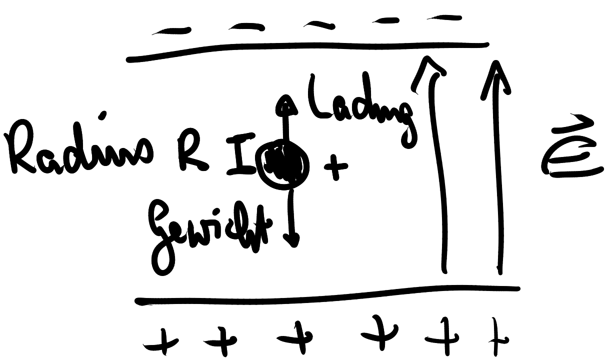
Quantelung der Ladung

$e = 1.602 \cdot 10^{-19}\,C$ Elementarladung

1910: Millikanversuch

Öltropfen (geladen) im Kondensator

a) Schweben:



$qE = mg$

b) Fallen:

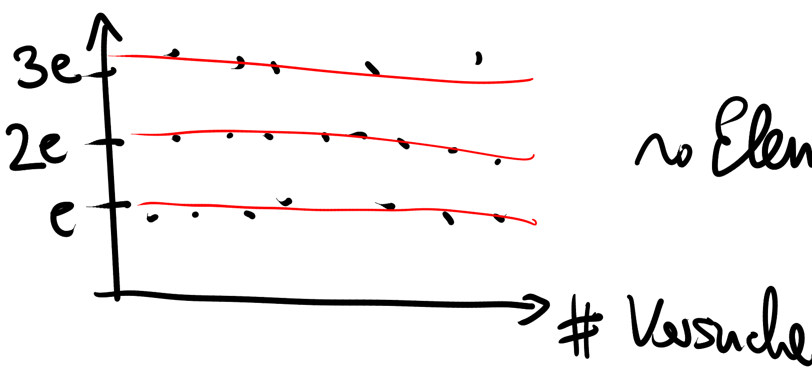
Vereinfacht ohne Luftauftrieb:

$$mg = 6\pi\eta R v$$

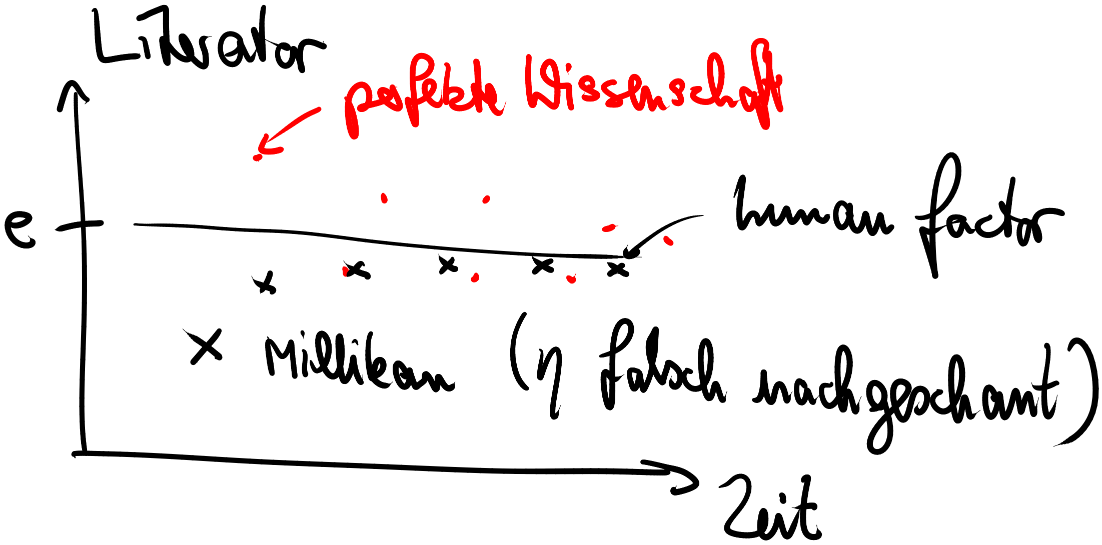
Geschw.

Viskosität der Luft

$$\leadsto R \leadsto m \underset{(a)}{\leadsto} q$$

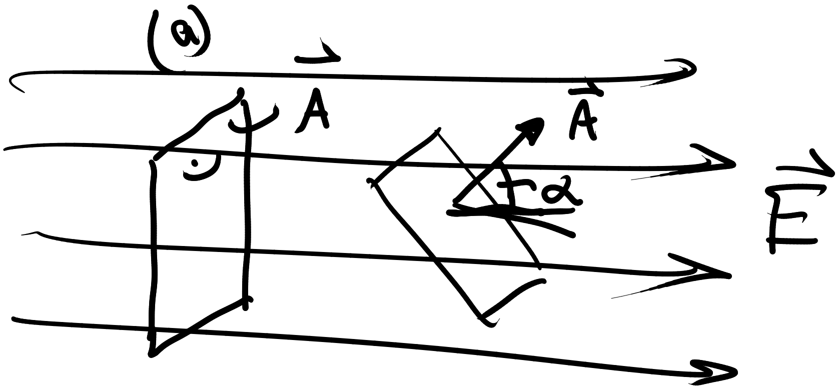


$\leadsto$ Elementarladung



Das Gauß'sche Gesetz

Fluß ϕ des elektrischen Feldes:



(a) $\phi = E \cdot A$

(b) $\phi = \vec{E} \cdot \vec{A}$

$= E \cdot A \cdot \cos\alpha$

Beispiel: Kugeloberfläche mit einer Ladung q im Mittelpunkt: $A \perp E$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \qquad A = 4\pi r^2$$

$$\phi = \iint_A \vec{E}\, d\vec{A} = \frac{1}{\cancel{4\pi}\varepsilon_0} \frac{q}{\cancel{r^2}} \cdot \cancel{4\pi r^2}$$

$$= q/\varepsilon_0$$

Dies gilt für jede beliebige, geschlossene Fläche:

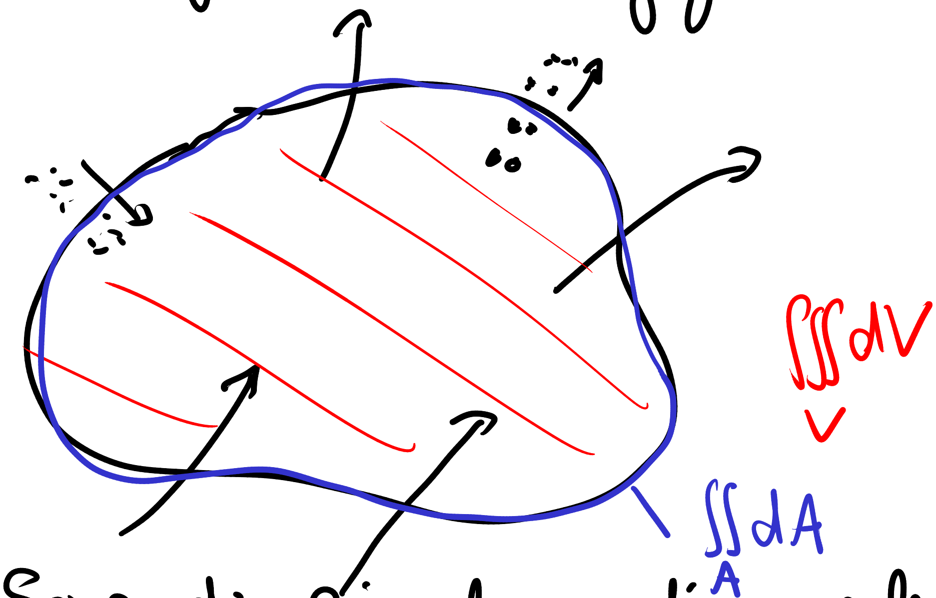
> Der Fluß des elektr. Feldes aus einer beliebigen, geschlossenen Fläche ist die Ladung im Inneren geteilt durch ε_0

Einheit: C/m^3

$$\phi = \iint_A \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0}\iiint_V \rho \, dV = \frac{q}{\varepsilon_0}$$

1. Maxwell'sche Gesetz (Gauß'sche Satz).

<u>Insbesondere:</u>
- $\phi = 0 \rightsquigarrow$ keine Ladung in geschloss. Fläche
- Gilt auch für Felder bewegter Ladungen
- Analogie mit der Saujagd



Spuren die Reingehen – die rausgehen = Zahl der Sauen in der Fläche. ($\hat{=}$ Ladung)

Integralsatz von Gauß

Für ein beliebiges Vektorfeld gilt:

$$\iint \vec{E}\, d\vec{A} \underset{\text{Mathe}}{=} \iiint_V \operatorname{div}\vec{E}\; dV \underset{\text{Physik}}{\overset{!}{=}} \iiint \frac{\rho}{\varepsilon_0}\, dV$$

Deshalb: $\boxed{\operatorname{div}\vec{E} = \rho/\varepsilon_0}$ 1. Maxwell-Gesetz in differentieller Form.

Wdh.: $\operatorname{div}\vec{E} = \vec{\nabla}\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$

Poisson-Gleichung: mit $\vec{E} = -\operatorname{grad}\varphi$

$$\rightsquigarrow -\underbrace{\operatorname{div}\operatorname{grad}}_{\Delta}\varphi = \frac{\rho}{\varepsilon_0} \rightsquigarrow \boxed{\Delta\varphi = -\rho/\varepsilon_0}$$

$$\Delta\rho = \frac{\partial^2\rho}{\partial x^2} + \frac{\partial^2\rho}{\partial y^2} + \frac{\partial^2\rho}{\partial z^2}$$

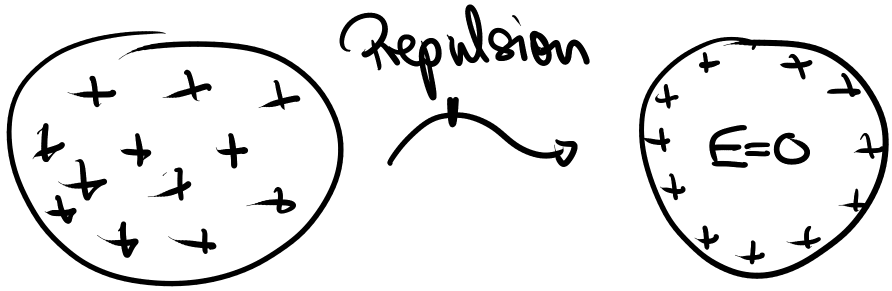
Laplace Operator.

Bemerkung: Dies alles ist die schlichte Folgerung von $F \sim 1/r^2$ (Analog: Gravitation)

Conductor in electrical field

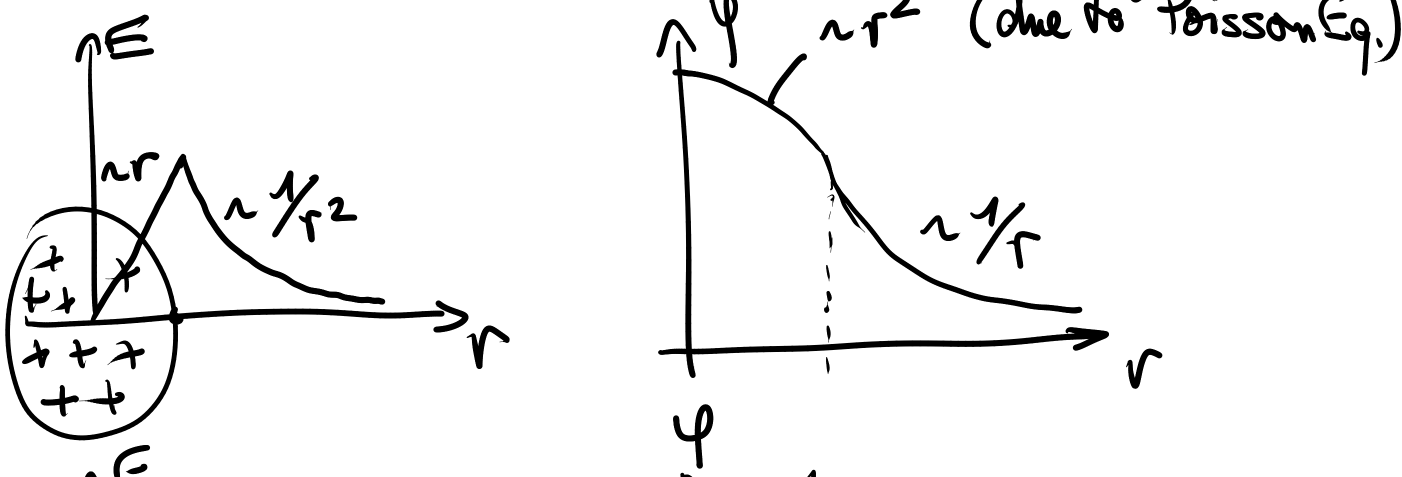
Charges are free to move in a conductor.
They move in the electrical field until no forces are exerted on them.

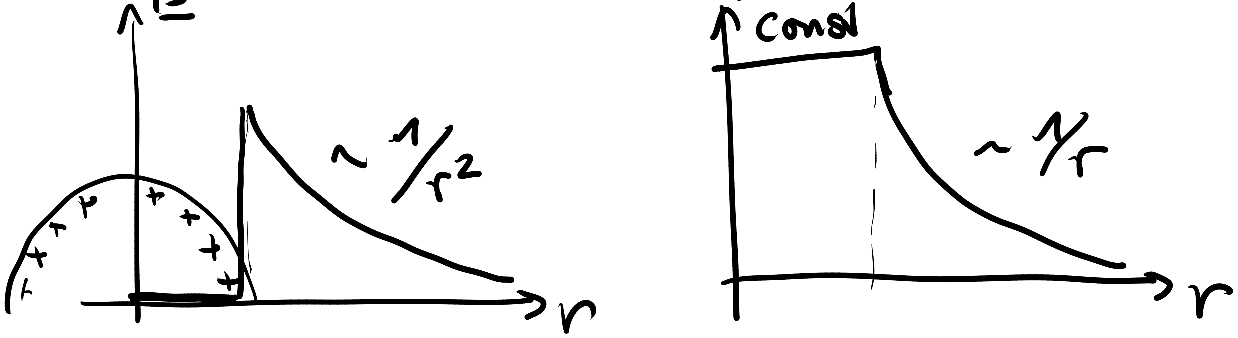
Example: Charges in a conductor move to its surface.



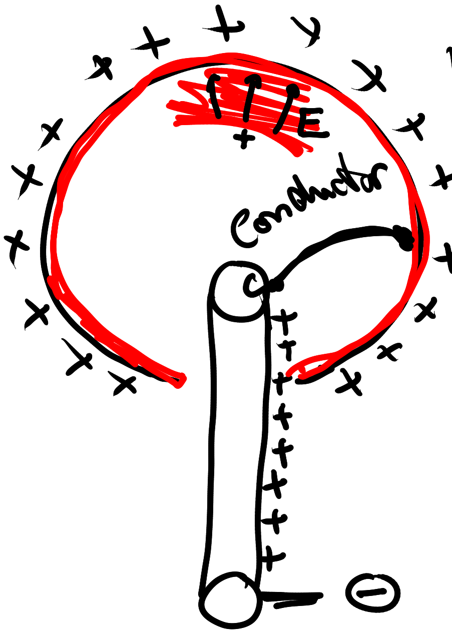
Inside a conductor, the field E and the charge density ρ are zero.

Before movement of the charges





Application

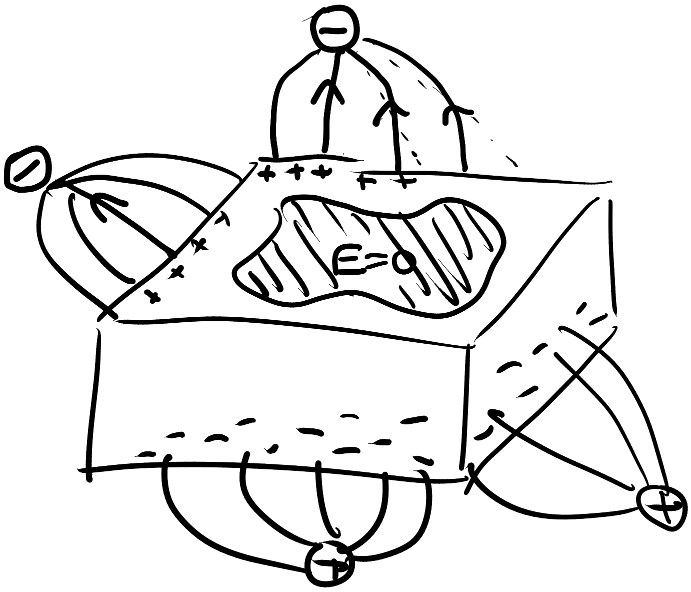


Van der Graaff Generator

Charging a sphere from the inside
⇝ High Voltages (10 MV)

2. Faraday cage

Even in external fields, $E=0$ inside a hollow conductor.



Reason: 1. Poisson equation $\Delta\varphi = -\rho/\varepsilon_0$

inside of cage: $\rho = 0 \rightsquigarrow \Delta\varphi = 0$

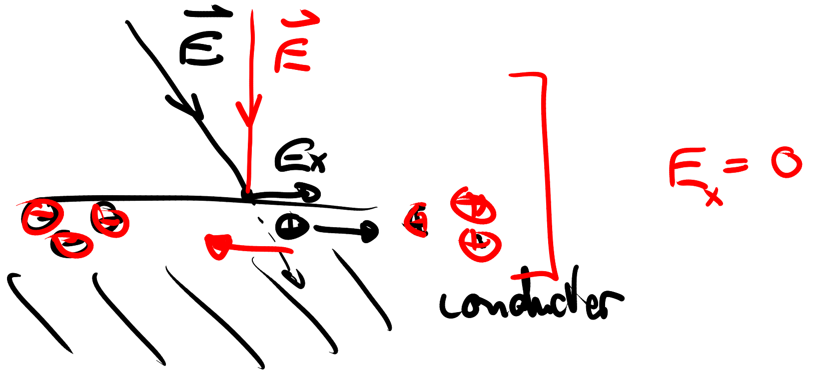
(still: E = const.)

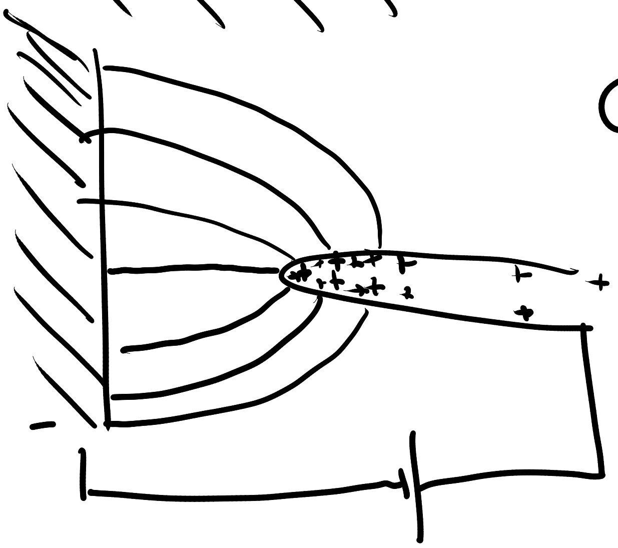
2. Boundary condition of the cage due to conductor: constant electrical potential φ = const at surface

with Continuity condition of Poisson equation.
~ Boundary condition $\varphi = const$ imposes also inside edge $\varphi = const$.

~ $E = 0$ inside

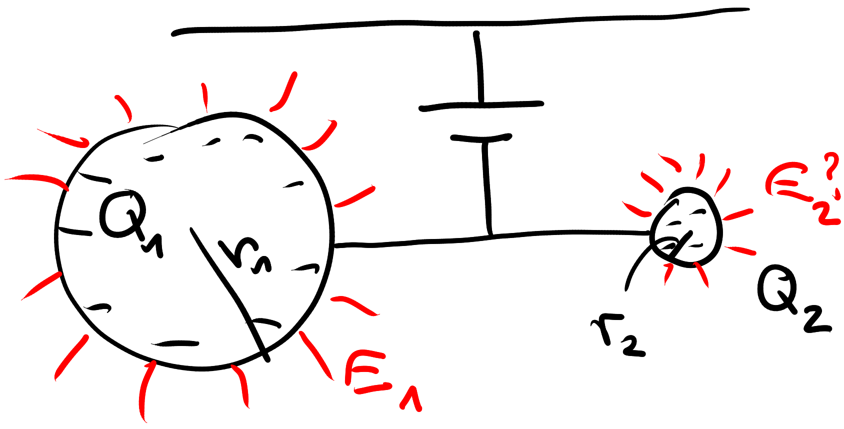
3. High electrical fields at sharp tips of a conductor





Condition of $\vec{E} \perp$ surface
~ movement of charges to the tip.

Simplest model:



Charged spheres, connected by a conductor, i.e. they have the same potential

$$\rightsquigarrow \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_1} = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_2}$$

$$\rightsquigarrow \frac{Q_1}{Q_2} \overset{(*)}{=} \frac{r_1}{r_2}$$

But: for the electrical field with Gauß law:

$$E \cdot 4\pi r^2 = Q/\varepsilon_0$$

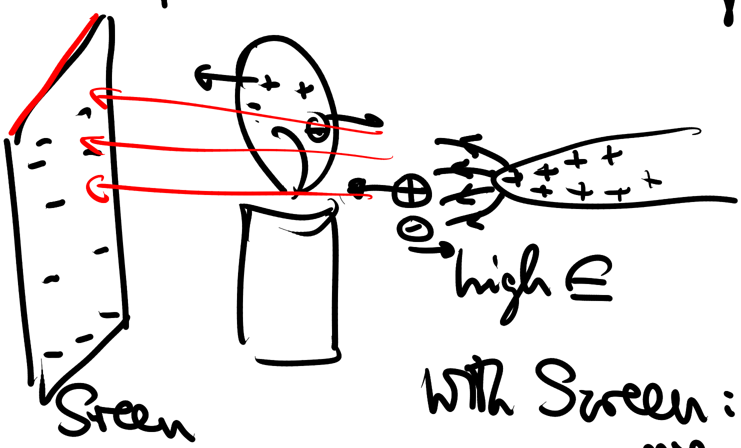
$$\sigma = \frac{Q}{A} = \frac{Q}{4\pi r^2} = \varepsilon_0 E$$

$$\rightsquigarrow \frac{E_1}{E_2} = \frac{Q_1 r_2^2}{Q_2 r_1^2} \overset{(*)}{=} \frac{r_2}{r_1}$$

$\rightsquigarrow$ Small radius means large field.

$\boxed{\text{Gries image}}$

- Blow out of a candle with sharp tip:

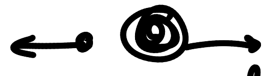


With Screen: directional movement $\rightarrow$ blow out.

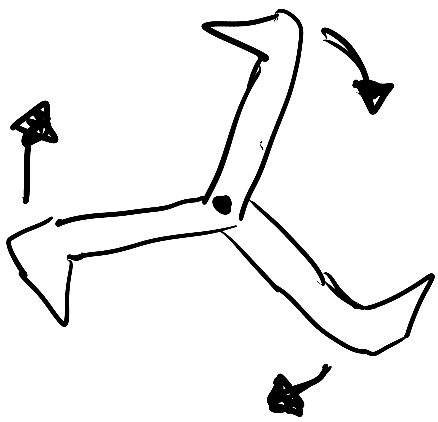
Electric Discharge

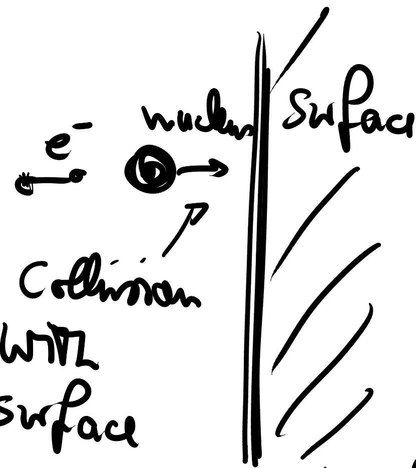
Discharge in air depends on the electrical Field, i.e. the forces necessary to create a Plasma

$$E_{max}^{air} \approx 3\cdot 10^{6}\,\frac{V}{m}$$



Electrical Windmill





~ momentum transfer