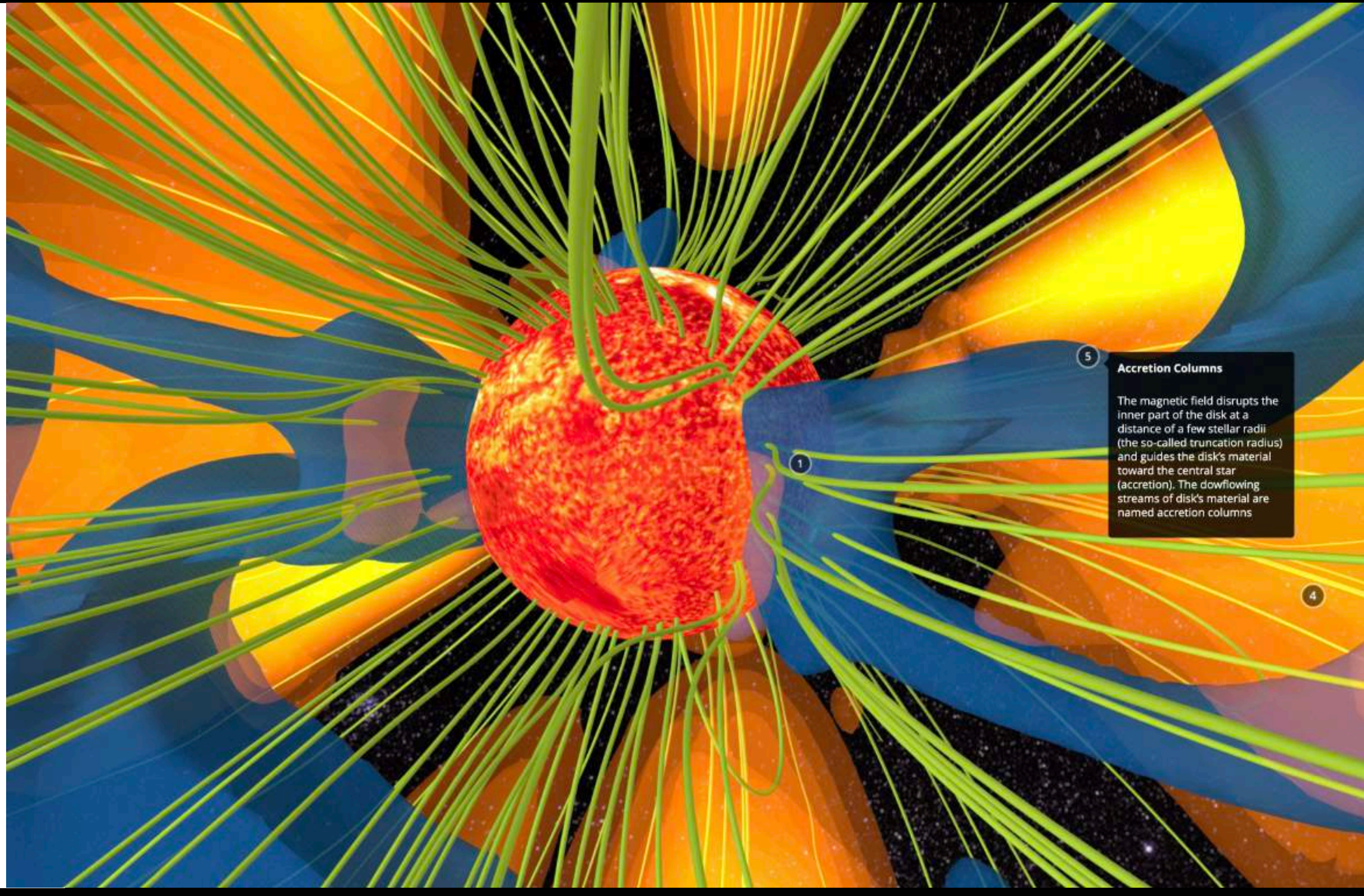


ACCRETION SIGNATURES

ACCRETION SIGNATURES

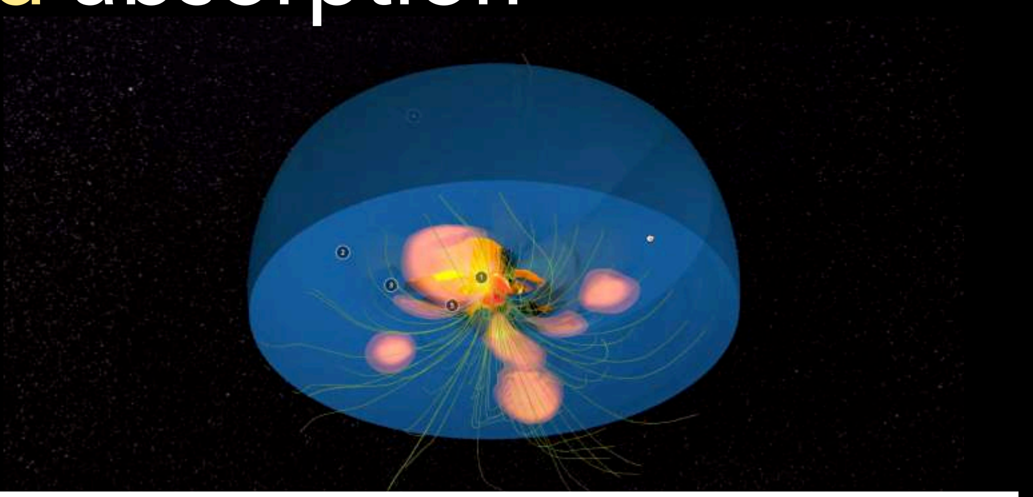
- Excess emission (veiling) over photosphere is strong evidence for accretion: $L_{\mathrm{acc}} = GM\dot{M}/R$
- Class II (T Tauri) stars have excess continuum emission arising from the accretion shock on the star, and emission lines from both the magnetosphere and the shock region.

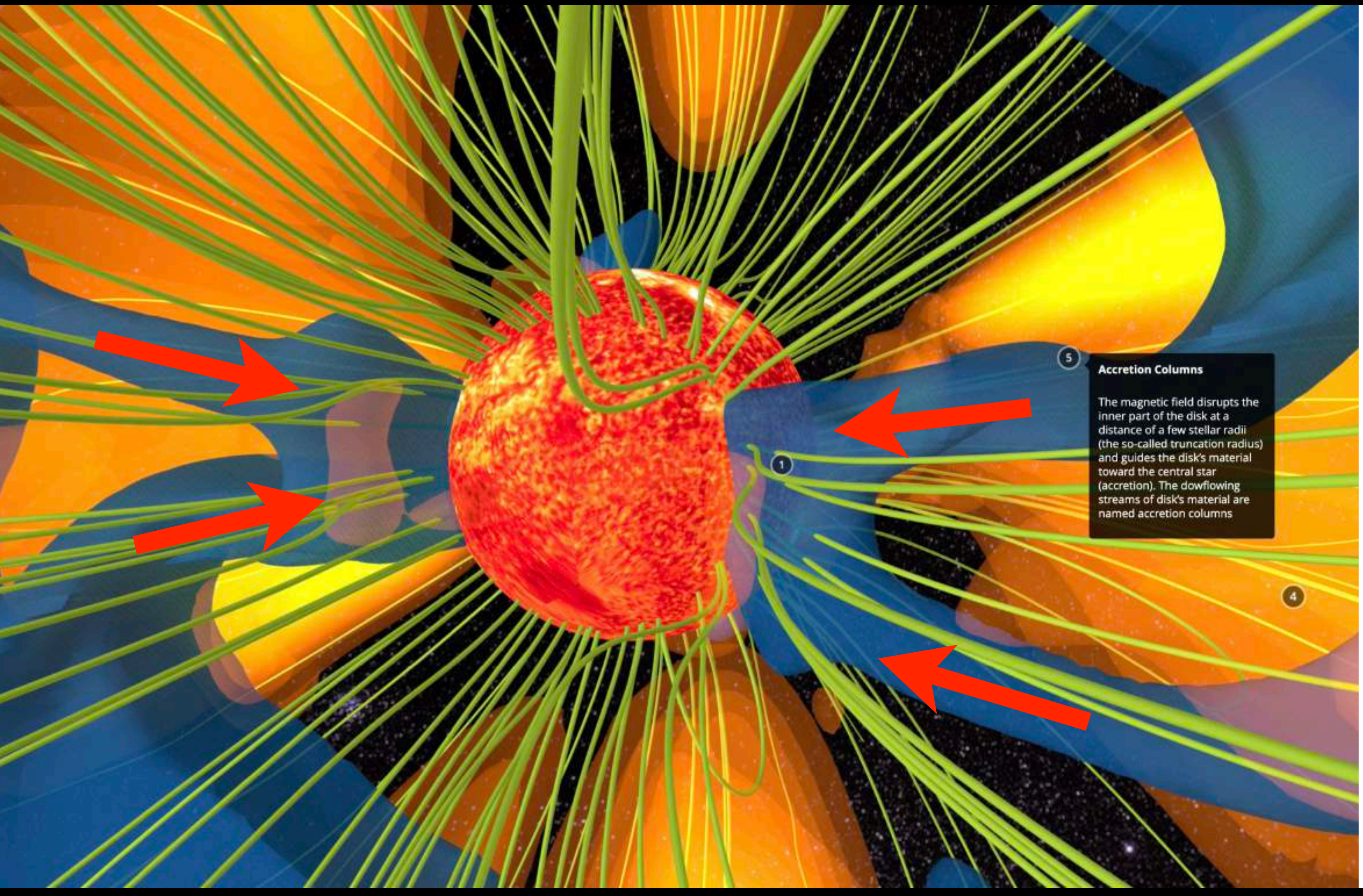


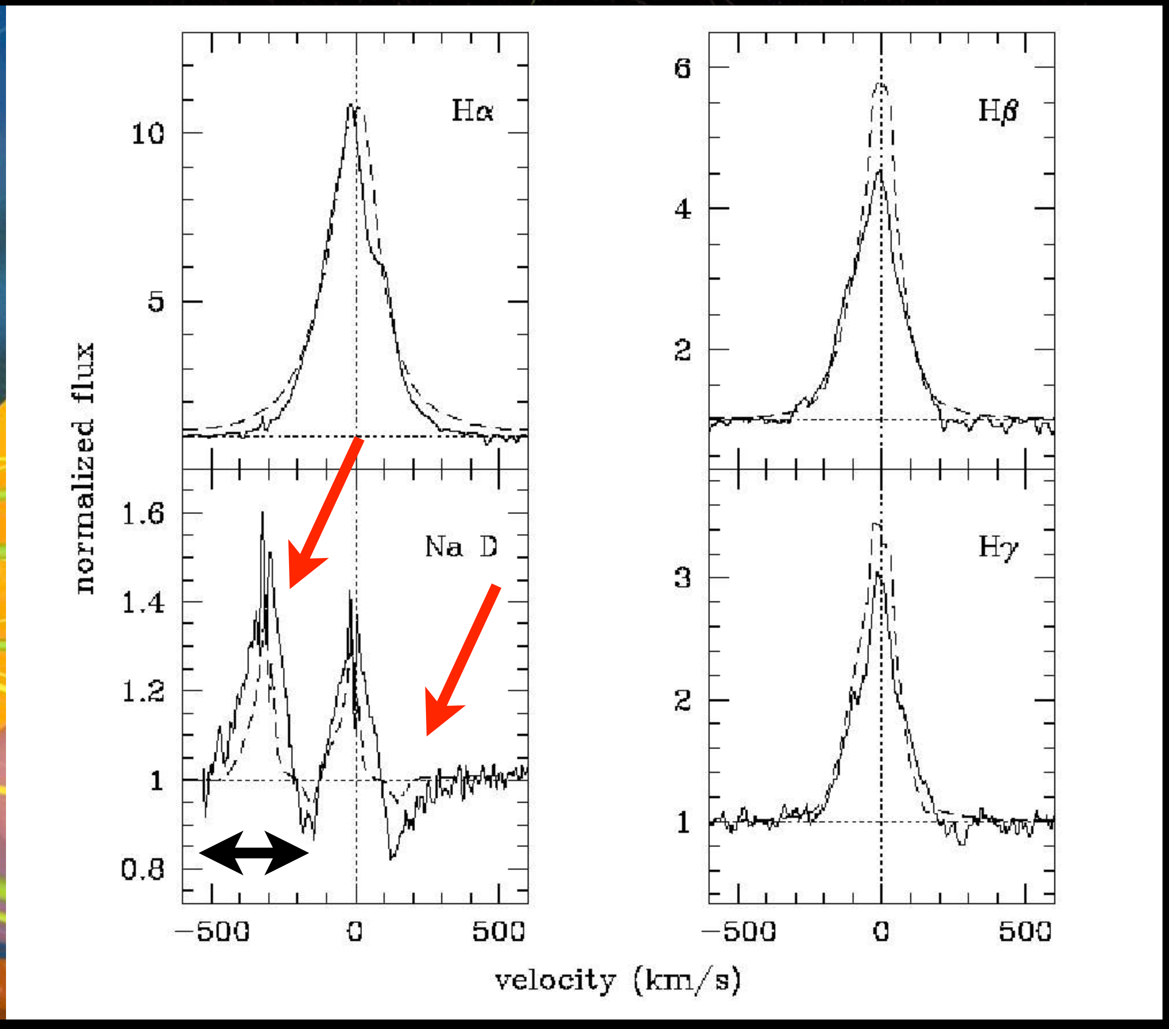


ACCRETION SIGNATURES

- Broad Emission lines ($\Delta v \sim 250$ km/s) from fast moving accretion flows show up as redshifted absorption
- Can only be seen at certain disc inclinations.







ANGULAR MOMENTUM

- Accretion requires angular momentum to be lost or redistributed in the disc.
- Specific angular momentum is approximately that of a Keplerian orbit: $l = R^2\Omega_\mathrm{K} = \sqrt{GM_*R}$.
  - Increasing function of radius.
- Two possibilities:
  - Viscous dissipation: predominant theory, but still not clear as to what causes the viscosity (friction).
  - Removed via outflows from the star-disc system.

VISCOUS EVOLUTION

- Within any shearing fluid, momentum is transported in the cross-stream direction because the random motion of molecules leads to collisions between particles that have different velocities.
- Assume a vertically thin axisymmetric sheet of viscous fluid to obtain a simple equation for the time evolution of the disk surface density $\Sigma(R, t)$.
- Large caveat: the molecular viscosity of the gas is much too small to lead to any significant dissipation.
    - …but remains approximately valid if the "viscosity" is reinterpreted as the outcome of a turbulent process.

VISCOUS EVOLUTION

- From the continuity equation (mass conservation) in cylindrical coordinates:

$$R\frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial R}\left(R\Sigma v_R\right) = 0$$

- From angular momentum conservation:

$$R\frac{\partial\left(\Sigma R^2 \Omega_{\mathrm{K}}\right)}{\partial t} + \frac{\partial}{\partial R}\left(R\Sigma v_R \cdot R^2\Omega_{\mathrm{K}}\right) = \frac{1}{2\pi}\frac{\partial \mathscr{T}}{\partial R}$$

- The right-hand side represents the net torque acting on the gas due to viscous stresses, where

$$\mathscr{T} = \underbrace{2\pi R}_{\text{circumference}} \cdot \underbrace{\overset{\text{kinematic viscosity}}{\nu}\Sigma R\frac{\partial \Omega_{\mathrm{K}}}{\partial R}}_{\text{viscous force per unit length}} \cdot \underbrace{R}_{\text{lever arm}}$$

VISCOUS EVOLUTION

- Eliminating v_R by expanding the derivatives and substituion gives us a diffusion equation:

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left[\sqrt{R}\frac{\partial}{\partial R}\left(\nu\Sigma\sqrt{R}\right)\right]$$

- Which is more obvious if we ν is constant (not actually true) and perform a change of variables:

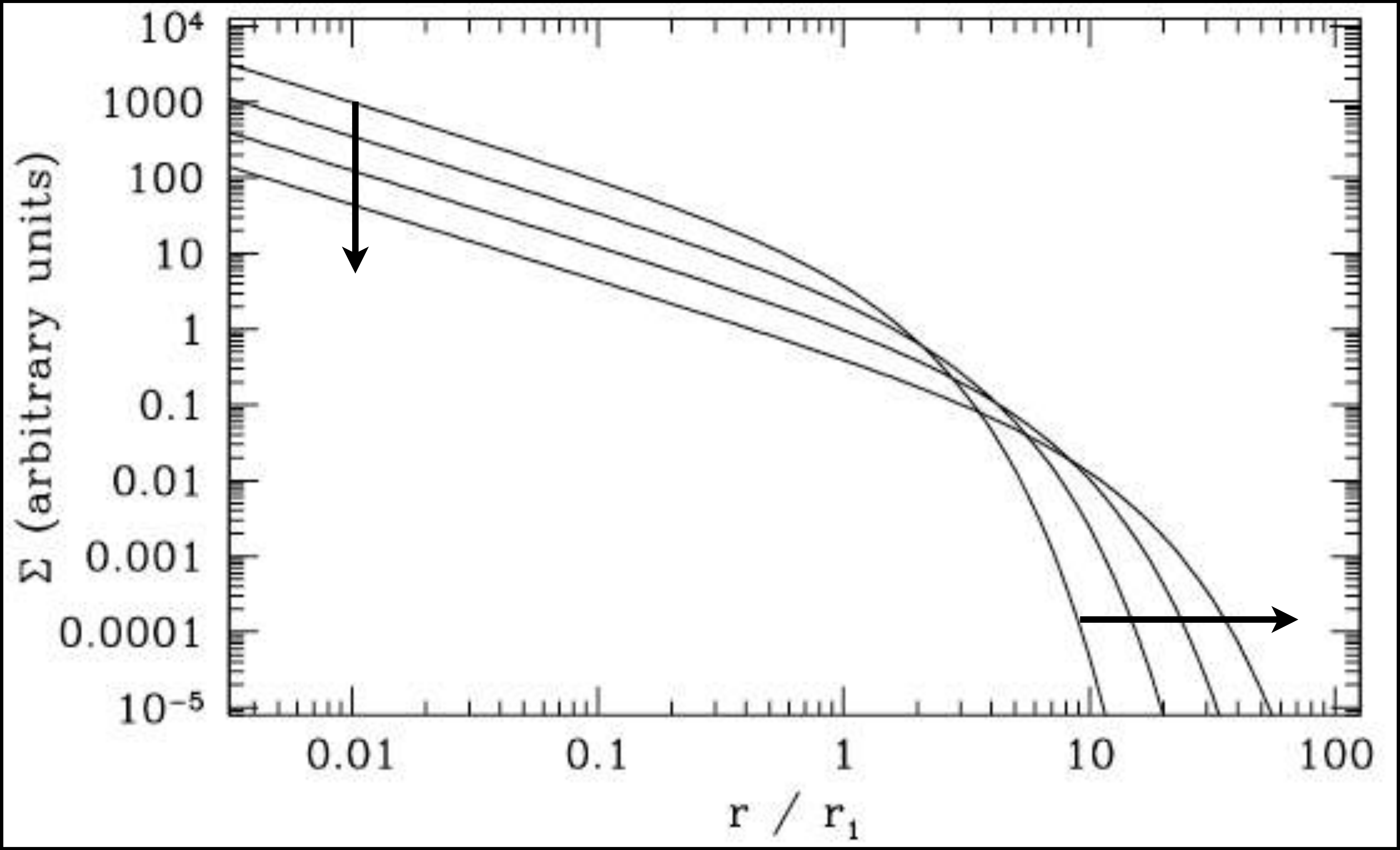
$$X \equiv 2\sqrt{R} \qquad\qquad f \equiv \frac{3}{2}\Sigma X$$

- Giving us the diffusion equation and diffusion coefficient:

$$\frac{\partial f}{\partial t} = D\frac{\partial^2 f}{\partial X^2} \qquad\qquad D = \frac{12\nu}{X^2}$$

- The timescale to diffuse across a length scale ΔX is: $\tau_\nu \approx \frac{R^2}{\nu}$

VISCOUS EVOLUTION



VISCOUS EVOLUTION

- Molecular viscosity alone yields timescales ~ 10^{13} yrs, longer than the age of the Universe! Instead, we think there is an underlying turbulence that "acts" like an effective viscosity.
- To avoid specifying the source of the turbulence, we often parameterise the viscosity as: $\nu = \alpha c_{\rm s} H$
  - The largest eddy $\lesssim H$
  - Turbulent velocity $\lesssim c_{\rm s}$ (otherwise a shock would form)
- Describes the leading order scaling expected in disks (so that the dimensionless Shakura-Sunyaev α-parameter varies more slowly with temperature, radius, etc. than ν)

VISCOUS EVOLUTION

- The condition for linear hydrodynamical stability (Rayleigh criterion) is that angular momentum increases outwards, i.e.:

$$\frac{d}{dR}\left(R^2\Omega\right) > 0$$

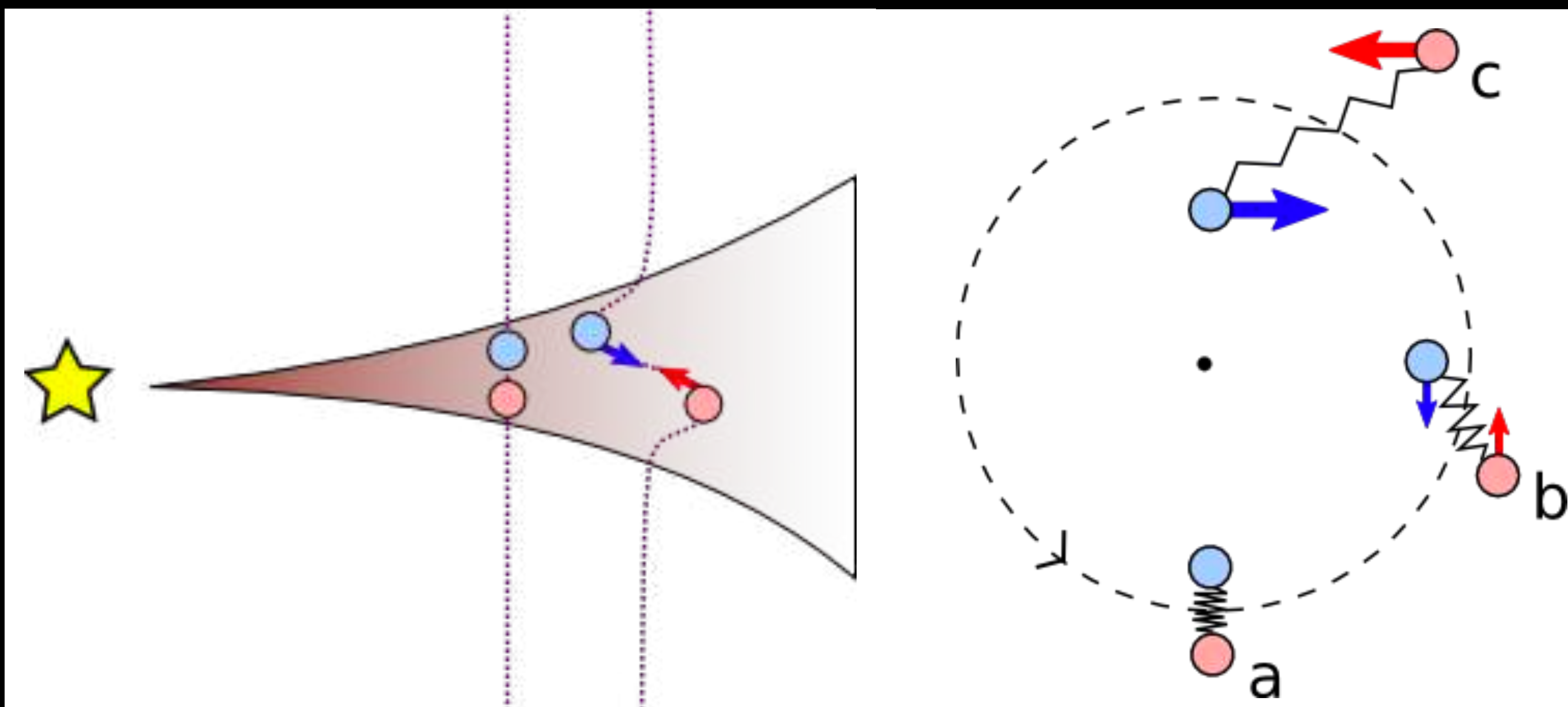
- This is always true in Keplerian discs where $R^2\Omega_\mathrm{K} \propto \sqrt{R}$.
- A magneto-hydrodynamical (MHD) flow requires a further stability condition, i.e. the angular velocity itself must increase with radius:

$$\frac{d}{dR}\left(\Omega^2\right) > 0$$

- Not true for Keplerian discs where $\Omega_\mathrm{K} \propto R^{-3/2}$.

VISCOUS EVOLUTION

- In ideal MHD, the fluid acts like a perfect conductor and field lines are frozen into the fluid (zero diffusion of magnetic field lines). In this case, even weak magnetic fields will generate a Magnetorotational Instability (MRI).



VISCOUS EVOLUTION

$$\beta = \frac{\text{(plasma pressure)}}{\text{magnetic pressure}}$$

$$= \frac{n k_{\mathrm{B}} T}{B^2/2\mu_0}$$

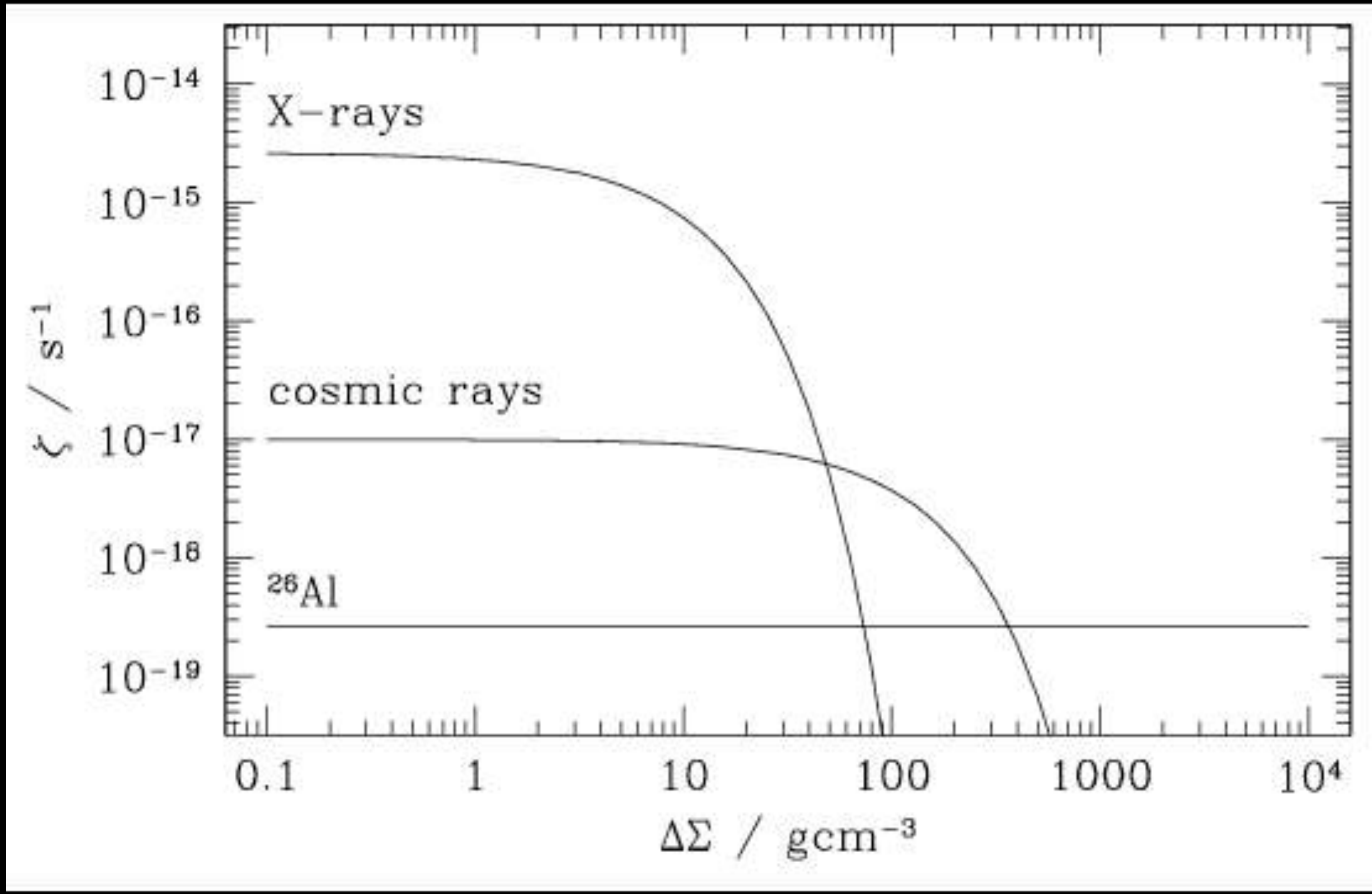
VISCOUS EVOLUTION

- Non-ideal MHD, the disc needs to be sufficiently ionised to overcome the effects of resistivity, which otherwise allows the field lines to diffuse back through the fluid.

- Two processes can ionise the gas in a disc:

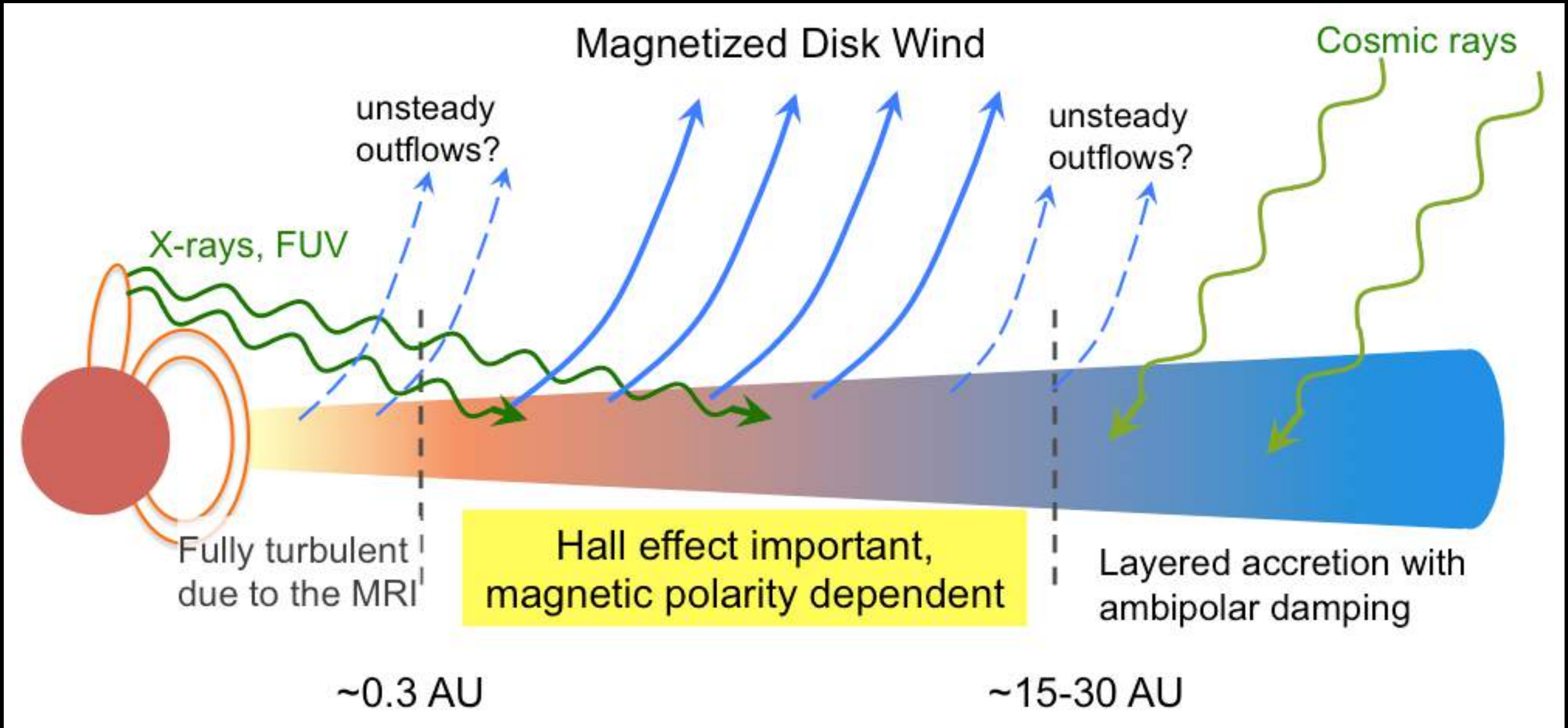
- Thermal (collisional) ionisation: requires $T \gtrsim 1000$ K, only occurs in inner 1 au of disc.

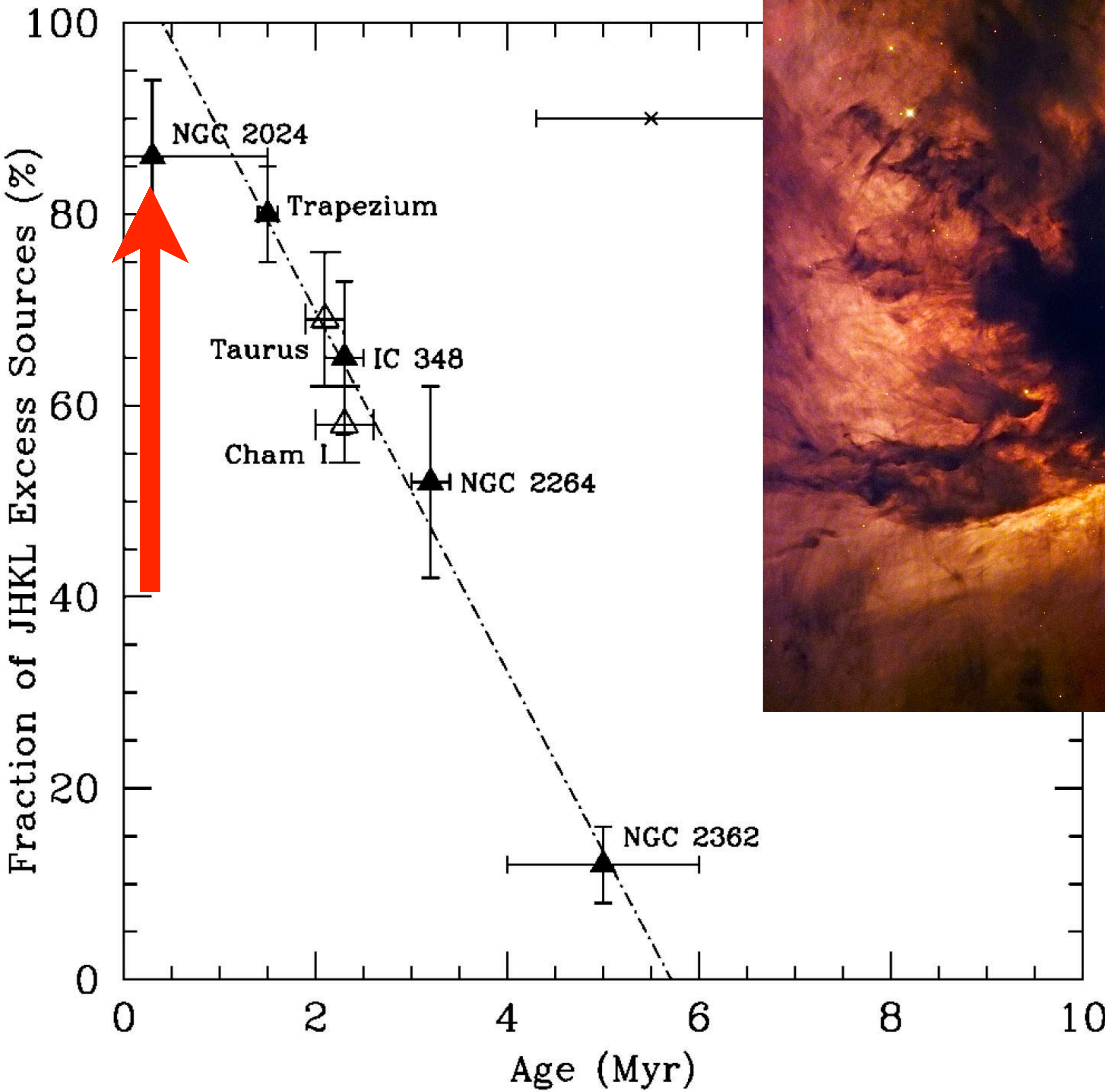
- Non-thermal (photo-) ionisation by UV, X-rays, and/or cosmic rays.



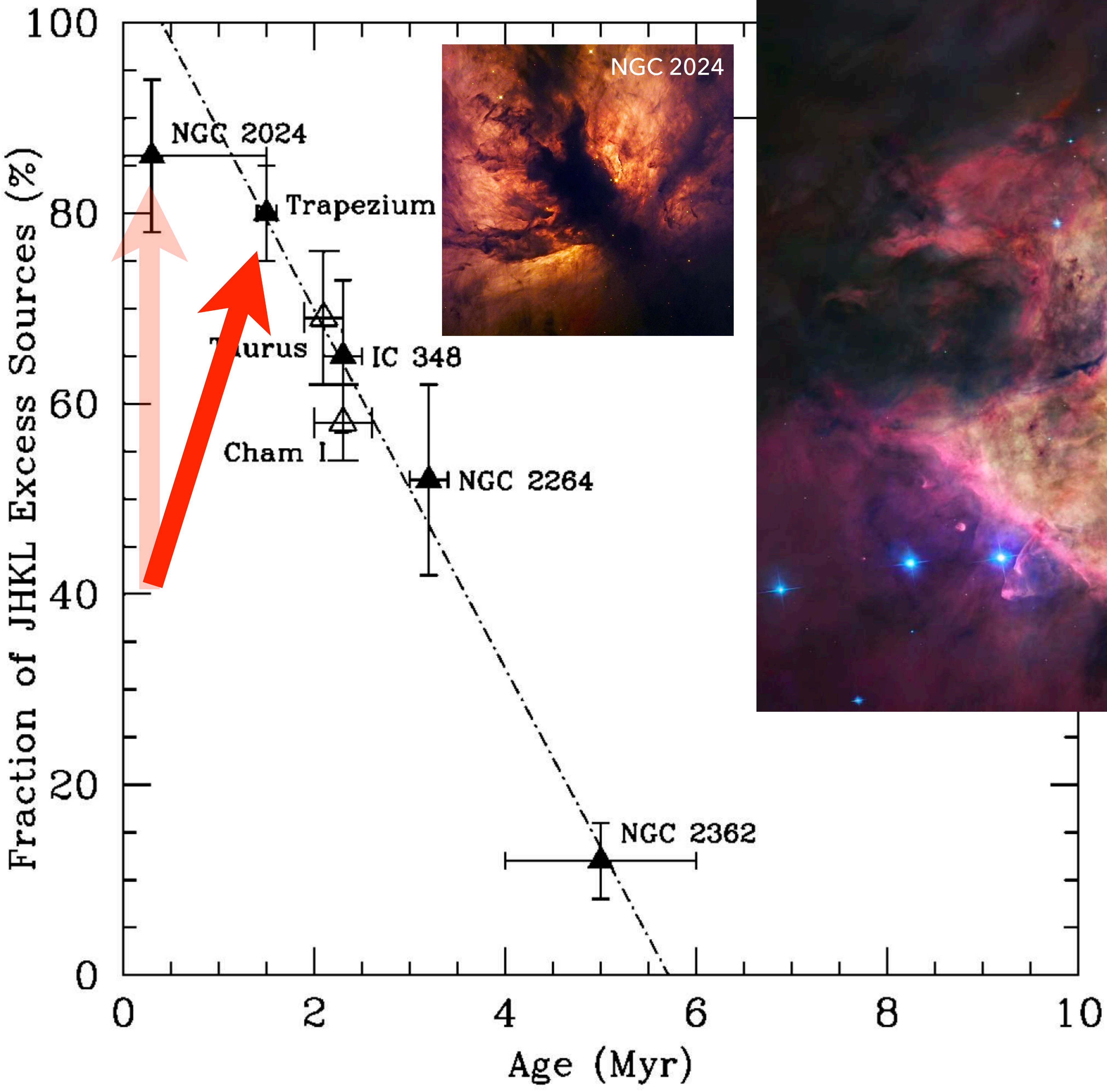
VISCOUS EVOLUTION

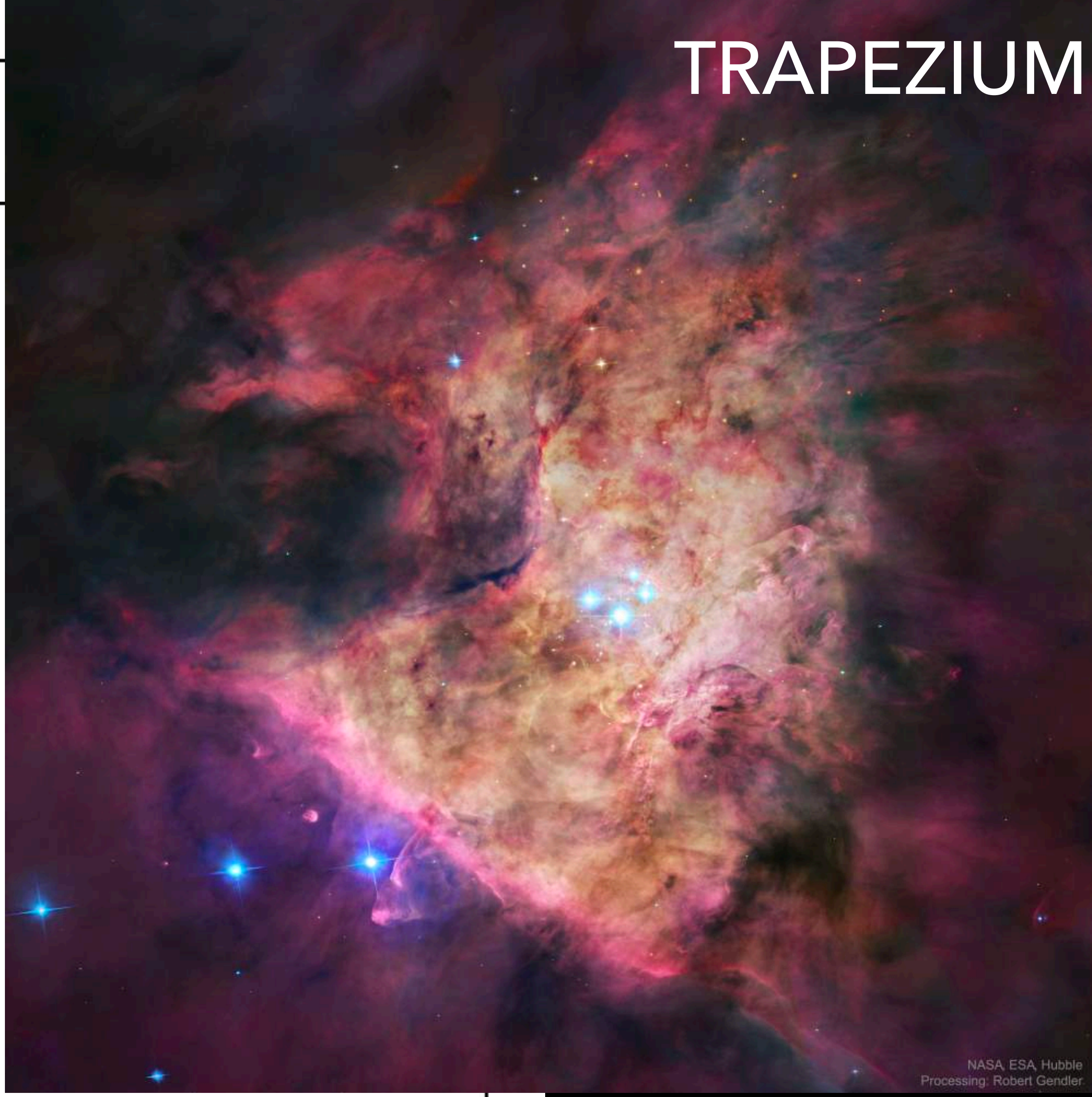
- For typical conditions, the MRI is likely damped between 0.1–10 au (dead zones).
- Important implications for dust dynamics, planetesimal formation, planet migration, and episodic accretion.



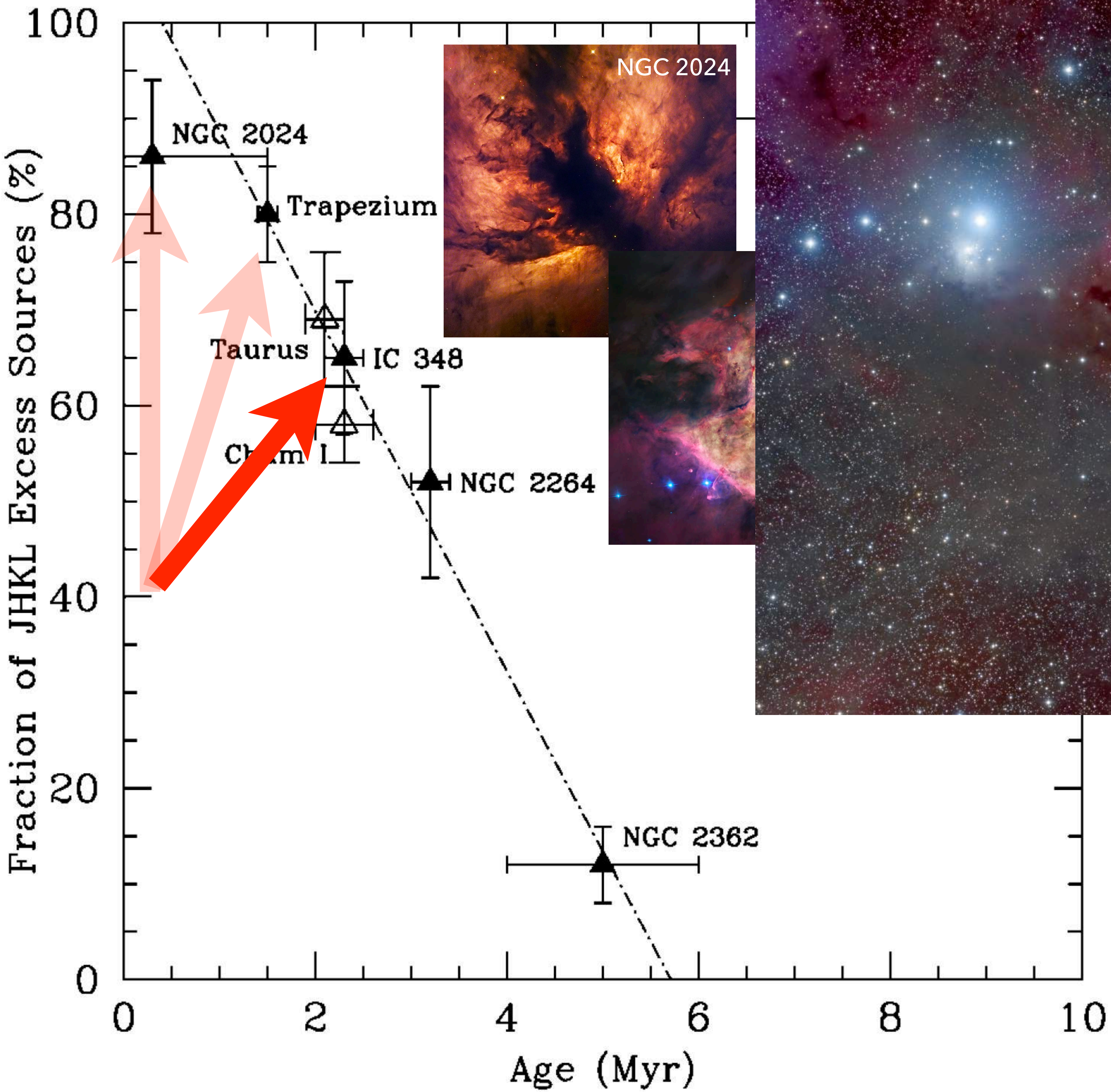


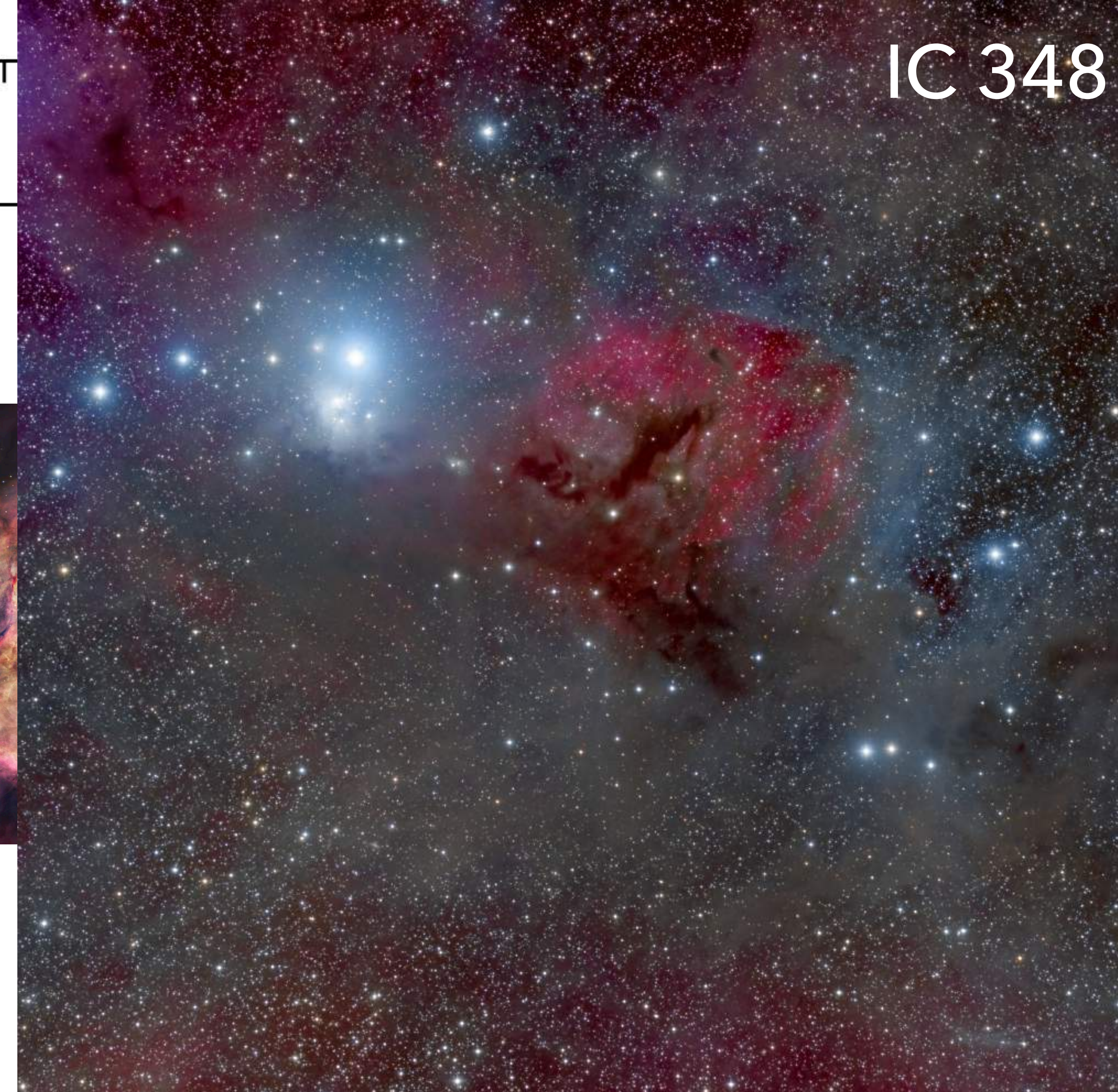
DISC LIFETIMES





DISC LIFETIMES





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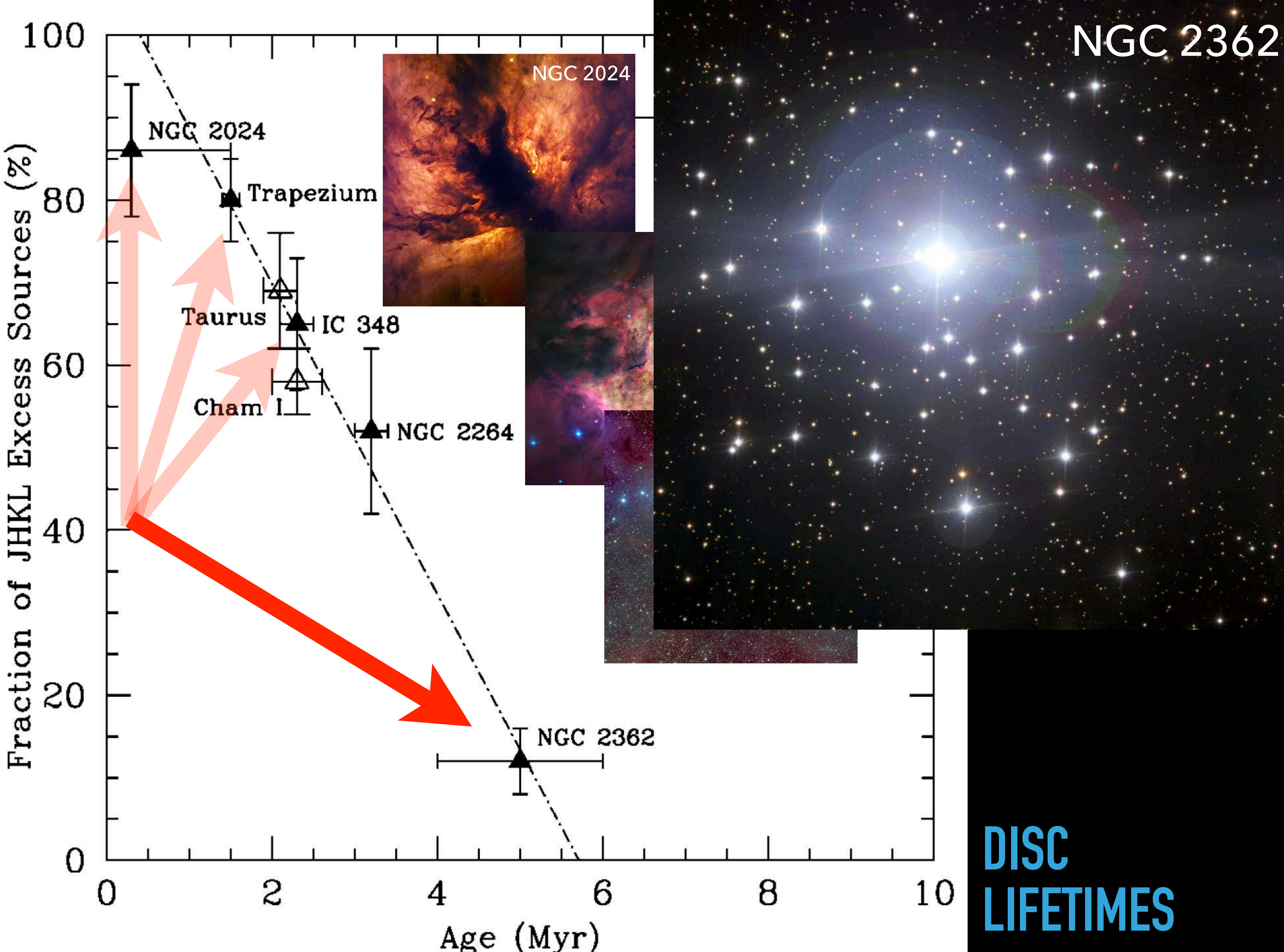

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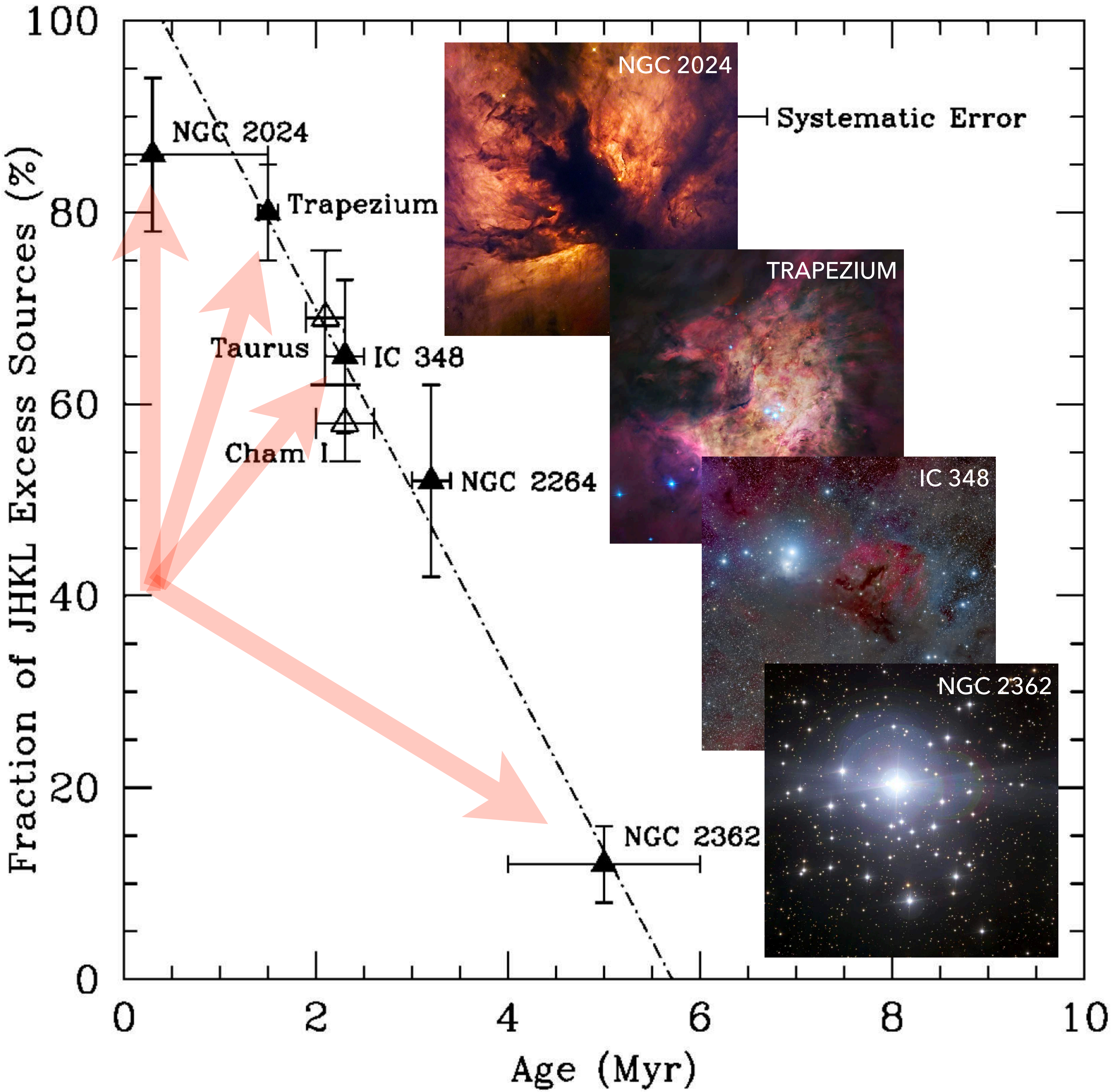
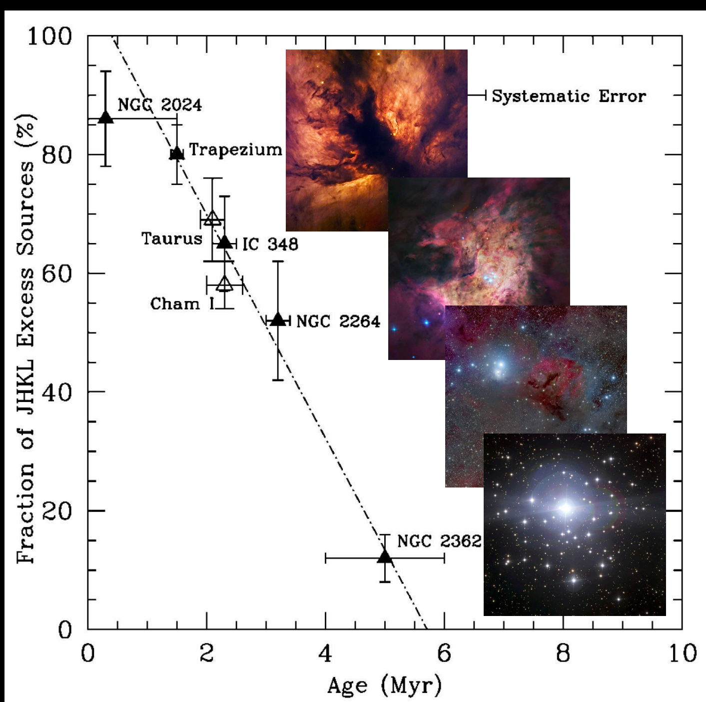
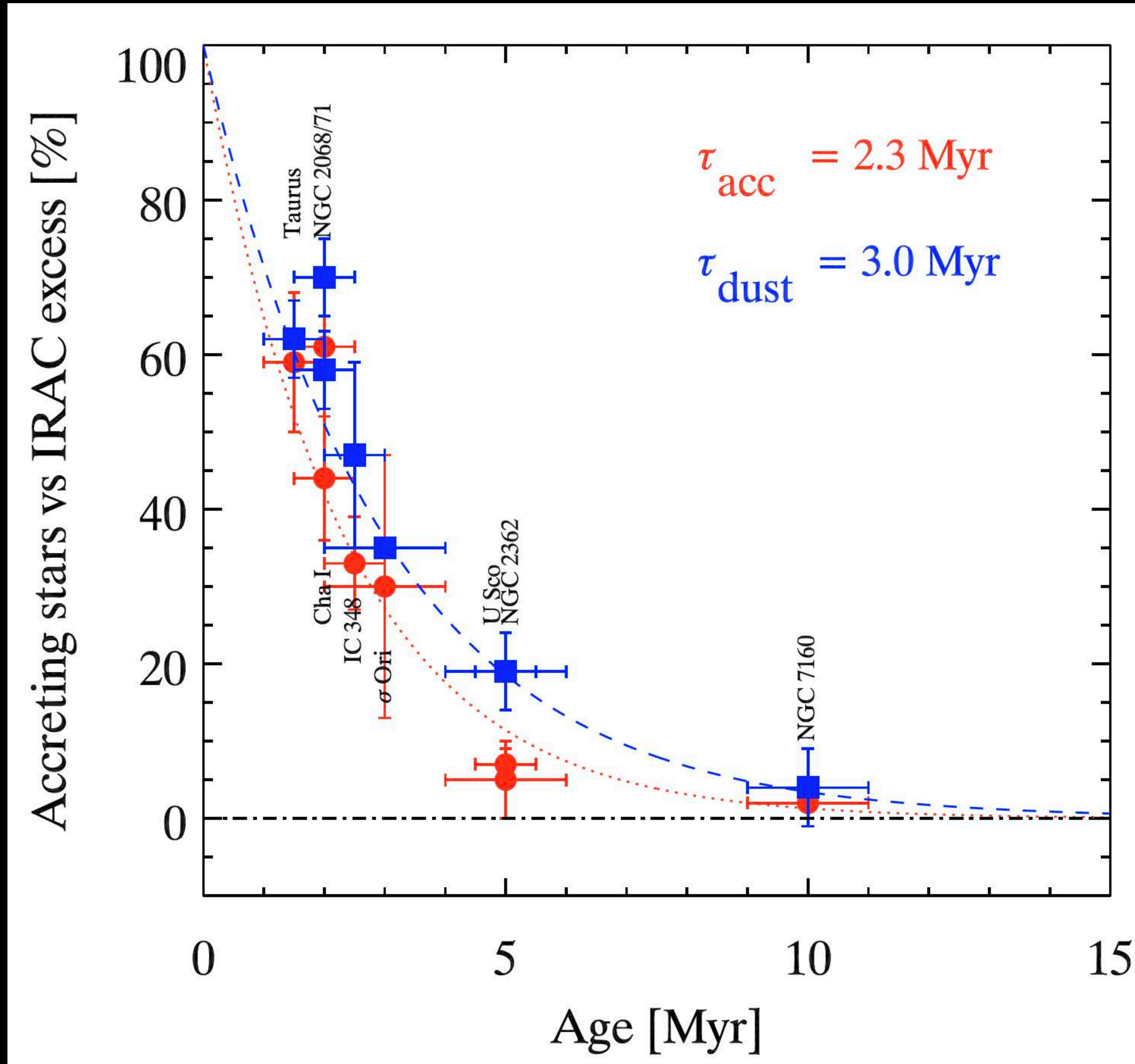


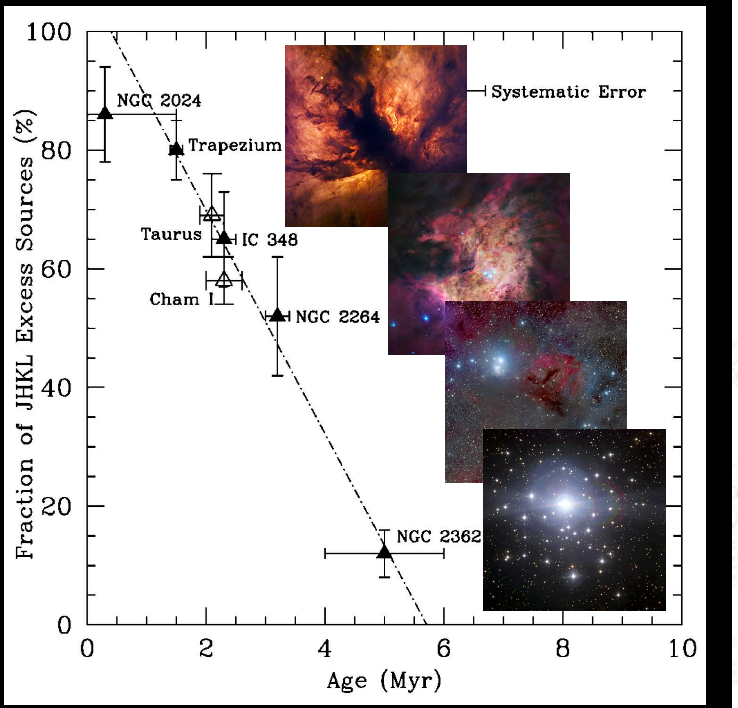
DISC LIFETIMES

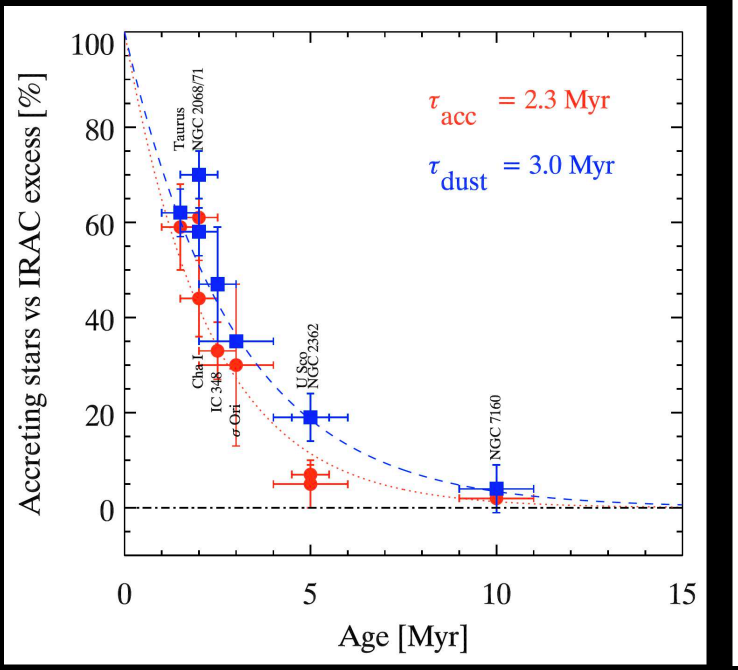
DISC LIFETIMES

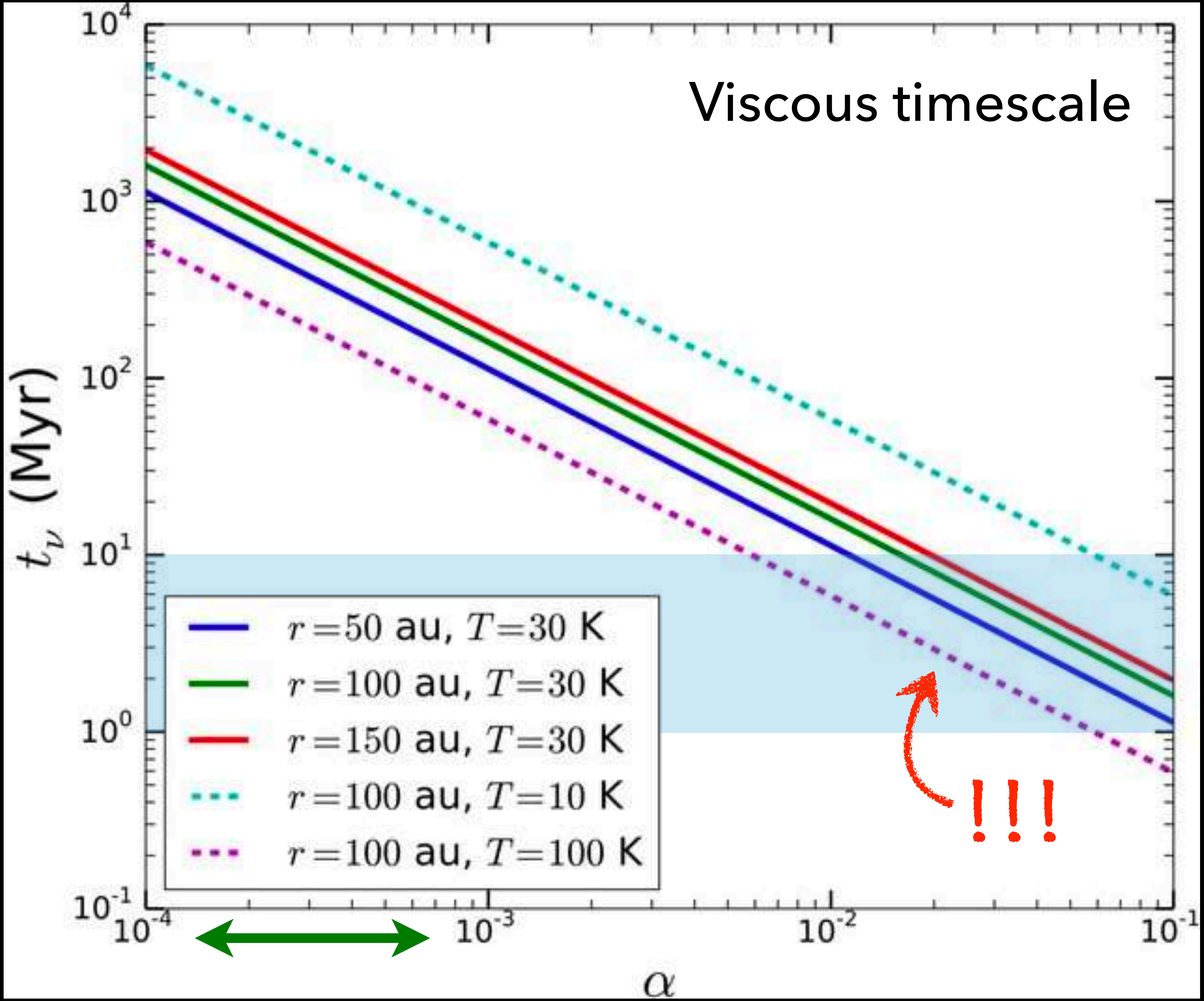


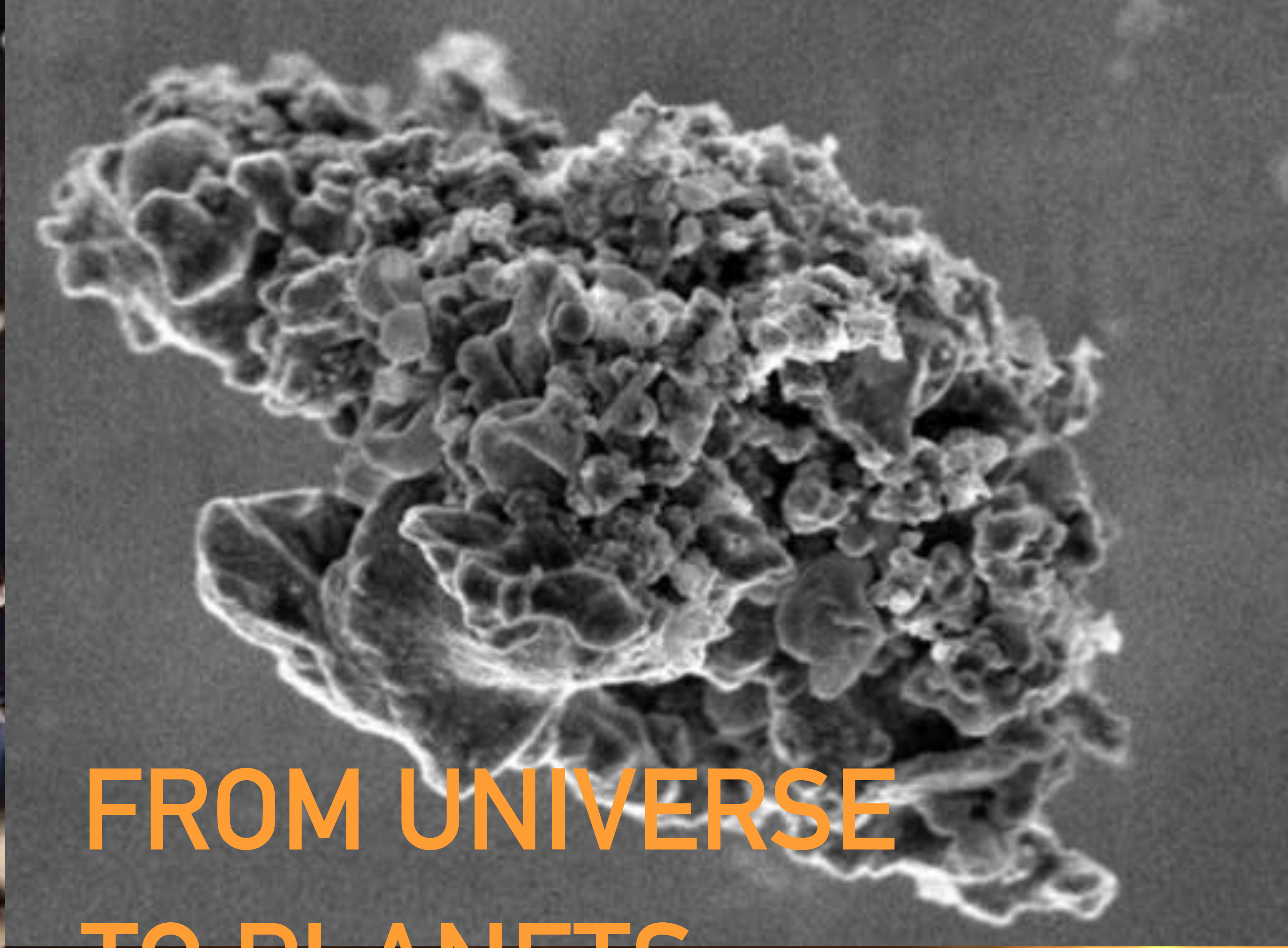


DISC LIFETIMES





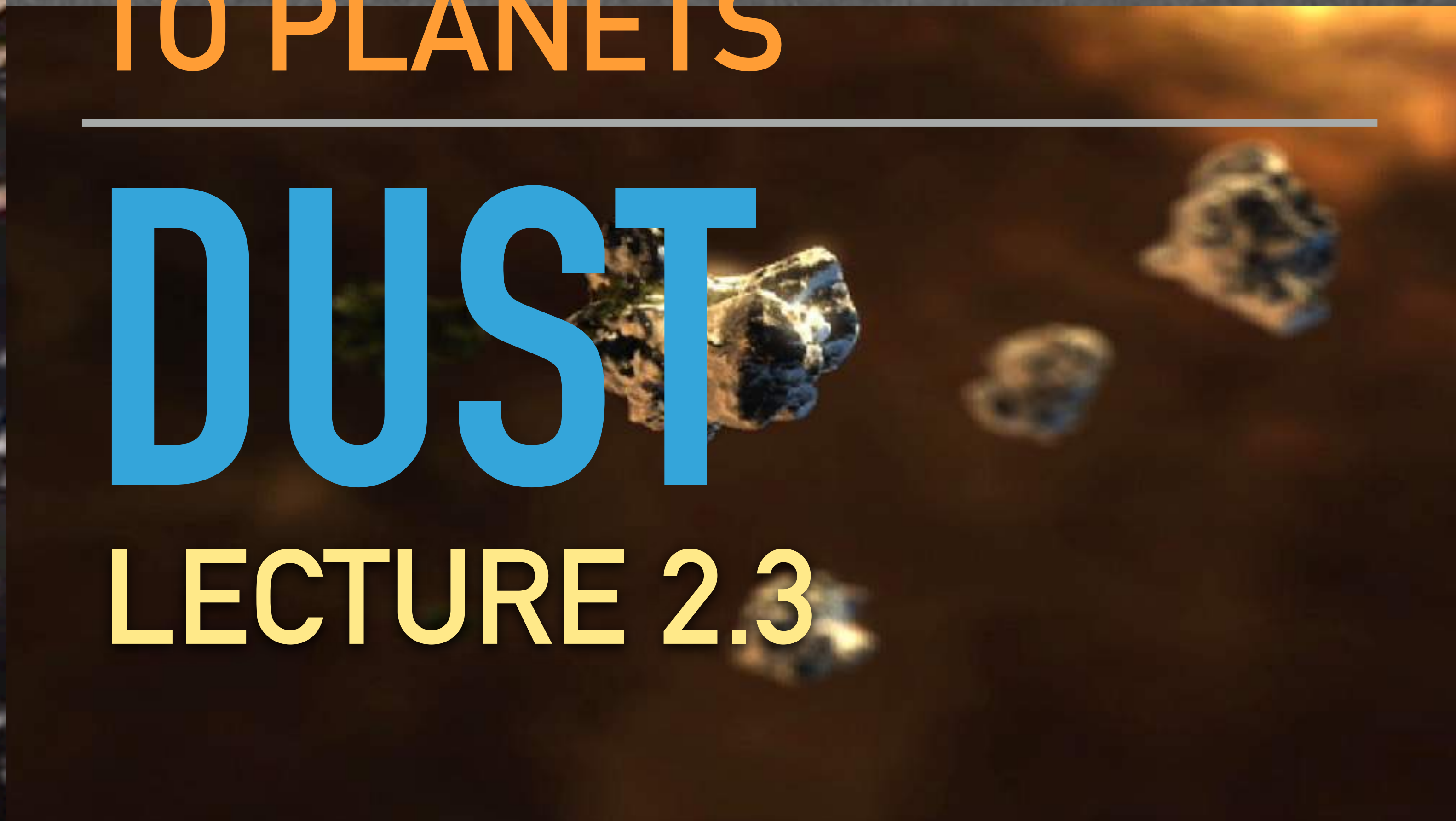




FROM UNIVERSE
TO PLANETS

DUST

LECTURE 2.3



DUST: SIZES AND MASSES

Samples

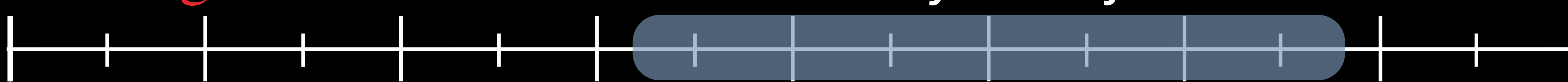
Lab & IDPs (interplanetary dust particles)

Meteorites

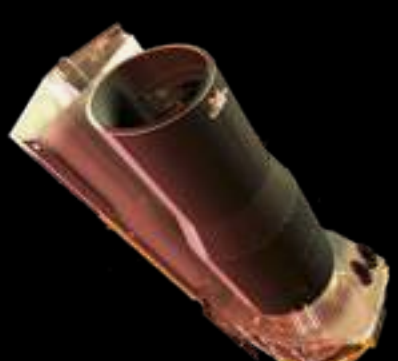
10^{-15} g

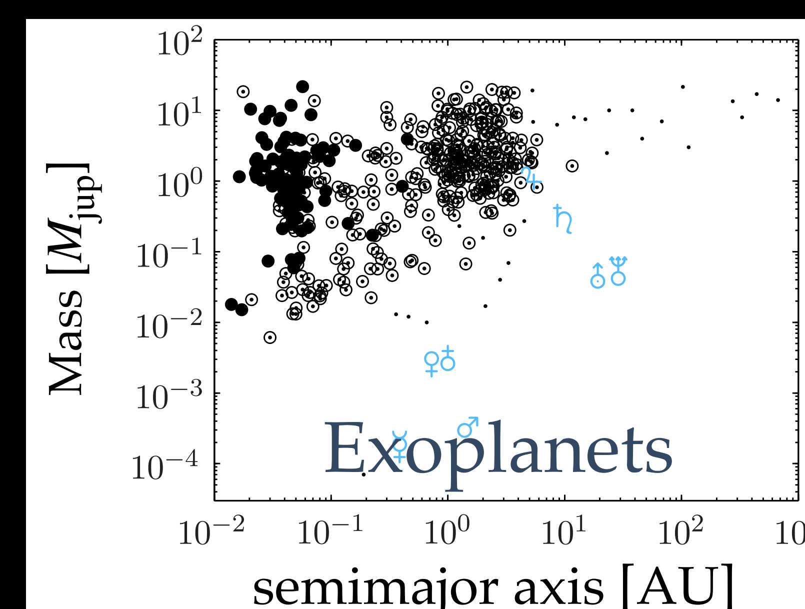
only theory

10^{27} g



Observations: IR and (sub-)mm

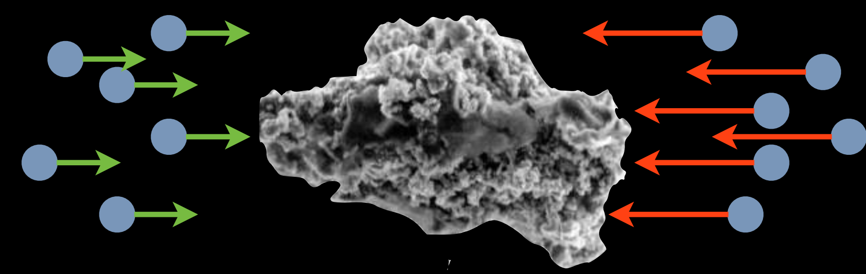




DUST: DRAG LAWS

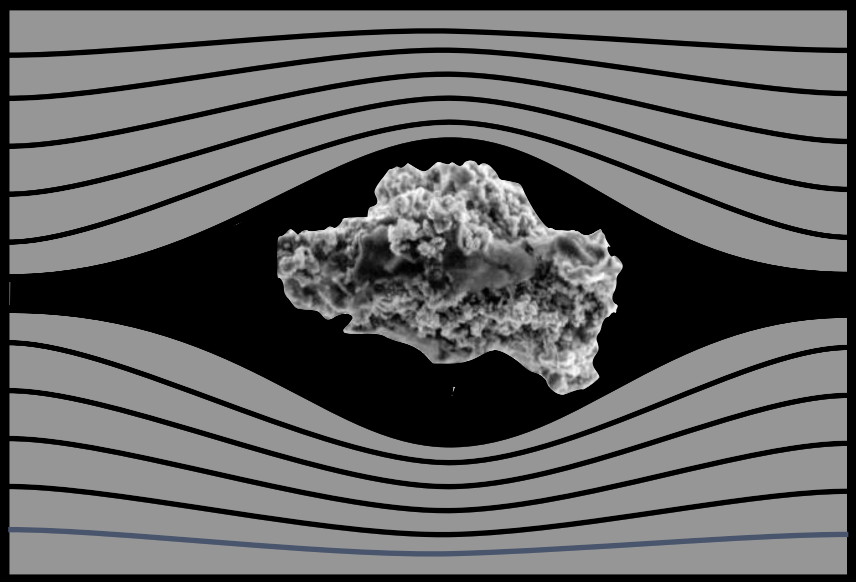
- Epstein regime: if particle size $\lesssim$ mean free path

$$F_{\mathrm{Epstein}} = -\frac{4\pi}{3}\rho_{\mathrm{grain}} a^2 v_{\mathrm{th}} \mathbf{v}$$



- Stokes regime: if particle size $\gtrsim$ mean free path

$$F_{\mathrm{Stokes}} = -\frac{C_{\mathrm{D}}}{2}\pi a^2 \rho_{\mathrm{grain}} v \mathbf{v}$$



  - C_{D} depends on the particle Reynolds number (the ratio of inertial forces to viscous forces).

DUST: DRAG LAWS

- Estimate the timescale for deceleration

$$t_{\rm stop} = \frac{v}{\dot{v}} = \frac{mv}{|F_{\rm drag}|}$$

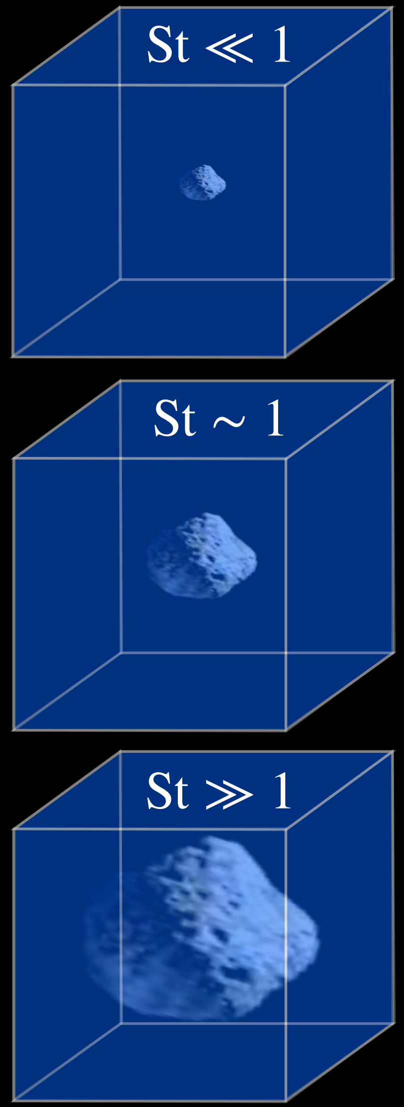
- For Epstein drag:

$$t_{\rm stop} = \frac{\rho_{\rm grain} a}{\rho_{\rm g} v_{\rm th}} \qquad \text{where} \qquad v_{\rm th} = \sqrt{\frac{8}{\pi}} c_{\rm s}$$

  - For example $R \sim 1$ au: $1\ \mu\text{m} \rightarrow 3$ s
    1 cm $\rightarrow 0.1$ yr

- Dividing by the orbital timescale, gives us the Stokes number:

$$\frac{t_{\rm stop}}{t_{\rm orb}} = t_{\rm stop}\Omega_{\rm K} \equiv \mathrm{St}$$



DUST: RADIAL DRIFT

- Force equation: drag, gravity, and pressure forces:

dust velocity drag coefficient gas density

$$\frac{d\mathbf{v}_\mathrm{d}}{dt} = -A\rho_\mathrm{g}(\mathbf{v}_\mathrm{d} - \mathbf{v}_\mathrm{g}) - \frac{GM_*}{R^3}\mathbf{R}$$

$$\frac{d\mathbf{v}_\mathrm{g}}{dt} = +A\rho_\mathrm{d}(\mathbf{v}_\mathrm{d} - \mathbf{v}_\mathrm{g}) - \frac{GM_*}{R^3}\mathbf{R} - \frac{\nabla P}{\rho_\mathrm{g}}$$

gas velocity dust density gas pressure

- The drag coefficient is is related to the Stokes number by:

$$A = \frac{v_\mathrm{th}}{\rho_\mathrm{grain}a} \qquad \longrightarrow \qquad \mathrm{St} = \frac{\Omega_\mathrm{K}}{A\rho_\mathrm{g}}$$

DUST: RADIAL DRIFT

- Split into components & linearise using: $\mathbf{u}_\mathrm{d} = \mathbf{v}_\mathrm{d} - \begin{pmatrix} 0 \\ R\Omega_\mathrm{K} \\ 0 \end{pmatrix}$

$$\frac{\partial u_\mathrm{d}^R}{\partial t} = -A\rho_\mathrm{g}(u_\mathrm{d}^R - u_\mathrm{g}^R) + 2\Omega_\mathrm{K} u_\mathrm{d}^\phi$$

$$\frac{\partial u_\mathrm{d}^\phi}{\partial t} = -A\rho_\mathrm{g}(u_\mathrm{d}^\phi - u_\mathrm{g}^\phi) - \frac{1}{2}\Omega_\mathrm{K} u_\mathrm{d}^R$$

$$\frac{\partial u_\mathrm{g}^R}{\partial t} = +A\rho_\mathrm{d}(u_\mathrm{d}^R - u_\mathrm{g}^R) + 2\Omega_\mathrm{K} u_\mathrm{g}^\phi - \frac{1}{\rho_\mathrm{g}}\frac{\partial P}{\partial R}$$

$$\frac{\partial u_\mathrm{g}^\phi}{\partial t} = +A\rho_\mathrm{g}(u_\mathrm{d}^\phi - u_\mathrm{g}^\phi) - \frac{1}{2}\Omega_\mathrm{K} u_\mathrm{g}^R$$

DUST: RADIAL DRIFT

- Solve for the stationary velocities (i.e time derivative = 0):

$$\cancelto{0}{\frac{\partial u_{\rm d}^R}{\partial t}} = -A\rho_{\rm g}(u_{\rm d}^R - u_{\rm g}^R) + 2\Omega_{\rm K} u_{\rm d}^\phi$$

$$\cancelto{0}{\frac{\partial u_{\rm d}^\phi}{\partial t}} = -A\rho_{\rm g}(u_{\rm d}^\phi - u_{\rm g}^\phi) - \frac{1}{2}\Omega_{\rm K} u_{\rm d}^R$$

$$\cancelto{0}{\frac{\partial u_{\rm g}^R}{\partial t}} = +A\rho_{\rm d}(u_{\rm d}^R - u_{\rm g}^R) + 2\Omega_{\rm K} u_{\rm g}^\phi - \frac{1}{\rho_{\rm g}}\frac{\partial P}{\partial R}$$

$$\cancelto{0}{\frac{\partial u_{\rm g}^\phi}{\partial t}} = +A\rho_{\rm g}(u_{\rm d}^\phi - u_{\rm g}^\phi) - \frac{1}{2}\Omega_{\rm K} u_{\rm g}^R$$

DUST: RADIAL DRIFT

- Using the substitutions:

$$\varepsilon = \frac{\rho_\mathrm{d}}{\rho_\mathrm{g}} \qquad \eta = -\frac{1}{2\rho_\mathrm{g} R \Omega_\mathrm{K}^2}\frac{dP}{dR} = -\frac{c_\mathrm{s}^2}{2v_\mathrm{K}^2}\frac{d\ln P}{d\ln R}$$

- We can finally solve for radial dust velocity:

$$u_\mathrm{d}^R = -\frac{2\eta v_\mathrm{K}}{\mathrm{St} + \mathrm{St}^{-1}(1+\varepsilon)^2}$$

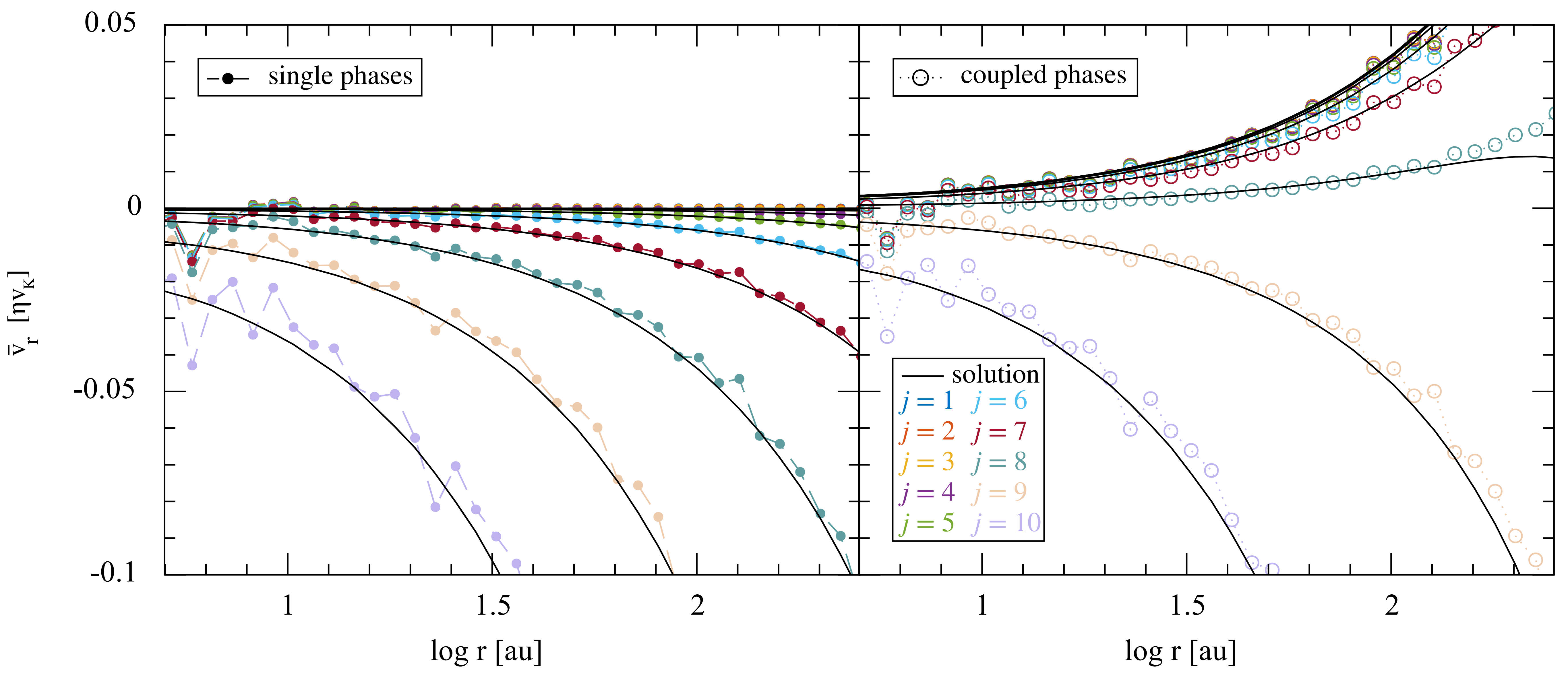
- Because the gas pressure decreases radially, the pressure force supports the gas disc and it rotates sub-Keplerian.
- Keplerian dust feels a headwind and exchanges momentum with the gas (i.e. dust slows down and gas speeds up).

DUST: RADIAL DRIFT

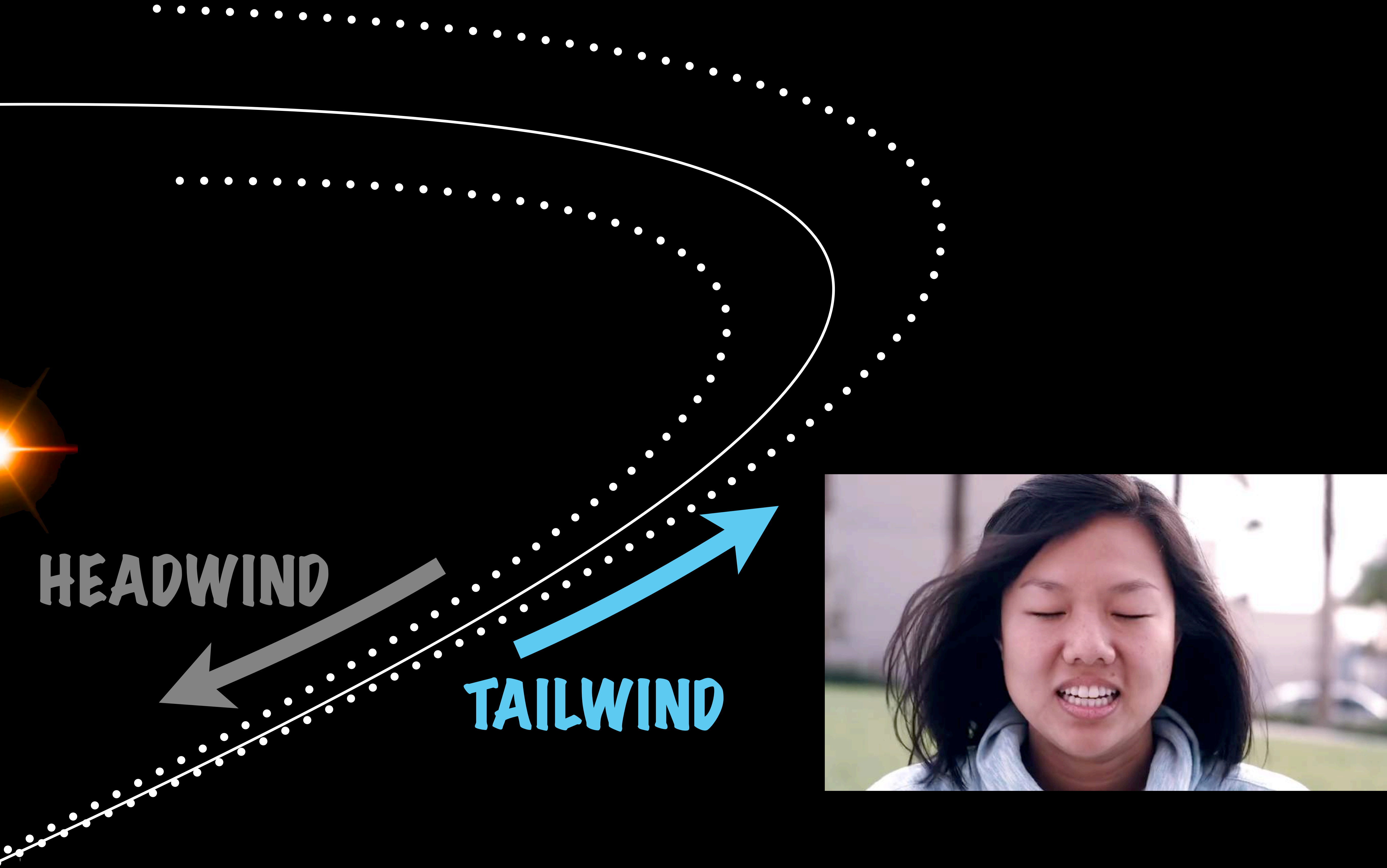
- For small dust-to-gas ratios, we can define some limits:
  - For $\mathrm{St} \ll 1$: $u_\mathrm{d}^R = -2\eta v_\mathrm{K}\mathrm{St}$
  - For $\mathrm{St} \gg 1$: $u_\mathrm{d}^R = -\dfrac{2\eta v_\mathrm{K}}{\mathrm{St}}$
  - Maximum velocity: $u_\mathrm{d}^R = -\eta v_\mathrm{K}$ $\left(\lesssim 60\ \mathrm{m/s}\right)$
    (when $\mathrm{St} = \mathrm{St}^{-1} = 1$)
- Velocity decreases as we move away from $\mathrm{St} = 1$!
- Large grains can “pile-up“ in and/or accumulate in pressure maximum.

Radial Migration

SINGLE-PHASE MULTI-PHASE

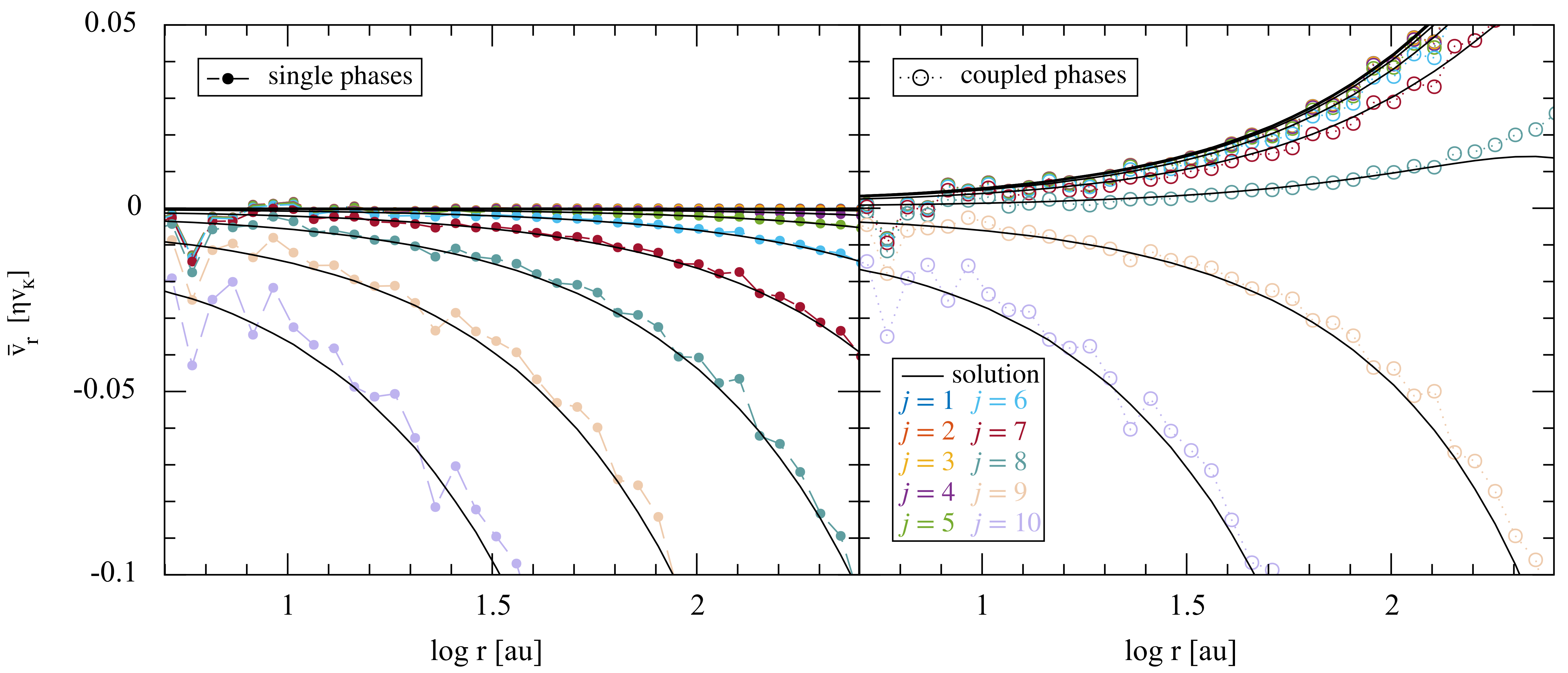


SINGLE-PHASE

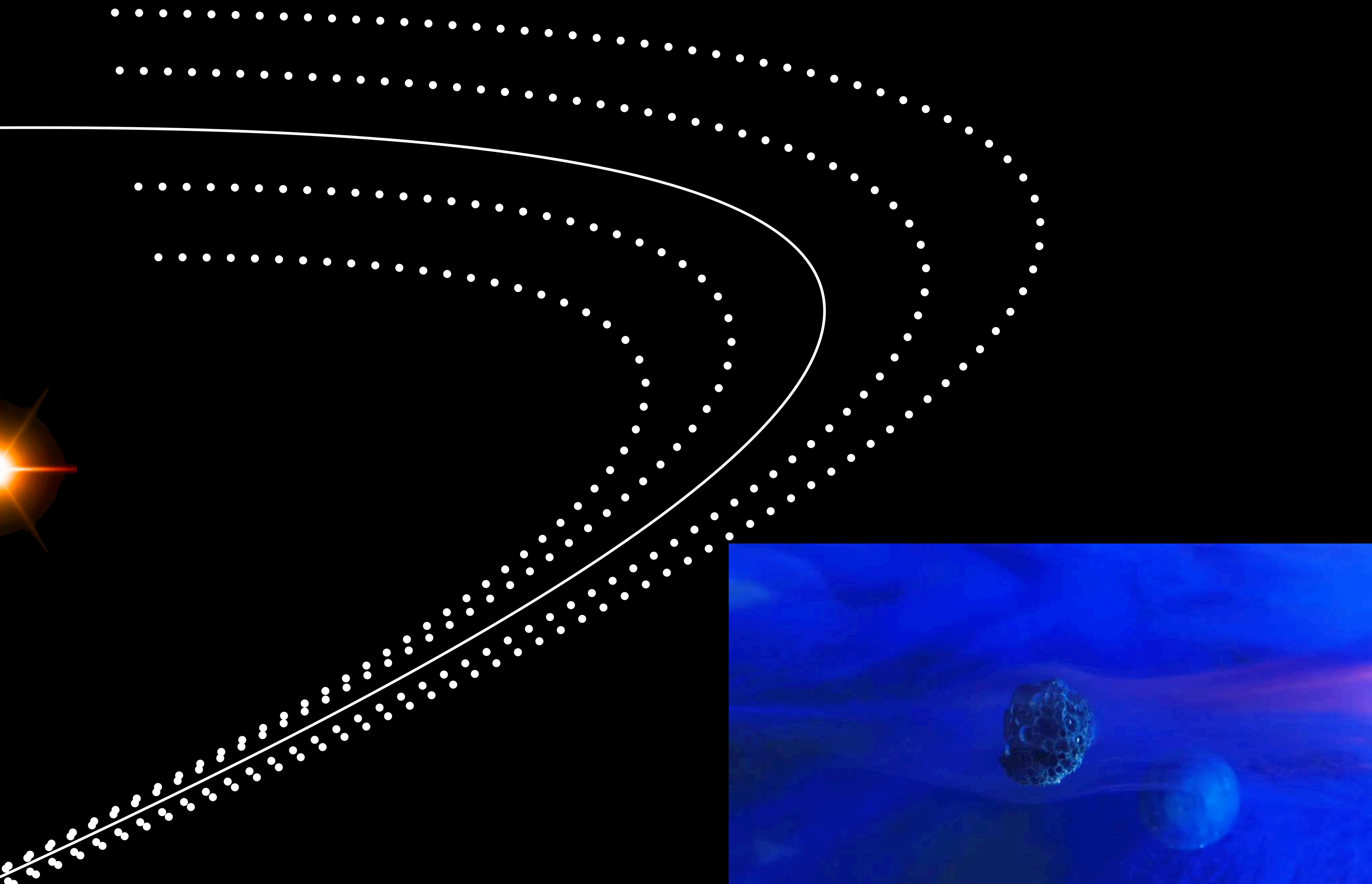


Radial Migration

SINGLE-PHASE MULTI-PHASE

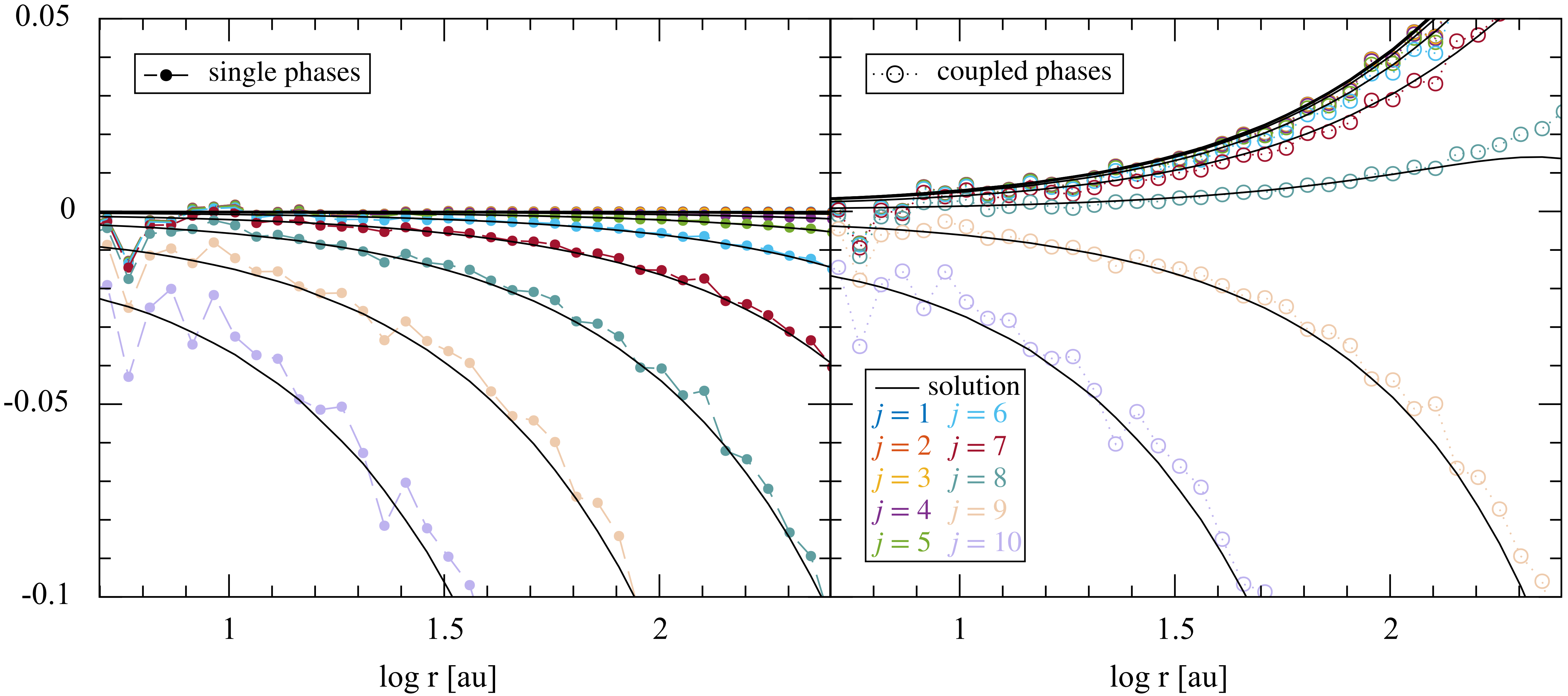


MULTI-PHASE

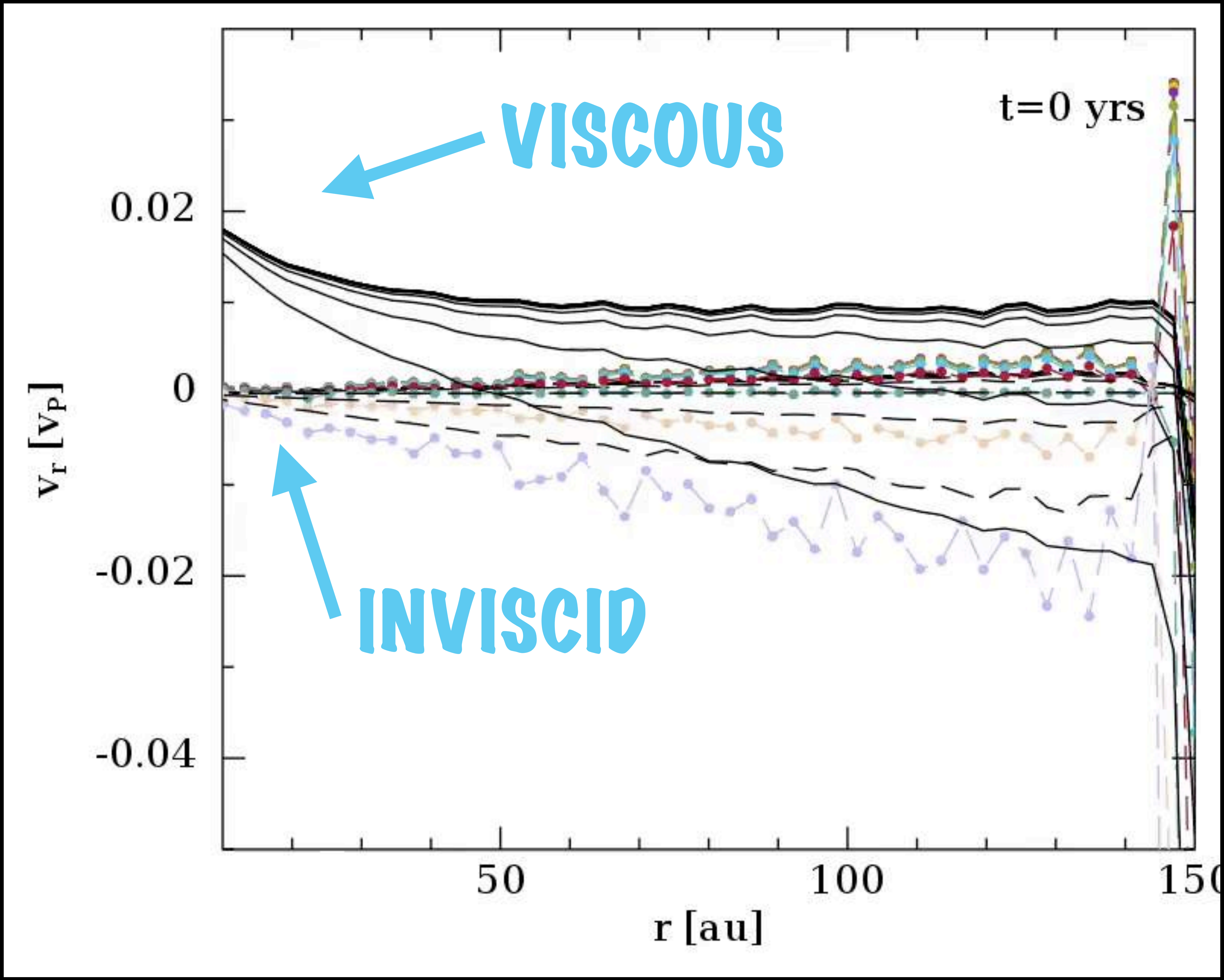


Radial Migration

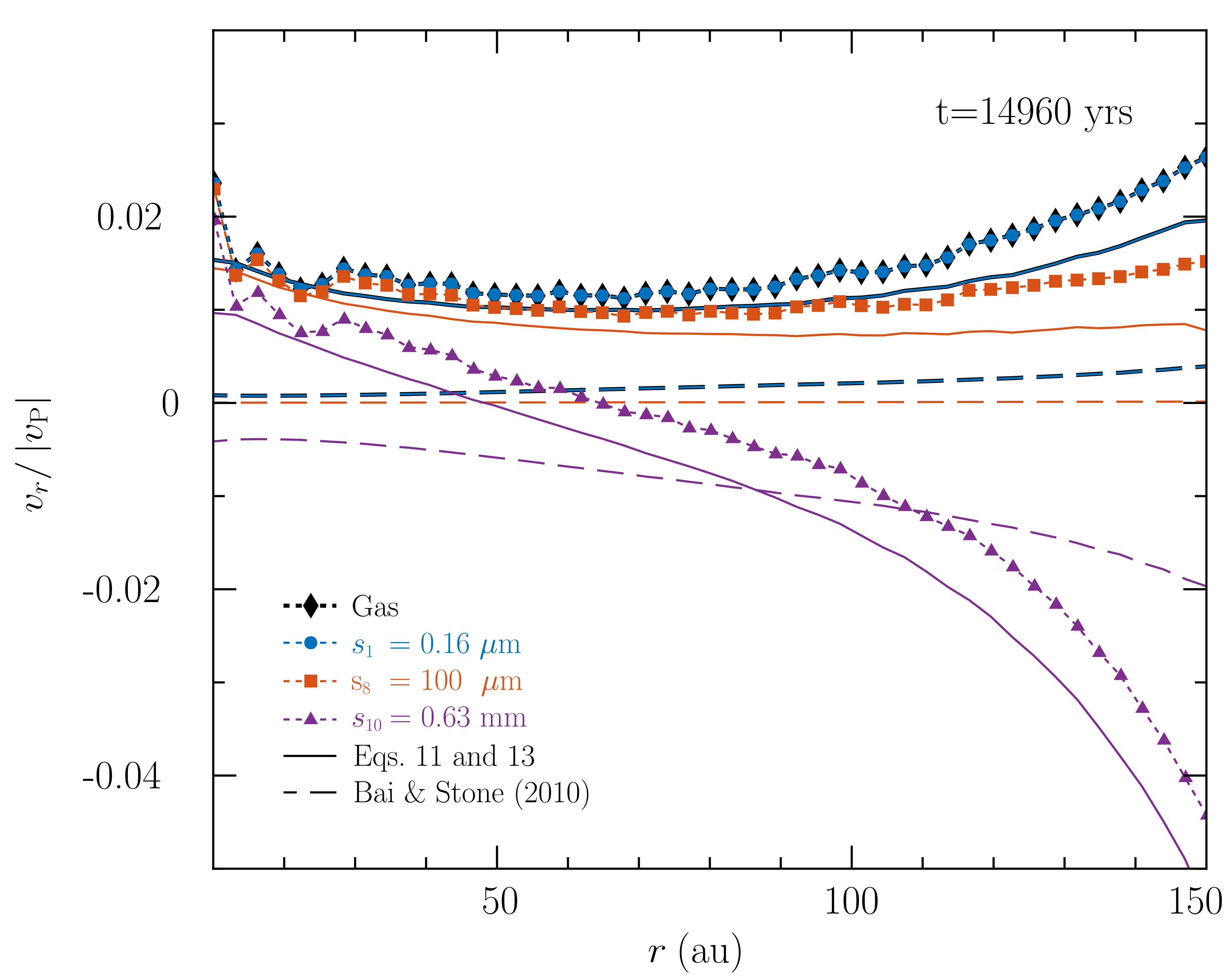
SINGLE-PHASE MULTI-PHASE



Radial Migration



Radial Migration



DUST: VERTICAL SETTLING

- Now let's consider the vertical component on its own. To simplify things, we'll ignore the back-reaction of the dust onto the gas:

$$\frac{\partial u_\mathrm{d}^z}{\partial t} = -A\rho_\mathrm{g}(u_\mathrm{d}^z - \underbrace{u_\mathrm{g}^z}_{0}) + z\Omega_\mathrm{K}^2$$

- Which is the equation for adamped harmonic oscillator. The steady state terminal velocity has a simple relation:

$$u_\mathrm{d}^z = -z\,\Omega_\mathrm{K}\mathrm{St} = -z\,t_\mathrm{stop}$$

- If we equating the vertical

DUST: VERTICAL SETTLING

- Now let's consider the vertical component on its own. To simplify things, we'll ignore the back-reaction of the dust onto the gas:

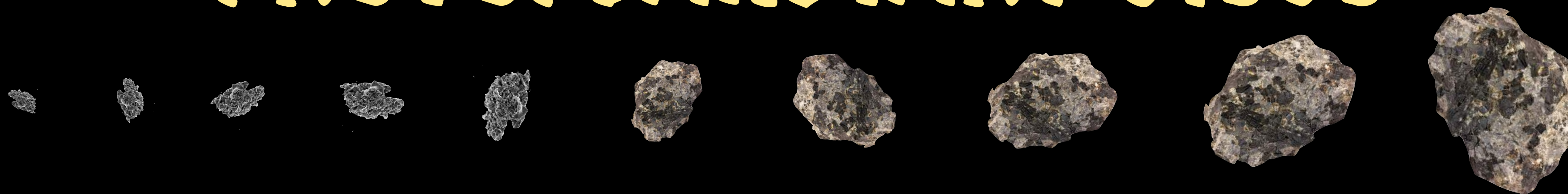
$$\frac{\partial u_{\mathrm{d}}^z}{\partial t} = -A\rho_{\mathrm{g}}(u_{\mathrm{d}}^z - \cancelto{0}{u_{\mathrm{g}}^z}) + z\Omega_{\mathrm{K}}^2$$

- Which is the equation for adamped harmonic oscillator. The steady state terminal velocity has a simple relation:

$$u_{\mathrm{d}}^z = -z\,\Omega_{\mathrm{K}}\mathrm{St} = -z\,t_{\mathrm{stop}}$$

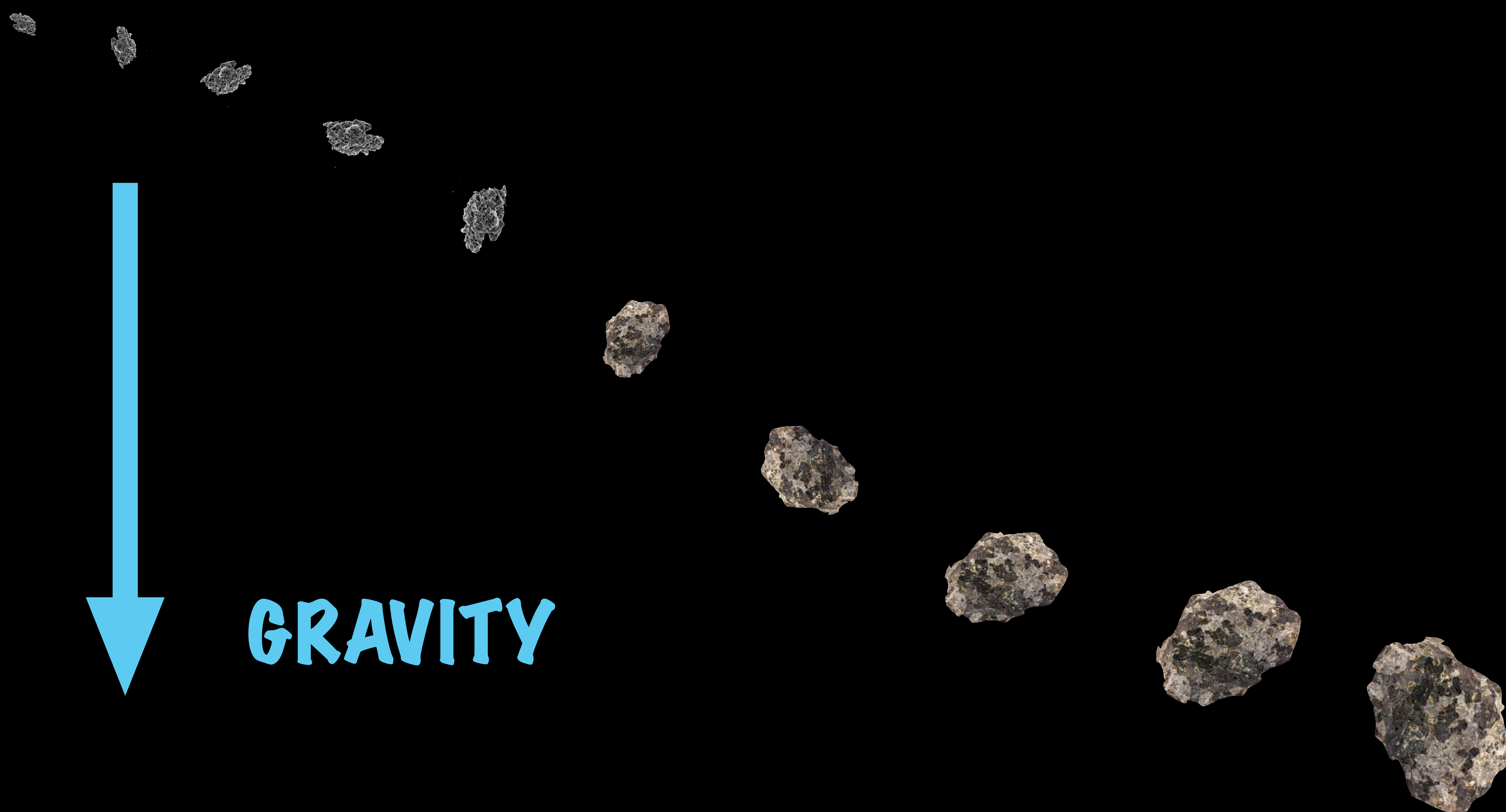
- Importantly, t_{stop} depends on ρ_{g} which increases towards the disc mid-plane. Small grains slowly settle to the mid-plane. Large grains (if lofted up), will oscillate about the disc mid-plane.

DUST SETTLING IN PROTOPLANETARY DISCS

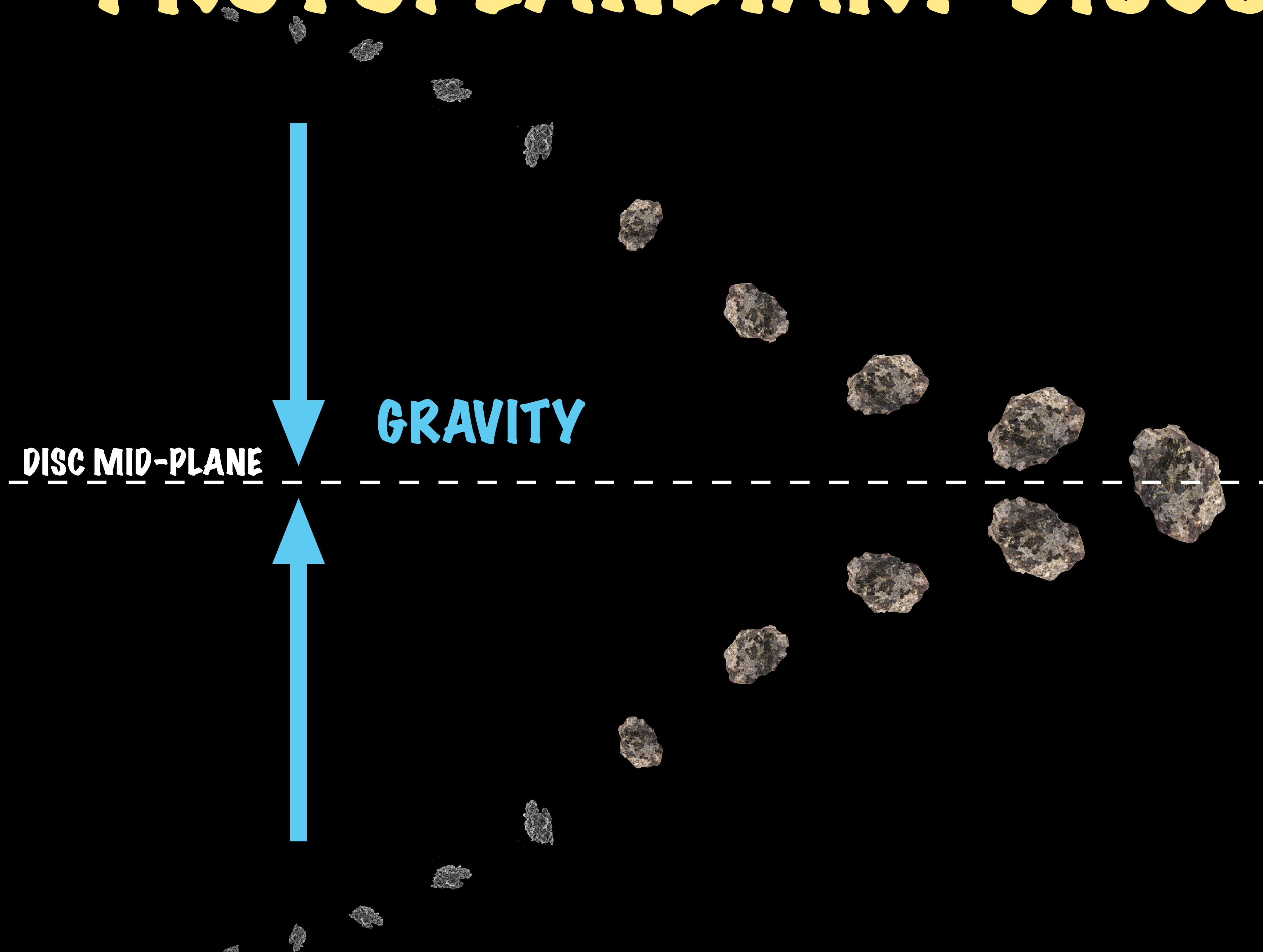


GRAVITY

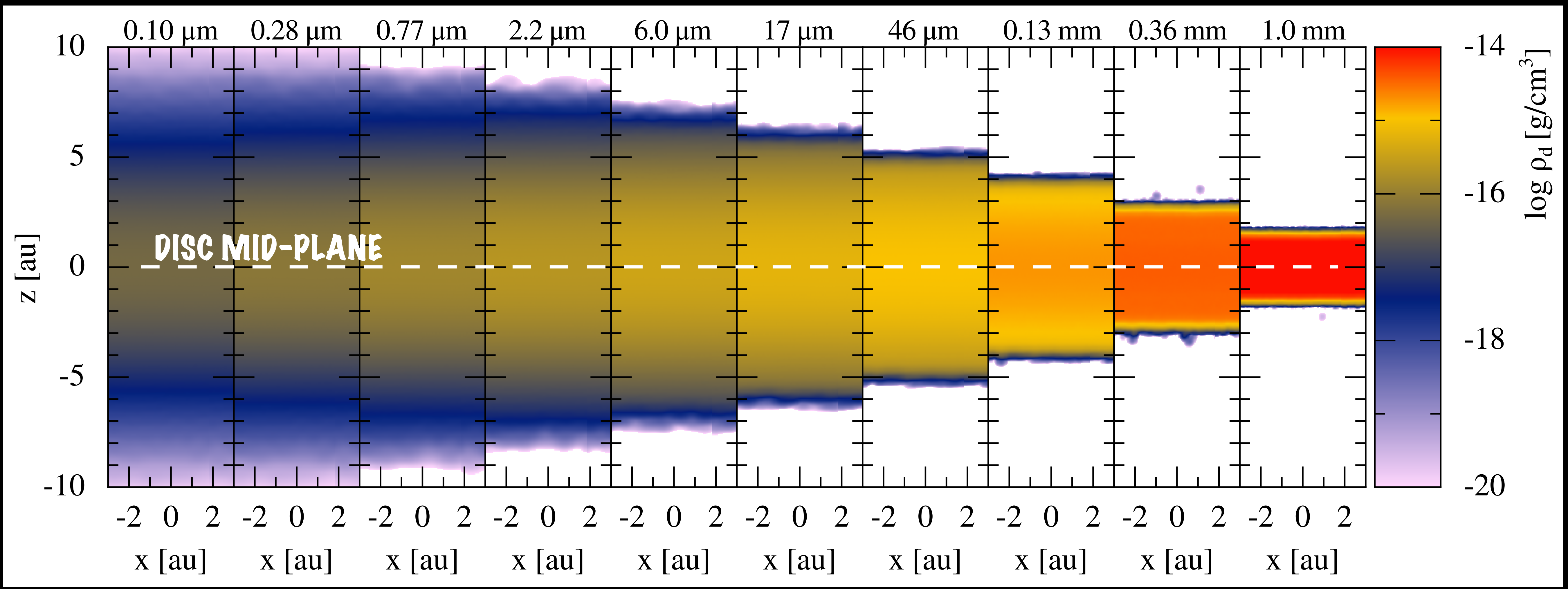
DUST SETTLING IN PROTOPLANETARY DISCS

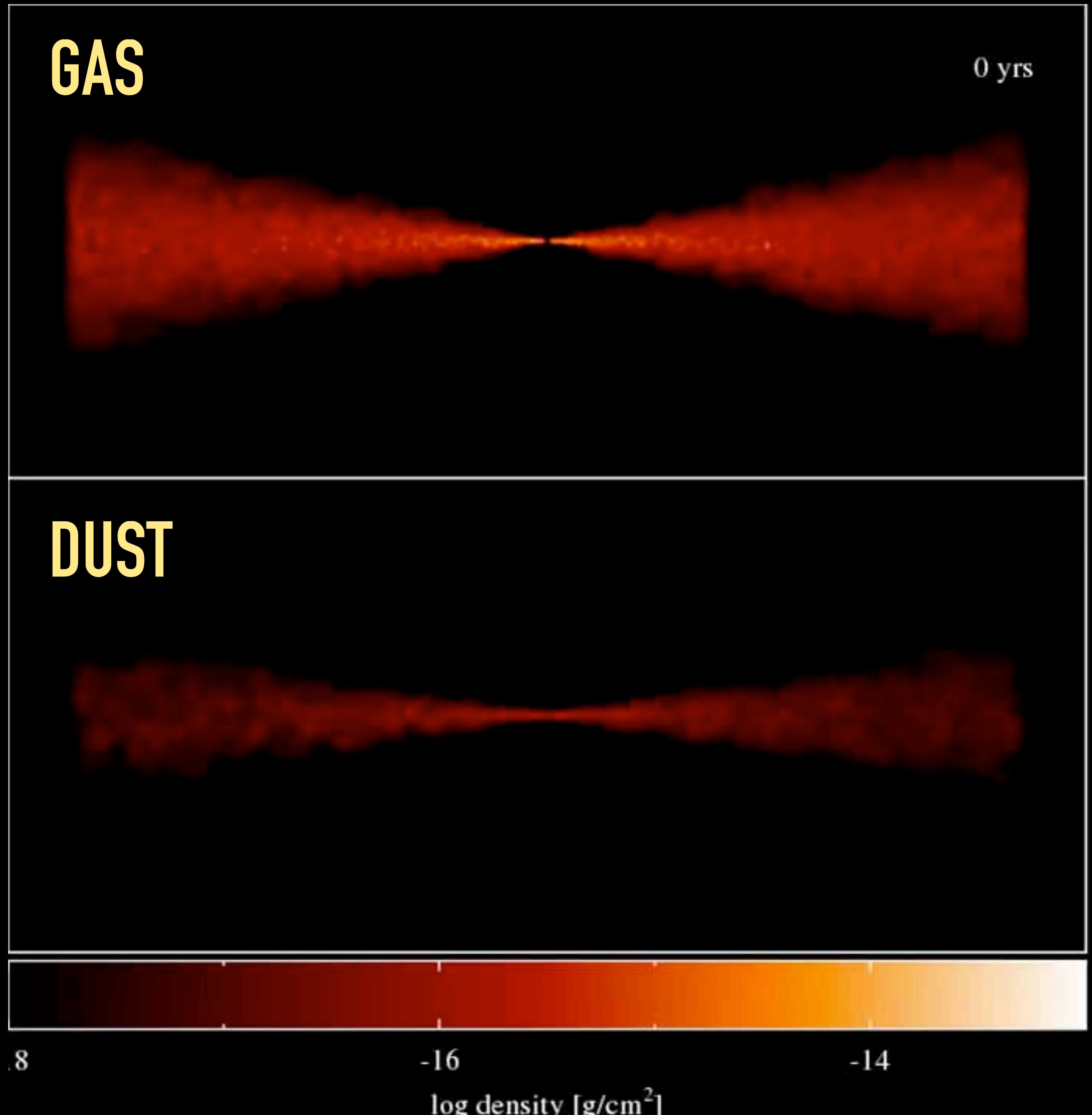


DUST SETTLING IN PROTOPLANETARY DISCS



DUST SETTLING IN PROTOPLANETARY DISCS




GAS
0 yrs
DUST
.8
-16
-14
log density [g/cm²]

DUST: VERTICAL SETTLING

- In a turbulent disc, turbulent eddies will kick-up dust vertically. Eventually, dust will reach a steady state defined by the following diffusion equation:

$$\frac{\partial \rho_{\mathrm{d}}}{\partial t} + \frac{\partial}{\partial z}\left[\rho_{\mathrm{d}} v_{\mathrm{d}} - \rho_{\mathrm{g}} D_{\mathrm{d}} \frac{\partial}{\partial z}\left(\frac{\rho_{\mathrm{d}}}{\rho_{\mathrm{g}}}\right)\right] = 0$$

- Where the diffusion coefficient is defined as: $D_{\mathrm{d}} \approx \frac{\alpha c_{\mathrm{s}} H}{\mathrm{Sc}}$ ($\mathrm{Sc} \sim 1 + \mathrm{St}$ is the Schmidt Number)

MAIN POINTS

- Discs are thin, but flared due to incident stellar radiation.
    - Inner disc and disc surfaces are hot (usually ionised). Mid-plane is cold and molecules condense out of the gas onto dust grains.
- Evolutionary stages can be distinguished by their SED.
- Discs redistribute angular moment through viscous dissipation, thereby allowing them to accrete.
    - Source of turbulence is still not clear, but likely is related to magnetic fields.
- Gas is pressure supported and rotates at sub-Keplerian velocities.
    - Dust experiences a headwind and drifts radially inwards (important for planet formation)
- Dust settles vertically, increasing the concentration of dust at the mid-plane where planets form.